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TESTS ON THE PERMEABILITY OF CONCRETE

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TESTS ON THE PERMEABILITY OF CONCRETE

I. INTRODUCTION

This bulletin is a report of a series of permeability tests made in the laboratory for testing materials at the University of Wisconsin during the summer and fall of 1908. The object of the tests was to determine the efficiency of some of the commercial compounds used for the water proofing of concrete.

Tests were made on fourteen compounds, each compound being subjected ordinarily to pressures of approximately 20 lbs/in^2 and 40 lbs/in^2. The duration of the test was usually three days, and a record was kept of the amount of water entering the concrete.

In order to ascertain the effect on the strength of concrete of the compounds which are added to the body of the concrete, compression specimens were made. Three of the compounds were tested in this manner, the specimens being broken at the ages of approximately 1, 2 and 10 months.

This work is a partial repetition and a continuation of a thesis completed by S. R. Hatch and H. E. Ketchum of the class of 1907. Due acknowledgment is made to them for their valuable assistance in designing the apparatus used for these tests. M. O. Withey and A. H. Miller, of the department of mechanics, also rendered effective service in performing the tests. J. Glaettli and A. E. Meinecke of the classes of 1909 and 1910, respectively, made the drawings and tables for this bulletin.

II. CONCLUSIONS

The following conclusions are drawn as the result of the experiments:

1. Unless extreme care is taken in proportioning, it is necessary that some form of water proofing be used for a 1:3:5 concrete for pressures from 20 lbs/in^2 to 40 lbs/in^2.

2. For nearly all specimens the rate of flow decreased rapidly with time. This was especially marked in the case of the mortar coatings and was due in part to their dry condition.

3. All of the surface paints were quite satisfactory at pressures of 20 lbs/in^2. When subjected to pressures of 40 lbs/in^2, Des Moines Elaterite No. 60 and Universal Damp-Proof Compound proved unreliable. The mixture of Flexible Compound and white lead gave good results at both pressures. Century Cement Fluid was not tested at the high pressure and while Antihydrine gave only a small flow at 40 lbs/in^2, the results can not be compared with those of the other compounds because the test was carried on under different conditions.

4. Water proofing materials composed of layers of felt, burlap, or tarred paper, cemented together with a compound, gave excellent results at the high pressure. Not one of the specimens treated with Hydrex Felt and Compound, Siastex Fabric No. 2 and Pitch, and Dehydratine No. 4 and tarred roofing paper showed any appreciable flow.

5. Coatine was practically impermeable at the two pressures.

6. The mortar coatings in the case of Antihydro gave good results at both pressures, there being practically no flow after the first 24 hours. It is possible, however, that its effectiveness was due partly to the method of applying the mortar. For waterproofing used in this manner, it would seem desirable to apply the mortar in two layers after first covering the concrete with a neat cement slush coat. The plain mortar with a neat cement wash was effective at a pressure of 20 lbs/in^2 but proved unreliable at 40 lbs/in^2. No doubt a plain mortar coating would prove satisfactory in water proofing concrete at high pressures providing it be applied as just described.

7. Waterproofing compounds that are added to the body of the concrete are worthless at pressures from 20 lbs/in^2 to 40 lbs/in^2 unless great care be used in proportioning.

8. Limate, when replacing 10 per cent to 22 per cent of the weight of cement, and Medusa Water-proof Compound equal to 2 per cent of the weight of the cement do not impair the compressive strength of 1:3:5 concrete. On the other hand, Aquabar equal to 4 1/6 per cent of the mixing water reduces the strength of 1:3:5 concrete by percentages varying from 36 per cent to 54 per cent.

III. DESCRIPTION OF APPARATUS AND MATERIAL

Apparatus:—The apparatus which was designed for these tests consists essentially of eight six-inch pipes filled with concrete and a pipe system connected with air and water reservoirs. Fig. 2 shows in detail the mold and attached casting and Fig. 1 is a general drawing of the pipe system for four specimens, the apparatus for the remaining four specimens being the same as shown.

The molds shown in elevation in Fig. 1 and in section in Fig. 2 were six-inch wrought iron pipe, 12½ inches long, with a cast-iron flange screwed to the upper end. In order to prevent the passage of water between the pipe and the cement lining ten or twelve V-shaped grooves were cut in each pipe, each groove extending around the inner surface of the pipe.

This flanged pipe was attached to the casting by means of six eye-bolts. A ¾-inch pipe, 4 feet 6 inches long, was screwed into this casting. Each of these ¾-inch pipes was joined to the main pipe, which in turn connected with the water main and with the air reservoirs. The shut-off globe valves for water and air are shown on the pipes connecting the main pipe with the water main and with the air reservoirs. Two cast-iron cylinders, 6½ inches in diameter and 4 feet 8 inches long, formed the air-reservoirs. They were connected with a large air-tank, not shown, by means of the pipe shown in Fig. 1, a shut-off globe valve being placed between the air-tank and air-reservoirs.

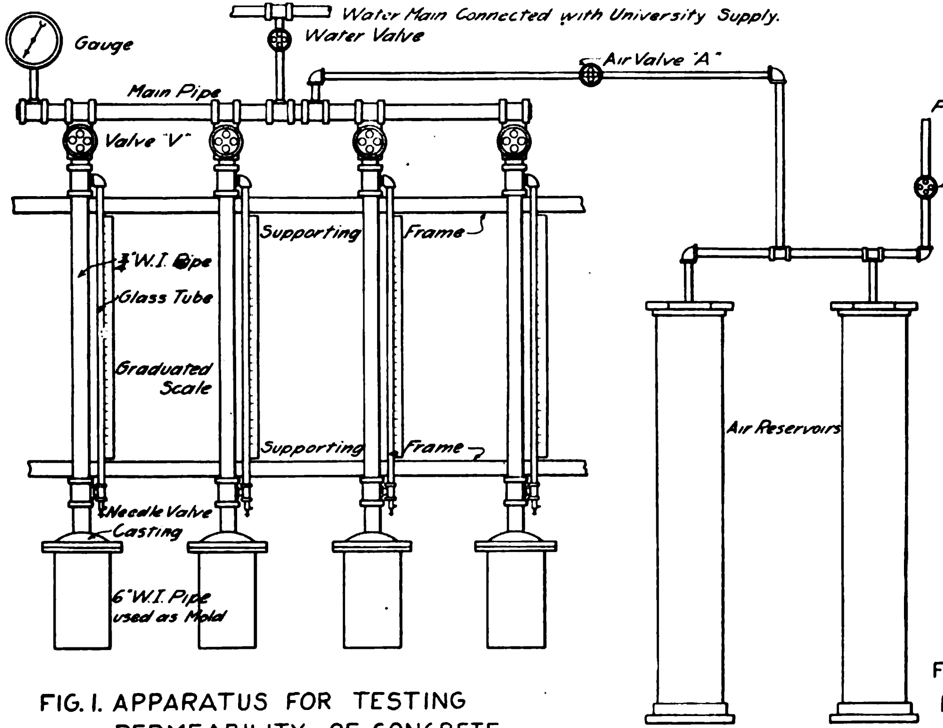

FIG. I. APPARATUS FOR TESTING PERMEABILITY OF CONCRETE

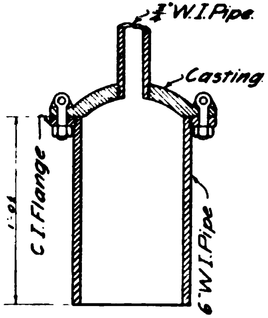

FIG. 2. SECTION OF MOLD CASTING.

A glass tube and attached scale graduated to hundredths of feet, were fastened to each ¾-inch pipe in order to obtain the water level in the pipe. The globe valve "V" was used to disconnect any specimen proving defective. The ¾-inch pipe and glass tube were drained by means of the needle valve. A gage registered the air pressure.

Concrete:—The proportions by volume of the concrete were 1:3:5, the required amount of the materials being weighed on a scale. No attempt was made to secure a waterproof concrete by proper proportioning. On the contrary, a lean mixture was desirable in order to bring out the waterproofing qualities of the compounds.

Stone and Sand:—Local stone and sand and Atlas cement were used. The stone was a rather sandy lime-stone while the sand was of the fine bank variety. The mechanical analyses of the stone and sand are shown in Tables 1 and 2, respectively.

Water-proofing Compounds:—The compounds tested, which were all secured through local dealers in order to obtain the ordinary commercial product, were Century Cement Fluid, Dehydratine No. 4, Des Moines Elaterite No. 60, Hydrex Waterproof Felt and Compound, Limate, Medusa Waterproof Compound, Siastex Waterproofing Fabric No. 2 and Pitch, Universal Damp-Proof Compound, Aquabar, Antihydro, Antihydrine, Wunner's Bitumen-Emulsion; Coatine, and Flexible Compound. In nearly all cases the manufacturers, at our request, gave special instructions in regard to methods of waterproofing against the pressures used, such instructions being usually followed in the tests. As the name indicates Universal Damp-Proof Compound is used for damp proofing and not for waterproofing.

IV. METHOD OF MAKING THE SPECIMENS AND OF PERFORMING THE TESTS

Making the Specimens:—Materials sufficient for two specimens were first weighed out. The sand was spread out evenly on the mixing floor and upon this was placed the cement in a thin layer. Two men with shovels mixed the sand and cement thoroughly to an even color when a crater was made in the center which was filled with water. After the water was absorbed the mass was turned several times until the mortar was in the form of a thick paste. The stone which had previously been dampened was now spread over the mortar and the whole turned by two men with shovels for five minutes, water being added at times with a sprinkler. The consistency of the concrete was such that it flowed readily from a pile on the floor. A quantity of neat cement was now mixed with sufficient water to form a thick paste.

The six-inch pipe was placed on the floor with flanged-end up and a loose fitting wood disk ½ inch in thickness was dropped into it. A wrought iron pipe 4½ inches in diameter and 5 inches long, which had been turned on the outside to a taper, was centered on the wood disk. A lining of neat cement paste varying from 6/10 to 7/10 inches in thickness was now tamped into the annular space between the pipes, care being taken to fill the grooves in the outside pipe. This lining was carried up to within one inch of the top of the inner pipe, which was then filled with concrete to the level of the cement lining, the concrete being tamped well with a trowel meanwhile. The inner pipe was now carefully drawn upward about three inches, and the cement lining and concrete were deposited as before. This operation was continued until the specimen was completed, the depth of concrete being about 12 inches. The manner of finishing the upper surface varied for the different types of specimens and will be described later.

The cement lining and grooves were very effective in preventing water from seeping along the surface of the pipe. This was shown by chipping out the concrete and the lining of several specimens that had been subjected to pressure. While damp spots were found here and there in the pipe, in no case were they continuous. Neither was there any unusual flow between the concrete and the cement lining.

A set of 8 specimens was usually made for each kind of waterproofing, 4 of the specimens being tested at a pressure of 40 lbs/in^2 and 4 at a pressure of 20 lbs/in^2. All specimens were first tested at pressure of 40 lbs/in^2, and, if satisfactory, no other specimens were made. The duration of the tests was about 3 days, although the time was extended to 7 or 10 days in a few cases. In order to prevent shrinkage cracks, the specimens were covered with damp cloths for a few days after being made. The specimens were stored in air and were 35 days old when tested. The pressure was at all times applied to the upper or waterproofed surface of the concrete.

Performing the Tests:—The specimens were securely bolted to the castings, an Eclipse rubber gasket being used between the finished faces of flange and casting. With the air valve "A" closed and the air valve "B" opened, air was admitted to the reservoirs until sufficient pressure was obtained. The water-valve was opened and water was allowed to fill the ¾-inch tubes. Care was taken that the pressure did not exceed that used in the test, this being regulated by opening the needle-valves. Air-valve "B" was closed and air-valve "A" connecting with the air reservoirs was opened, thus subjecting the specimens to pressure. Usually it was necessary to drain the ¾-inch pipes through the needle valves until the water level was visible in the glass tubes.

The rate of flow of the water through the concrete was obtained by noting the scale readings and the time. Pressures were also noted, but they showed very little decrease as the volume of the air-reservoirs was very large compared to the volume of the ¾-inch pipes. No readings were taken for five minutes after the pressure was on. As the rate of flow rapidly decreased readings were taken at intervals of 10 or 15 minutes for the first few hours and then at intervals gradually increasing from 2 to 8 hours. The bottoms of the specimens were frequently examined and any dampness noted.

The threaded joint between the 6-inch pipe and the flange was the only unsatisfactory part of the apparatus. In some specimens considerable leakage was noticed at this joint, the cause no doubt being due to the springing of the joint when bolting the flange to the casting. When readings were taken these joints were always carefully examined and any leaks noted. The effect of this leakage has been eliminated as much as possible in reducing the data. Occasionally it has been necessary to throw out the entire set of readings on a specimen. This defect may doubtless be remedied by using a heavier flange.

V. RESULTS OF EXPERIMENTS

The compounds that were tested may be classified as follows according to the manner in which they were used:

(1) Compounds that were applied to the surface of the concrete. These may be sub-divided into three classes: (a) Compounds which were applied as surface paints, as Century Cement Fluid, Des Moines Elaterite No. 60, Universal Damp-Proof Compound, Antihydrine, and Flexible Compound. (b) Compounds which were applied in layers of felt, burlap, or tarred paper with a cementing material, as Hydrex Waterproof Felt and Compound, Siastex Waterproofing Fabric No. 2 and Pitch, and Dehydratine No. 4 and common tarred roofing paper. (c) Coatine which is a surface coating used in the form of a thick layer.

(2) Foreign ingredients that were added to the body of the concrete as Limate, Medusa Water-Proof Compound, and Aquabar.

(3) Foreign ingredients that were added to a mortar coating as Medusa Water-Proof Compound, Aquabar, Antihydro and Wunner's Bitumen–Emulsion.

A set of specimens was made using for waterproofing an ordinary mortar coating and a neat cement wash, this method of waterproofing being frequently made use of in reservoirs under a low head. In order to determine the effectiveness of waterproofing materials in general, a set of specimens was made in which the concrete was not waterproofed.

Different methods, as illustrated in Fig. 3, were used in finishing the upper surface of the specimens, depending upon the kind of waterproofing compound used. As shown at A and B in Fig. 3, the concrete was finished flush with the top of the pipe, the upper surface of the concrete being well troweled. Before troweling, the specimens were allowed to stand a half hour in order that the free water on the concrete might be absorbed.

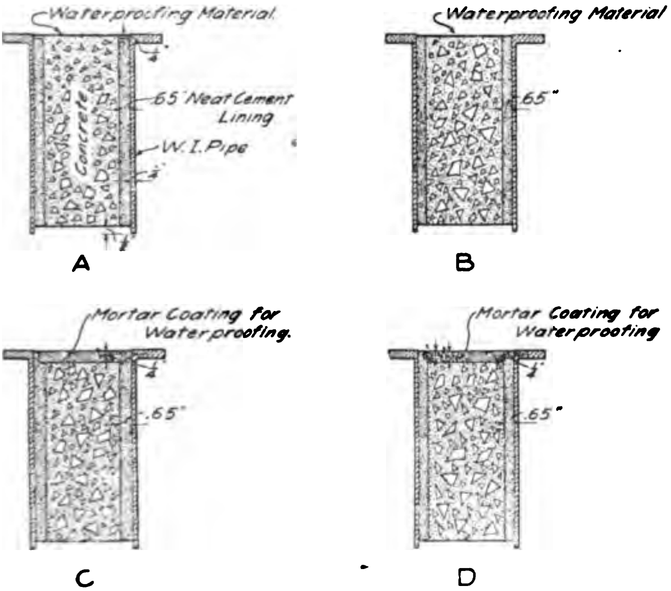

FIG. 3. SHOWING METHOD OF FINISHING UPPER SURFACE OF CONCRETE.

The cement lining extends to the top of the concrete in B, while in A it is cut off ¼ inch below. In the specimens that were coated with mortar (see C and D of Fig. 3) the surface of the concrete was ¾ of an inch below the top of the pipe. After the concrete had absorbed the standing water the mortar top was added. the surface of the concrete and the mortar being thoroughly troweled.

In Tables III to XVI, inclusive, are tabulated the results of the tests. In column 1 is given the time in hours counted from the beginning of the test and in columns 2, 3, 4 and 5 the corresponding flow for the different specimens in millionths of cu. ft. of water per sq. ft. of concrete surface. The corresponding flow in pounds of water may be obtained by multiplying the values in the columns by the constant .0000102. Tables III, IV, VI, VIII, XI, XII, XIV and XVI show only a part of the data because of the length of the test but the data selected is typical.

The experiments will be discussed in the same order in which the waterproofing compounds have been considered. In applying all surface preparations care was taken to secure a dry clean surface and to have the preparation well brushed in.

Century Cement Fluid:—This is a gray substance resembling caked putty, which was thinned with benzine to the consistency of a thick cream before being used. It is sold by the Century Cement Fluid Co., Madisonville, Ohio. The compound was applied in three coats with a brush at the ages of 14, 24 and 30 days. Only four specimens of the type shown at B in Fig. 3 were made because of a lack of time. These were subjected to a pressure of approximately 20 lbs/in^2 for 263 hours and no appreciable flow was noted.

Des Moines Elaterite No. 60:—This is a black tar-like liquid furnished by The Elaterite Paint & Manufacturing Co., Des Moines, Iowa. It is of the consistency of linseed oil and was applied cold. Eight specimens were made similar to that shown at B in Fig. 3. Three coats of the compound were applied at the age of 15, 23 and 31 days for the specimens tested at 20 lbs/in^2 and at the age of 10, 17 and 23 days, for those at 40 lbs/in^2. The specimens subjected to a pressure of 20 lbs/in^2 showed no measurable flow during the 72 hours that they were under pressure. The flow of the specimens subjected to a pressure of 40 lbs/in^2 was quite variable. (See Table III.) The duration of the test was 68 hours. Specimen No. 53 was practically impermeable, the flow of No. 54 and No. 52 was moderate but No. 51 showed excessive flow. While considerable leakage was noted at the flanged joint of No. 51 and No. 52, this leakage would not account for the increased flow of No. 51. Furthermore, while the bottoms of No. 53 and No. 54 were dry at the end of the test, the bottom of No. 51 was wet and that of No. 52 was damp.

Universal Damp-Proof Compound:—This is a heavy tar-like paint which was secured from the Universal Compound Co., No. 88 Maiden Lane, New York City. The 8 specimens were of the type sketched at B in Fig. 3. The compound was applied at the age of 15, 23 and 30 days for the pressure of 21 lbs/in^2 and at the age of 10, 16 and 26 days for the high pressure specimens. As will be seen by referring to Table IV the results at the 20 lbs/in^2 pressure for a period of 251.5 hours were satisfactory but extremely variable. Specimens No. 127 and No. 130 were practically impermeable, while specimens No. 128 and No. 129 showed moderate flow. Three of the four specimens that were tested at a pressure varying from 35 lbs/in^2 to 37 lbs/in^2 showed excessive flow, while considerable water entered the fourth specimen. (See Table V.) Moisture was noted on the bottom of No. 57 within 6 hours after the pressure was on, while No. 58 and No. 56 were wet within 16 hours. Specimen No. 59 remained dry on the bottom during the test which continued for 69 hours. This compound was unsatisfactory at this high pressure.

Antihydrine:—This compound is manufactured by The Antihydrine Company, New Haven, Conn. It is thin tar-like liquid which is applied cold. Because of a lack of time only 4 specimens of the type shown at B in Fig. 3, were made. Three coats of Antihydrine were applied at the age of 15, 23 and 31 days. Under a pressure of 20½ lbs/in^2 one of the specimens was impermeable, while the flow for the remaining three was moderate. All the specimens were dry on the bottom at the end of the test, which lasted about 168 hours. (See Table VI.) These specimens were removed from the apparatus and stored for 26 days when they were tested at a pressure of 40 lbs/in^2 for 79 hours. As will be seen by referring to Table VII the flow for each of the specimens was very small.

Flexible Compound:—This material, which resembles linseed oil in color and consistency, was secured from S. P. Holmes & Co., Chicago, Ill. Before applying the compound it was mixed with white lead, the proportions being 25 pounds of lead to one gallon of the compound, the mixture being of a creamy consistency. A set of specimens was made similar to that shown in Fig. 3 at B. Three coats of the waterproofing were used; applied at the age of 15, 23 and 30 days for the low and at 10, 16 and 26 days for the high pressure specimens. The results at each pressure were very satisfactory (see Tables VIII and IX), the flow being small and uniform. The first set of specimens was tested at a pressure of approximately 21 lbs/in^2 for 251.5 hours, and the second set at a pressure varying from 35 lbs/in^2 to 37 lbs/in^2 for 69 hours.

Hydrex Waterproof Felt and Compound:—These materials were purchased from the Hydrex Felt and Engineering Co., New York City. Four specimens, similar to those shown in Fig. 3 at A, were treated with these compounds at the age of 19 days. The compound was heated so that it ran readily from a stick and was then quickly applied with a swab in an even layer to the upper surface of the concrete. A sheet of felt was now placed on the compound and well smoothed with a brush. Alternate layers of compound and felt were applied until the specimen was covered with three thicknesses of felt and four layers of the compound. The sheets of felt were of sufficient size to extend outside of the flange. This waterproofing gave excellent results, the specimens showing no flow whatever when subjected to a pressure of 40 lbs/in^2 for 163 hours.

Siastex Waterproofing Fabric No. 2 and Pitch:—These materials were obtained from the Sicilian Asphalt Paving Co., New York City. The form of specimen used and the manner of applying the waterproofing were exactly the same as for Hydrex Waterproofing Felt and Compound. The specimens were treated at the age of 11 days. The test continued for 68 hours at a pressure of 39 lbs/in^2 and the specimens showed no permeability whatever.

Dehydratine No. 4 and Tarred Roofing Paper:—Dehydratine No. 4 is manufactured by the A. C. Horn Co., New York City. It is a black tar-like liquid of medium consistency which was applied cold. Tarred roofing paper similar to that found in ordinary building construction was used with this compound.

The four specimens were of the type sketched at A in Fig. 3, and the materials were applied in the same manner as was the Hydrex Felt and Compound with this exception, Two coats of Dehydratine were applied to the surface of the specimens and to the upper sheet of roofing paper, the first coat being quite dry when the second coat was added. As will be seen by referring to Table X the flow at a pressure of 40 lbs/in^2 was practically zero for the 66 hours that the test continued.

Coatine:—This is a gray fibrous material resembling soft putty and was obtained from the H. B. Morgan Co., Grand Crossing, Chicago. It does not appear to harden and was still quite plastic 35 days after applying. Eight specimens were made of the form shown at D in Fig. 3 but with Coatine used in place of mortar. In order to secure adhesion between the Coatine and the concrete and cement lining, the surface of the latter was roughened with a trowel and allowed to set thoroughly before the Coatine was applied. The layer of Coatine was placed on the specimens at the age of 5 days, care being taken to secure a good joint by proper troweling. Table XI shows the flow for three specimens at a pressure of 20 lbs/in^2 for 74 hours. Specimen No. 193 was impermeable while No. 192 and No. 195 gave a very slight flow. In Table XII are given the results for the specimens tested at an average pressure of 38 lbs/in^2. Specimen No. 200 was impermeable, No. 202 practically so, while No. 203 showed a slight flow and No. 201 a moderate flow. However, the needle valve of No. 201 leaked throughout the test, and this will account in part for the increased flow of the specimen.

Antihydro:—This is a thin liquid, greasy in consistency and containing a brown sediment. It was secured from the F. M. Hausling Co., New York City. Eight specimens were made of the type shown at D in Fig. 3. After the concrete had absorbed the water standing on its surface, the mortar coating was applied in three parts, the total thickness being ¾ of an inch. The concrete was first covered with a slush coat or grout of neat cement to a thickness of ⅛ of an inch. This was followed by a scratch coat of mortar ¼ to ⅜ of an inch thick, the proportions being one part of cement to two parts of sand. After the initial setting of the scratch coat, the finish coat of mortar was applied, its [illegible] an inch and the proportions one part [illegible] sand. Each coating was gaged with a [illegible] anhydro, the proportions by volume be- [illegible] to 20 parts of water. Care was taken [illegible] coating. The results at a pressure of 40 [illegible] satisfactory as will be seen by referring to [illegible] was small and uniform for the different [illegible] exception of specimen No. 190, which [illegible] flange joint (see Table XIV), the speci- [illegible] impermeable for a pressure of 20 lbs/in² [illegible]

[illegible] specimens were made similar to that [illegible] A ¾-inch mortar coat of the propor- [illegible] parts sand, was applied to the con- [illegible] well troweled. At the age of 14 and 23 [illegible] each specimen was painted with a coating of [illegible] consistency. The cement and mixing [illegible] treated with any waterproofing compounds. [illegible] that were tested at a pressure of 39 [illegible] XV gave satisfactory results. Specimen No. [illegible] flow, dampness being noted on the bottom 24 [illegible] pressure was applied. The flow was moderate for the [illegible] that were subjected to a pressure of 19½ lbs/in² [illegible] XVI. Owing to defective apparatus it was not pos- [illegible] specimen No. 135.

[illegible] pressures of 40 lbs/in² and 20 lbs/in² were also made [illegible] Water-Proof Compound, Limate, Aquabar, Wunner's [illegible] Emulsion and plain concrete, but the results obtained [illegible] conflicting. Additional tests on these materials are de- sirable.

[illegible] these two facts were brought out:

[illegible] when Medusa Water-Proof Compound, Limate and [illegible] were added to the body of the concrete and not in the [illegible] mortar coating, unsatisfactory results were obtained in [illegible] of 18 specimens.

[illegible] all specimens of plain concrete were wet on the bottom [illegible] or at the end of 3 days, thus showing the effective- [illegible] of the waterproofings discussed in this bulletin.

VI. COMPRESSION TESTS ON CONCRETE.

Concrete:—The proportions by volume of the concrete were 1:3:5. The cement, sand, and stone used were the same that were described under the permeability tests. The concrete was machine-mixed, the following method being used except in the case of Medusa Water-Proof Compound. Sufficient sand, cement, and stone were weighed on a scale to make 2½ cu. ft. of concrete. The sand and cement were thrown into the mixer in the order named. The mixer was run about one minute, sufficient water being slowly added to form a stiff mortar. The stone was now dumped into the mixer and the whole mass turned for 1½ minutes, the water being again gradually added until the concrete was of a wet consistency.

Waterproofing Compounds:—The compounds tested were Limate, Medusa Water-Proof Compound, and Aquabar.

Making of Specimens:—The compression specimens were of a cylindrical form, 6 inches in diameter and 18 inches high, cast iron molds being used. After the molds were set up on a horizontal plate, the concrete was deposited in them in layers, it being well stirred with a steel rod. Sufficient concrete was used so that after settling, its surface was slightly below the top of the cylinder. In a few days the upper surface of the concrete was plastered with 1:1 mortar, the surface of the mortar being finished flush with the top of the cylinder. Cylinders treated with each of the waterproofing compounds were made with 6 in each set. and 8 cylinders of plain concrete were made.

Performing the Tests:—The cylinders were broken in a Riehlé Universal testing-machine of 100,000 lbs. capacity at the ages of 35, 62. and 292 days. Three sheets of blotting paper were used at the tops and bottoms of the specimens when tested.

Results of Experiments:—In Table XVII are given the results of the tests which will now be considered.

(a) *Plain Concrete:*—For some unexplained reason the strength of the plain concrete specimens, marked P, is about 6 per cent less at 292 days than at 62 days.

(b) *Limate:*—This material is a hydrated lime obtained from the Western Lime and Cement Co., Milwaukee, Wis. Four sets of specimens were made, the ratio by weight of Limate to the cement for the different cylinders being 10 per cent, 14 per cent, 18 per cent and 22 per cent, respectively. In each case the Limate replaced an equal weight of the cement. The Limate and cement were first thoroughly mixed dry by hand. This mixture with the sand was then thrown into the mixer and the mixing process carried on as before described. The specimens marked L10, L14, etc. (see Table XVII) indicate those in which the Limate is equal to 10 per cent, 14 per cent, etc. of the weight of the cement. As shown in this table, the use of Limate usually increased the strength of the concrete, this effect being especially marked with the L10 and L14 specimens.

(c) *Medusa Water-Proof Compound*:—This is a dry powder furnished by the Sandusky Portland Cement Co. of Sandusky, Ohio. A quantity of the compound equal to 2 per cent of the weight of the cement was thoroughly mixed dry with the cement. Water was slowly added to this mixture while at the same time it was well troweled. The addition of water and the troweling were continued until the mixture was of a damp consistency. It was now thrown into the mixer with the sand, and the mixing completed as before outlined. By referring to Table XVII it will be seen that the cylinders (mark M) treated with this compound were stronger than those of plain concrete at corresponding ages. Furthermore, there is a decided increase in strength with age.

(d) *Aquabar*:—This compound was obtained from the Aquabar Co., Philadelphia, Pa. This is a material in gelatine form which, when properly diluted, is used to temper the mortar or concrete. The compound was thoroughly mixed with water in the proportions of 2 gallons of the compound to 48 gallons of water. The concrete for these cylinders was mixed as before explained with this exception, that the Aquabar solution instead of water was used to temper the mortar and the concrete. The results (see Table XVII) were very unsatisfactory, the Aquabar specimens (mark Aq) showing from 46 per cent to 64 per cent of the strength of plain concrete at corresponding ages.

TABLE 1.

ANALYSIS OF LOCAL LIMESTONE.

Per cent Voids=43.5. Wt. per cu. ft.=90.5 lbs.

Diameter of Mesh, ins.	Amount Retained, lbs.	Per cent Passing.
1.25	0	100
1.00	1.0	97.6
.75	2.5	91.6
.50	15.0	55.6
.33	11.0	29.4
.25	2.25	25.0
.14	4.5	13.2
Residue	5.5	10.8

TABLE II.

ANALYSIS OF SAND.

Per cent Voids = 37. Wt. per cu. ft. = 94¼ lbs.

Sieve Number.	Diameter of Mesh.	Per cent Passing.
¼"	.25	100
6	.131	99.5
10	.073	94.3
16	.042	87.5
20	.034	83.9
30	.022	70.0
40	.015	58.0
50	.011	26.0
74	.0078	19.5
100	.0045	6.7

TABLE III.

PERMEABILITY OF CONCRETE SPECIMENS TREATED WITH DES MOINES ELATERITE NO. 60.

Pressure, 40 lbs/in^2. Age, 35 days.

TIME IN HOURS FROM BEGINNING OF TEST.	FLOW IN MILLIONTHS OF CU. FT. OF WATER PER SQ. FT. OF CONCRETE SURFACE.				REMARKS.
	No. 53.	No. 54.	No. 51.	No. 52.	
0.0	0	0	Apparatus not in order.	0	
0.2	0	130		0	
1.3	0	530		0	
3.3	130	930		0	
4.1	130	1060		130	No. 52 leaking.
14.3	530	3440		530	" " "
15.9	530	3580	0	930	No. 51 leaking.
16.8	530	3980	3180	1460	
20.9	530	4500	4510	4110	" " "
25.7	530	5040	5430	6500	
26.7	530	5160	5570	6630	No. 52 leaking.
36.9	530	5960	6760	7020	" " "
41.2	530	6230	8090	7160	
46.7	530	6500	9550	7820	
50.2	530	6630	12590	8880	" " "
60.7	530	7160	22800	10200	
67.9	530	7550	27800	10600	No. 51 wet on bottom. No. 52 damp on bottom

TABLE IV.

PERMEABILITY OF CONCRETE SPECIMENS TREATED WITH UNIVERSAL DAMP-PROOF COMPOUND.

PRESSURE 21 lbs /in². Age, 35 days.

TIME IN HOURS FROM BEGINNING OF TEST.	FLOW IN MILLIONTHS OF CU. FT. OF WATER PER SQ. FT. OF CONCRETE SURFACE.				REMARKS.
	No. 128.	No. 129.	No. 130.	No. 127.	
0.0	0	0	No flow	0	Small leak in No. 128 & 129.
0.3	0	115		0	
1.0	230	345		0	
11.3	3680	2740		230	Small leak in No. 129.
16.4	4370	3450		345	" " " " "
20.5	4720	4140		460	" " " " "
24.8	5060	4850		575	
40.4	5980	7140		805	
50.5	6440	8050		920	
63.5	6790	9090		920	" " " " "
84.0	7360	10340		1035	
10[illegible].3	7940	11500		1035	
131.5	[illegible]290	12400		1035	
155.5	8630	12880		1150	
179.5	8860	13100		1150	
203.8	9090	13210		1150	
227.5	9440	13330		1150	
251.5	9550	13330		1150	

TABLE V.

PERMEABILITY OF CONCRETE SPECIMENS TREATED WITH UNIVERSAL DAMP-PROOF COMPOUND.

Pressure, 36 lbs/in². Age, 35 days.

TIME IN HOURS FROM BEGINNING OF TEST.	FLOW IN MILLIONTHS OF CU. FT. OF WATER PER SQ. FT. OF CONCRETE SURFACE.				REMARKS.
	No.57.	No. 58.	No. 56.	No. 59.	
0.0	0	0	0	0	
0.4	1720	130	1720	0	
1.0	5570	795	1720	265	
2.9	14430	1325	3180	530	
5.9	27500	17100	16300	2520	No. 56 leaking freely. No. 57 wet on bottom.
15.7	43100	38300	39200	15620	No. 56 leaking freely. No. 56 & No. 58 wet on bottom.
19.9	48000	43200	43900	19090	
28.4	55900	5 000	Shut off specimen because of leakage of apparatus.	24500	
39.2	63500	59400		31200	
43.6	66000	61900		32350	
46.7	67500	63500		33550	
48.3	68600	64300		34100	
62.5	76100	71700		39800	
67.5	78400	73850		41500	
69.3	79000	74500		42000	

TABLE VI.

Permeability of Concrete Specimens Treated with Antihydrine.

Pressure, 20¼ lbs/in². Age, 35 days.

Time in Hours from Beginning of Test.	Flow in Millionths of Cu. Ft. of Water per Sq. Ft. of Concrete Surface.				Remarks.
	No. 175.	No. 176.	No. 177.	No. 178.	
0.0	0	No flow.	0	0	
0.2	0		0	0	
0.4	0		130	0	
1.3	0		265	0	
1.6	130		400	0	
.7	400		660	530	
15.6	1855		1325	1855	No. 178 leaking,
27.7	3180		1990	2920	
39.5	3640		2780	3710	
49.7	4110		3180	4240	
63.9	4510		3710	4910	
84.3	4770		3980	5560	
107.5	5300		4110	6360	
132.5	5970		4770	7560	
156.1	6500		4910	7950	
179.7	6900		5040	8210	
204.2	7160		5300	8750	
227.9	7700		5560	9290	

TABLE VII.

PERMEABILITY OF CONCRETE SPECIMENS TREATED WITH ANTIHYDRINE.

Pressure, 40 lbs /in². Age, 64 days.

TIME IN HOURS FROM BEGINNING OF TEST.	FLOW IN MILLIONTHS OF CU. FT. OF WATER PER SQ. FT. OF CONCRETE SURFACE.				REMARKS.
	No. 178.	No. 177.	No. 175.	No. 176.	
0.0	0	0	0	0	
0.2	0	0	0	0	
1.0	0	0	0	115	
1.8	0	115	0	115	
6.6	230	115	460	230	No. 175 leaking.
25.8	920	920	2070	690	" " "
31.3	1150	1035	2300	805	
47.3	1610	1380	2760	1265	
55.3	1840	1610	2990	1495	
73.8	2300	1840	3450	2070	
79.1	2530	1955	3565	2185	

TABLE VIII.

PERMEABILITY OF CONCRETE SPECIMENS TREATED WITH FLEXIBLE COMPOUND AND WHITE LEAD.

Pressure, 21 lbs/in^2. Age, 35 days.

TIME IN HOURS FROM BEGINNING OF TEST.	FLOW IN MILLIONTHS OF CU. FT. OF WATER PER SQ. FT. OF CONCRETE SURFACE.				REMARKS.
	No. 132.	No. 133.	No. 134.	No. 131.	
0.0	0	0	0	0	
0.3	130	0	0	130	
1.0	400	0	0	130	
11.3	1190	400	530	530	
16.4	1190	530	660	660	
20.5	1325	530	795	660	
24.8	1460	795	930	795	
40.4	1720	1190	1460	1060	
50.5	1720	1325	1590	190	
63.5	1990	1590	1855	1325	
84.0	2250	2120	2385	1590	
108.3	2520	2650	2920	1990	
131.5	2780	3040	3180	2250	
155.5	3050	3440	3440	2520	
179.5	3320	3840	3980	2650	
203.8	3580	4240	4370	2915	
227.5	3840	4510	4760	3180	
251.5	3980	49.0	5160	3310	

TABLE IX.

Permeability of Concrete Specimens Treated wieh Flexible Compound and White Lead.

Pressure, 36 lbs/in^2. Age, 35 days.

Time in Hours from Beginning of Test.	Flow in Millionths of Cu. Ft. of Water per Sq. Ft. of Concrete Surface.				Remarks.
	No. 62.	No. 6 .	No. 61.	No. 55.	
0.0	0	0	0	0	
0.4	230	0	0	230	
1.0	230	0	0	460	
2.9	690	0	0	805	
5.9	1035	230	115	1380	
15.7	1840	345	230	2760	
19.9	2185	575	345	3335	
28.4	2875	920	345	4140	
39.2	3680	1380	575	5290	
43.6	4025	1610	575	5635	
46.7	4255	1725	575	5980	
48.3	4370	1725	575	6095	
62.5	5405	2185	690	7245	
67.5	6210	2300	690	7590	
69.3	6555	2415	805	7820	

TABLE X.

PERMEABILITY OF CONCRETE SPECIMENS TREATED WITH DEHYDRATINE No. 4 AND TARRED ROOFING PAPER.

Pressure, 40 lbs /in^2. Age, 35 days.

TIME IN HOURS FROM BEGINNING OF TEST.	FLOW IN MILLIONTHS OF CU. FT. OF WATER PER SQ. FT. OF CONCRETE SURFACE.				REMARKS.
	No. 110.	No. 107.	No. 108.	No. 109.	
0.0	0	0	No flow.	0	
0.5	0	0		0	
1.5	0	0		0	
2.8	0	0		0	
4.8	0	0		0	
14.3	230	0		115	
17.3	230	0		115	
20.0	230	0		115	
25.0	230	0		115	
30.4	230	0		115	
41.5	345	115		115	
44.0	460	230		230	
52.4	575	345		230	
64.0	690	345		345	
66.8	690-	460		345	

TABLE XI.

PERMEABILITY OF CONCRETE SPECIMENS TREATED WITH COATINE.

Pressure, 20 lbs/in^2. Age, 35 Days.

TIME IN HOURS FROM BEGINNING OF TEST.	FLOW IN MILLIONTHS OF CU. FT. OF WATER PER SQ. FT. OF CONCRETE SURFACE.				REMARKS.
	No. 192.	No. 194.	No. 193.	No. 195.	
0.0	0	No test made.	No flow.	0	
0.8	0			130	
1.1	130			130	
2.8	265			265	
4.2	400			265	
5.0	400			265	
6.0	400			265	
8.5	400			400	
20.9	665			530	
29.7	795			530	
32.4	795			665	
44.0	1060			795	
53.8	1190			1060	
56.5	1190			1060	
68.0	1190			1060	
74.0	1325			1060	

TABLE XII.

PERMEABILITY OF CONCRETE SPECIMENS TREATED WITH COATINE.

Pressure, 38 lbs /in^2. Age, 35 days.

Time in Hours from Beginning of Test.	Flow in Millionths of Cu. Ft. of Water per Sq. Ft. of Concrete Surface.				Remarks.
	No. 203.	No. 200.	No. 202.	No. 201.	
0.0	0	No flow.	0	0	
0.2	130		130	0	
0.8	130		265	130	
1.5	265		530	265	
5.0	530		795	1060	
8.8	530		795	1460	
22.5	530		795	2920	No. 201 leaking.
24.2	795		795	3050	No. 201 continued to leak throughout the test.
40.2	795		795	3580	
48.1	930		795	3840	
62.6	930		795	4110	
72.1	1195		795	4770	
96.9	1195		795	5960	
120.8	1460		795	7290	
144.7	1590		795	8210	

TABLE XIII.

PERMEABILITY OF CONCRETE SPECIMENS TREATED WITH ANTIHYDRO.

Pressure, 40 lbs/in². Age, 35 days.

TIME IN HOURS FROM BEGINNING OF TEST.	FLOW IN MILLIONTHS OF CU. FT. OF WATER PER SQ. FT. OF CONCRETE SURFACE.				REMARKS.
	No. 106.	No. 104.	No. 105.	No. 103.	
0.0	0	0	0	0	
0.5	930	660	460	1190	
1.5	1990	1590	1325	2250	
2.8	2520	1990	1720	2520	
4.8	2920	2250	2250	2780	
14.3	3320	2520	2915	3040	
17.3	3320	2520	3040	3040	
20.0	3320	2520	3040	3040	
25.0	3440	2520	3180	3040	Small leak in No. 105.
30.4	3440	2650	3310	3180	
41.5	3580	2650	3580	3310	" " " " "
44.0	3580	2650	3710	3310	" " " " "
52.4	3710	2650	4640	3440	" " " " "
64.0	3840	2720	5430	3580	" " " " "
66.8	3840	2720	5560	3580	

TABLE XIV.

PERMEABILITY OF CONCRETE SPECIMENS TREATED WITH ANTIHYDRO.

Pressure, 20 lbs/in². Age, 35 days.

TIME IN HOURS FROM BEGINNING OF TEST.	FLOW IN MILLIONTHS OF CU. FT. OF WATER PER SQ. FT. OF CONCRETE SURFACE.				REMARKS.
	No. 189.	No. 191.	*No. 190.	No. 188.	
0.0	0	0	0	0	
0.2	345	345	460	345	
0.4	575	690	690	345	
0.8	920	920	1150	1265	
2.2	1380	1150	1725	1610	
3.7	1610	1150	1840	1840	
5.5	1725	1150	....	1955	
16.7	1955	1150	....	2300	
21.7	2070	1150	3220	2415	
26.2	2070	1150	3340	2415	
29.2	2070	1150	3450	2415	
40.6	2070	1150	3910	2415	
50.2	2070	1150	4260	2530	
64.7	2070	1150	4950	2530	
66.7	2070	1150	5060	2530	
140.8	2070	1150	5060	2530	

*No. 190 leaked throughout the test.

TABLE XV.

PERMEABILITY OF CONCRETE SPECIMENS TREATED WITH PLAIN MORTAR.

Pressure, 39 lbs/in^2. Age, 35 Days.

TIME IN HOURS FROM BEGINNING OF TEST.	FLOW IN MILLIONTHS OF CU. FT. OF WATER PER SQ. FT. OF CONCRETE SURFACE.				REMARKS.
	No. 63	No. 64.	No. 65.	No. 66.	
0.0	0	0	0	0	
0.4	805	460	460	1495	
1.7	1950	1380	1380	4370	
4.9	2530	1950	1840	9200	
14.9	3105	2640	2185	19200	
19.4	3340	2880	2300	24400	
24.2	3450	3110	2300	25250	No. 66 damp on bottom.
28.0	3450	3220	2415	27000	
37.4	3680	3450	2415	30400	No. 66 wet on bottom.
42.1	3680	3680	2415	31600	
47.8	3800	3800	2530	33000	
50.8	3800	3910	2530	33700	
62.1	3910	4140	2530	35900	
67.8	4030	4160	2530	36900	

TABLE XVI.

PERMEABILITY OF CONCRETE SPECIMENS TREATED WITH PLAIN MORTAR.

Pressure, 19¼ lbs /in². Age, 35 days.

TIME IN HOURS FROM BEGINNING OF TEST.	FLOW IN MILLIONTHS OF CU. FT. OF WATER PER SQ. FT. OF CONCRETE SURFACE.				REMARKS.
	No. 135.	No. 136.	No. 137.	No. 138.	
0.0	Apparatus not in order.	0	Apparatus not in order.	0	
0.3		660		660	
0.8		660		1325	
1.2		795		1590	
3.0		1190		1990	
4.6		1460		2250	
5.9		1720	0	2520	
9.1		1850	1460	2780	
20.3		2120	1720	3440	
28.3		2250	1720	3710	
32.5		2250	1990	3710	
43.5		2250	2120	3980	
48.2		2250	2250	4110	
56.3		2250	2380	4240	
67.7		2250	2520	4510	
80.3		2250	2780	4770	
93.1		2250	2780	5040	

TABLE XVII.

EFFECT OF WATERPROOFING COMPOUNDS ON THE COMPRESSIVE STRENGTH OF 1: 3: 5 CONCRETE.

SPECIMENS.	ULTIMATE COMPRESSIVE STRENGTH IN LBS/IN².					
	Age, 35 days.		Age, 62 days.		Age, 292 days.	
		Average.		Average.		Average.
P	940	....	1110	....	1330	
P	780	....	1160	....	880	
P	....	....	....	....	930	
P	....	860	....	1135	1140	1070
Aq.	570	....	480	....	620	
Aq.	530	550	570	525	....	620
L10	1110	....	1200	....	1240	
L10	960	1035	1180	1190	1210	1225
L14	1130	....	1270	....	1130	
L14	1050	1090	1320	1295	1300	1215
L18	1040	....	1000	....	1040	
L18	860	950	1190	1095	1020	1030
L22	1000	....	1250	....	1340	
L22	750	875	10.0	1140	1140	1240
M	970	....	1090	....	*1270	
M	890	930	1160	1125	1220	1245

*This specimen was broken by mistake at age of 224 days.

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THE STRENGTH OF THE ALLOYS OF NICKEL AND COPPER WITH ELECTROLYTIC IRON

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INTRODUCTION

Of the properties of binary alloys of iron with the common metals comparatively little is known. This is attributed to the fact that pure iron with which to alloy copper or nickel, or tin, or others of the metallic elements, has been a rare laboratory product, and interest has been directed, naturally, to alloys made from commercial materials.

Based upon the discovery of a simple method of refining iron electrolytically, an extensive line of laboratory investigation was begun five years ago in the Applied Electrochemistry Laboratory of the University of Wisconsin, and has been carried on almost without interruption since that time. This investigation has consisted in the production and study of the properties of alloys of iron, eliminating as far as possible the common impurities, sulphur, phosphorus, silicon, manganese and carbon.

This work has been conducted through facilities afforded by the College of Engineering of the University of Wisconsin and through grants from the Carnegie Institution of Washington.

From the large amount of data and information derived from this investigation, a small portion, dealing with the alloys of copper and nickel with iron, is presented in this bulletin. This selection of material has been made on account of the interesting and possibly important influence of small percentages of copper and of nickel on the properties of iron.

It is appreciated by no one more than by the authors that laboratory experiments carried out on the small scale which has been necessary here, can by no means embody all the factors attendant upon large scale production, and the conclusions arrived at can be only suggestive when applied to commercial production of alloys.

The study of iron alloys of the degree of purity which we have attained is not alone of scientific interest, since recent improvements in commercial materials make it possible to secure remarkably pure iron at reasonable cost.

The fact which impresses itself most emphatically upon one who undertakes to study the properties of iron alloys, is that he has an unlimited field to deal with and one which has not been cultivated intensively or even superficially.

Not only are there new alloys to be produced, but much of the older work on iron and steel, now considered as on a fairly settled basis, can be repeated with profit in the light of a modern knowledge as to the influence on the properties of material, of variations in heat treatment, and other factors.

In presenting this bulletin, the writer acknowledges indebtedness to numerous of his colleagues who have assisted in this work, and especially, to Mr. Carl Hambuechen, who worked out the details for refining iron electrolytically, to Dr. Oliver P. Watts and Mr. Otto Kowalke, who conducted the laboratory work of this investigation for two years, and to Mr. James Aston who as joint author has done much more than his share in writing this report on the alloys of nickel and copper with iron.

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THE STRENGTH OF THE ALLOYS OF NICKEL AND COPPER WITH ELECTROLYTIC IRON

Electrolytic Iron

The basis of the alloys dealt with here is iron of a high degree of purity, obtained by double electrodeposition. In the first refining, high grade Swedish bar iron is used as anode material, and the metal is deposited upon both sides of a sheet lead cathode. These cathodes are then used as anodes in a second set of tanks, and a double refined product is obtained upon a sheet aluminum cathode. This electrolytic iron is stripped off in sheets ¼ inch to ⅜ inch in thickness about 10 inches square, and is broken up for use in the crucibles.

The character of deposited material is indicated in the following table. The analyses given in the first three columns are those of Messrs. Booth, Garrett and Blair, of Philadelphia. Also, all of the analytical work on the alloys proper, including those appearing later in this discussion, are the work of the above laboratory, and acknowledgment is due for their liberality and interest in the work. The most recent analysis of the double refined iron is given in the last column, and was reported by Mr. Alvan C. Davis, of Edgeworth, Pa.

	Swedish Anode.	Single Refined.	Double Refined.	Double Refined.
Carbon	0.260	0.013	0.012	0.000
Sulphu	0.007	0.007	None.	0.003
Silicon	0.109	0.003	0.013	0.002 or less.
Phosphorus	0.007	0.020	0.004	0.003
Manganese	0.021	None.	None.	0.020
	0.404	0.043	0.029	0.028
Iron (difference)	95.596	99.957	99.971	99.972

The analyses of the final product differ somewhat in the percentages of the various impurities found. Whether this is due to the difference of the material at the different times of sampling, or to the limiting error of the analytical method in detecting the small percentages actually found, is problematical. However, the initial product used in the preparation of the alloys may be considered of negligible impurity, with an iron content of 99.97 per cent.

Preparation of Alloys

In the preparation of the alloys, the brittle electrolytic iron was broken into small pieces, and the pre-determined amounts needed for each charge, together with the addition elements, were weighed out. The charges were placed in specially prepared magnesia crucibles, which were in turn protected by a graphite jacket. The crucibles were buried in an electric furnace of the resistor type; the charge being prevented from contamination by a magnesia lid luted on the crucible, and a graphite cover for the jacket.

The charges were brought to fusion, and held at this temperature for a few hours after which the current was cut off from the furnace and the latter was allowed to cool down. There was no agitation of the charge, the resulting alloy being entirely due to diffusion of the elements. The ingots weighed about 500 grams (1 lb.)

In spite of every precaution, there was some absorption of carbon by the charge. There is every indication that this was due to the interaction between the iron and the carbon monoxide furnace atmosphere, whereby carbon was set free by the reaction $Fe + CO = FeO + C$, and then taken up by the iron by cementation. The amount absorbed varied somewhat with the conditions of the melt, and probably also with the nature of the alloying element. Numerous analyses, however, showed the amount to be well under 0.10 per cent; sulphur, silicon, phosphorus and manganese are negligible. A charge of electrolytic iron melted into an ingot and forged under the same conditions employed in the preparation of the alloys showed by analysis carbon 0.047, sulphur 0.005, silicon 0.062, phosphorus 0.016, manganese 0.0.

In the tests, therefore, the properties observed may be ascribed to the addition elements.

The ingots were heated in a forge fire and drawn out under a steam hammer into rods about ½ inch in diameter and 20 inches long. These rods were cut into appropriate lengths and machined for the tensile strength tests into bars approximating 0.3 to 0.4 inch in diameter over a free length of 2 inches. Qualitative observations were made of forgeability and of hardness in machining.

An equal number of samples were reserved for testing as forged and after annealing at 900°C. The tests were made upon a Riehle 1,000,000 pound power-driven machine. The load was applied very slowly, and the elastic limit, or rather, the yield point, was determined by means of dividers, being taken as that load at which a perceptible stretch occurred in a marked 2-inch length of the bar. This was checked by the drop of the beam. In the later tests, multiple dividers were used, enabling the yield point to be observed very closely, as well as the elongation at this point. The data recorded included the load at the yield point, and the maximum necessary to rupture the bar; also the elongation in two inches and the reduction of area at the fracture.

This particular discussion may be divided into three general parts—(1) the nickel-iron series; (2) the copper-iron series; (3) a series with iron alloyed with both nickel and copper.

NICKEL-IRON ALLOYS

Much has been written regarding certain nickel steels for commercial usage and there has been considerable investigation of these particular alloys. Also, there is much literature on the metallography and the theory of nickel steels, with which phase this paper is not particularly concerned. There is comparatively little published data of research, especially systematic research on the effect of the addition of varying percentages of nickel to steels of otherwise comparable compositions.

Of the latter, to which the present paper is largely supplementary or correlative, may be mentioned the following:

Between 1892 and 1902, the Prussian Society for the Encouragement of Industry published several reports dealing with the mechanical properties of the nickel-iron alloys (Berichte des Sonderausschüsses für Eisen-Nickel Legierungen. 1892–1902).

R. A. Hadfield (Inst. Civ. Engrs., 1898, Volume CXXXVIII, page 1) gives data of a very complete study of a series of nickel-iron alloys, covering the mechanical and physical properties. The nickel varies from 0 to 50 per cent, with other impurities as follows: C, 0.13 to 0.23 per cent; Si, 0.20 to 0.38; S, 0.08 to 0.11; P, 0.05 to 0.09; Mn, 0.65 to 1.08.

L. Guillet (Bul. Soc. d'Encour pour l'Industrie Nationale, May, 1903) publishes the results on three series of steels with nickel from 2 to 30 per cent, manganese low and carbon about 0.12, 0.22 and 0.82 per cent in the several series. The work includes the mechanical tests and the metallographic study with a view of finding the relation between the physical properties and the structure.

Carpenter, Hadfield and Longmuir (7th Report, Alloys Research Committee, Inst. Mech. Engrs., Vol. 2, 1905, page 857) give the data of a thorough investigation of a series of alloys with nickel from 0 per cent to 20 per cent, carbon 0.41 to 0.52; manganese, 0.79 to 1.03. This research was intended as a supplement to Hadfield's work mentioned above, the steels being practically of the same composition, except that there was an increase of the carbon content from a mild steel proportion of about 0.20 per cent to that of a medium steel with 0.45 per cent.

In the present research the effort has been to determine the effect of additions of varying percentages of nickel to iron in the absence of other impurities, especially carbon. As mentioned heretofore, the iron is of such purity that the results observed may be ascribed to the nickel addition alone. The added nickel was electrolytic material of high grade.

ANALYSIS

Ingots were prepared of all compositions from 0 to 100 per cent of nickel, but the mechanical tests here enumerated are confined largely to the range of 0 per cent to 20 per cent of nickel. The analyses given below are of a series of random selection which is representative of the entire range.

ANALYSES.

Mark.	Per cent Ni.		Per cent C.
	Added.	Analysis.	
144C	0.25	0.27	0.088
144D	0.50	0.56	
144E	1.0	1.07	
144F	2.0	1.93	
144J	7.0	7.05	
157A	8.0	8.17	
157D	10.0	10.20	
157E	11.0	11.29	
157H	12.0	12.07	
144M	13.0	13.11	
144P	19.0	19.21	
166G	22.0	22.11	
154S	25.0	25.20	
166I	26.0	26.40	
166C	28.0	28.42	
166L	35.0	35.09	
166O	50.0	47.08	
166Q	75.0	75.06	

As might be expected from the close analogy of the properties of nickel and iron, the agreement of the analytical results with the percentage of nickel added to the charge, is exceedingly close and when it is considered that the samples for analysis were from turnings of test bars after the ingots had been forged into rods, it may be taken as evidence of the perfect alloying of iron and nickel and the entire absence of segregation in the ingot.

FORGING

Ingots were generally sound and the lower percentages forged about the same as iron and at normal temperatures. In fact, there was no trouble in forging any of the alloys within the range of 0 to 20 per cent of nickel. Alloys of 25, 26, 28, 35, 50 and 75 per cent of nickel forged without special difficulty, although in an earlier trial one ingot with 75 per cent smashed. A 100 per cent nickel was not forgeable. In later tests an ingot of 30 per cent nickel smashed and one of 34 per cent was partially destroyed. With extreme care and high welding heats, and with the liberal use of flux a good bar was drawn out of an 85 per cent ingot. A high heat is probably necessary for all of the high nickel alloys.

The discrepancies noted in the forgeability might be due to oxidation such as Hadfield experienced, and which caused such red shortness that he was compelled to use manganese and aluminum as deoxidizers; or there might be a critical range at about 34 per cent of nickel at a composition corresponding to a definite compound of Fe_2Ni. It is at about this composition that the greatest difficulty is experienced in forging, and other measurements of various characters tend to support this supposition of a compound.

WELDING

There was no difficulty in welding low nickel bars during forging. The high nickel alloys, including one of 85 per cent nickel, which tended to crumble at lower heats, welded nicely at very high temperatures of working and with the liberal use of borax to prevent oxidation. Bars of all percentages of nickel were welded in the electric welder, although some failures were noted between 18 per cent and 35 per cent of nickel. These welds were not subjected to tests other than those indicated by grinding the juncture and testing with the hands.

HARDNESS

The hardness tests are such qualitative results as were noted in the machining, sawing and filing necessary in preparing the test bars. In the unannealed condition, working was easy for alloys up to and including 7 per cent of nickel; the alloys were rather stiff from 8 per cent to 10 per cent, inclusive, and hard between 11 per cent and 20 per cent, with the 20 per cent alloy less hard than some of those of intermediate compositions. All alloys were machined in the unannealed state with lathe tools, although this was very difficult in the range between 10 and 20 per cent of nickel. The higher nickel bars were again soft.

After annealing there was a somewhat noticeable hardness, beginning at about 11 to 13 per cent, but it was not extreme in any case, and there was no difficulty in sawing or machining.

BENDING

These tests were again only qualitative, being the observations in breaking bars sawed half through and in the unannealed condition. Alloys from 0 to 2 per cent nickel cracked at a 60° angle; from 3 per cent to 8 per cent, inclusive, at 45°; 9 to 10 per cent, at 30°, and from 11 per cent to 20 per cent, inclusive, the bars broke off short. The fracture changed from a fibrous to a granular at about 10 per cent of nickel.

The above hardness and bending tests are in agreement with the tensile tests noted below.

TENSILE STRENGTH TESTS

The summary of results is included in Tables 1 and 2 and in Plates I and II. The detailed data are not given; also no records are noted of abnormal results due to defective bars, etc.

In the summary are given the yield point and ultimate strength in pounds per square inch, the percentage of elongation in 2 inches, and the percentage of reduction of area at fracture. There are also included the maximum, minimum and average values of each of the above items, together with the number of samples from which such average is calculated. This furnishes an indication of the consistency of the data. In the plates, the yield point, the ultimate strength and the elongation and the reduction of area for varying percentages of nickel are designated by appropriate symbols, and are the values given in the tables in the columns of averages.

With these points as a guide, smooth curves are drawn in each case. With the many factors affecting somewhat the strength of a metal, it is not expected that the points will fall consistently on a curve. It is intended merely that the curve will show the tendency of the effect of increasing the percentage of nickel in the alloy.

UNANNEALED

The results of the tests in the unannealed state are enumerated in Table 1 and the general summary is best noted by referring to the curves on Plate 1. Between 0 and 8 per cent of nickel there is a gradual increase of the ultimate strength and elastic limit which are approximately linear functions of the percentage increase of nickel. At a nickel content of 0.25 per cent the figures are 66,730 for the ultimate strength and 57,180 for the yield point, these values rising to 91,740 lbs. and 70,770 lbs., respectively, for an increase of nickel to 8 per cent. The elongation and reduction of area indicate a decrease of ductility between 0 and 2 per cent. From this point they rise to a maximum at 5 per cent of nickel, above which percentage there is a gradual falling off until the composition of 8 per cent of nickel is reached. At 5 per cent nickel we note values of 64,030 for the elastic limit and 77,150 for the ultimate strength, with the high values of 29.2 for the elongation and 65.2 per cent for the reduction of area. The elastic ratio, however, is high, being 0.83.

Beyond 8 per cent of nickel there is a decided upward bend in the curves for the ultimate strength and elastic limit and a corresponding drop in those for the reduction of area and the elongation, indicating, therefore, a brittle zone. This is most pronounced at 11 per cent of nickel, where the yield point is 148,240 and the ultimate strength 181,230, while the elongation falls off to 7 per cent and the reduction of area to 25.5 per cent. On increasing the nickel above 12 per cent there is a return of ductility indicated by the gradual rise of the elongation and the marked increase of the reduction of area. At 20 per cent nickel we have the very high values of elastic limit 115,500, ultimate strength 186,000, with the accompanying elongation of 16 per cent and reduction of area of 59.5 per cent. The brittle zone may be said to extend from 10 per cent to 15 per cent of nickel.

The data for percentages of nickel above 20, which are not plotted, show a still further increase of ductility; at 50 per cent of nickel we have values of elastic limit of 75,700, ultimate strength 108,500, elongation 26.5 per cent, reduction of area 63.77 per cent.

These facts are in accord with the bending and hardness tests noted heretofore where the maximum brittleness lay between 10 per cent and 20 per cent in bending and the greatest difficulty in machining was in the same range.

ANNEALED

Results of the tests on the annealed bars are given in Table 2 and Plate II. The effect of annealing has been to lower the ultimate strength and the elastic limit throughout the entire range with the accompanying increase of elongation and reduction of area; this, however, without materially altering the general influence of the nickel. We note the same gradual increase of maximum stress and yield point over a range which is now extended to 10 per cent of nickel instead of 8 per cent. At this point the same sharp increase takes place and the elongation and reduction of area fall off in accordance. The point of minimum ductility is now at 13 per cent, where the elongation has fallen off to 11.6 per cent and the reduction of area to 34.5 per cent. The pronounced maximum of strength is now not present, the elastic limit of 97,730 and the ultimate strength of 121,810 being below the maximum values of the tests. At 11 per cent of nickel, the composition of the former alloy of least ductility, the deviations of maximum and minimum values are large, lying between extremes of 23 per cent and 0 per cent for the elongation and between 65.2 per cent and 0 per cent for the reduction of area.

The brittle zone is quite as pronounced as it was in the unannealed condition, but is now more limited in extent and may be said to cover the range from 12 per cent to 15 per cent of nickel. Above 15 per cent there is the same gradual increase of elongation and marked rise in the reduction of area, but we do not note the marked ductility at the 20 per cent nickel alloy which was observed for the unannealed bar.

COMPARISONS

The safe values to be assumed for nickel steels of the compositions used in commercial practice may be taken from Waddell's elaborate paper before the American Society of Civil Engineers in September, 1908. For bridge structures he recommends a composition of 3½ per cent of nickel and 0.38 per cent of carbon, with values of elastic limit of 60,000 lbs. per square inch. and ultimate strength of 105,000 lbs. per square inch.

These figures are for sections as rolled, and with a ductility somewhat lower than for ordinary steels, as is evidenced by the elongation of 15 per cent for nickel against 27 per cent for carbon steel (in 8 inches) and a reduction of area of 41 per cent and 55 per cent, respectively. For a lower carbon content with the same nickel the values become—elastic limit 45,000, ultimate strength 70,000, elongation 25 per cent; and for the increase of carbon to 0.45 per cent the elastic limit is taken as 65,000, maximum stress 115,000 and the elongation 12 per cent.

Values noted from our research for an alloy of 3 per cent of nickel are—yield point 62,920, ultimate strength 74,860, elongation (in 2″) 24.5 per cent, reduction of area 63.9 per cent; and for nickel = 4 per cent, the elastic limit is 65,000, maximum stress 75,670, elongation 27.8 per cent and the reduction of area 66.5 per cent. These values become, after annealing—yield point 55,400, ultimate strength 70,670, elongation 27.5 per cent, reduction of area 67.6 per cent for the alloy of 3 per cent nickel; and for the nickel content of 4 per cent the elastic limit is 52,100, maximum stress 70,070, elongation 28.4 and reduction of area 66.7. The ultimate strengths are well in accord with Waddell's assumptions for low carbon steels, with the elastic limit approaching that of his higher carbon materials.

Of the other tests recorded it is possible to make direct comparisons with Hadfield's results (Inst. Civ. Engrs. 1888–9). His alloys had a carbon content from 0.13 per cent to 0.23 per cent and manganese in the appreciable amounts of 0.65 per cent to 1.08 per cent. For the alloy of 3.82 per cent of nickel the results were as follows: as unannealed, elastic limit 62,700; maximum stress 82,900; elongation 35.8; reduction of area 55.6. These figures are in very close agreement with the 4 per cent alloy of our tests.

In Guillet's tests, for his low carbon series ($C = 0.12$) and for bars in the unannealed state, the maximum stresses and elastic limits are lower throughout than ours for like percentages of nickel. For his 5 per cent alloy the elastic limit is 43,200, maximum stress 52,800, elongation 25 per cent; our values are, respectively, 64,030, 77,150 and 29.2 per cent. However, the same general influences of the addition of nickel are observed.

A most interesting comparison is that of the effect of increasing percentages of nickel. The references are to articles mentioned at the beginning of this section of the paper.

In Hadfield's tests there was a gradual increase of strength and a decrease of ductility with the smaller additions of nickel. The maximum breaking stresses were at 11 to 16 per cent of nickel in the unannealed state with a minimum ductility at 15.5 per cent of nickel. After annealing the maximum strength is at 11.4 per cent of nickel with the lowest ductility at a composition of 9.5 per cent. There is a fair return of ductility at about 24 per cent of nickel after passing through the brittle zone.

Guillet's tests were on three series of alloys, each of a different percentage of carbon, and all with low manganese. He found in each series a brittle zone. For the alloys of lowest carbon = 0.12 per cent, the lowest minimum ductility was at 15 per cent of nickel; for carbon = 0.22 it was at 10 per cent nickel; and for carbon = 0.82 per cent it was at a nickel content of 7 per cent.

In the Report of the Alloys Research Committee, where the carbon was that of a medium steel, being from 0.41 to 0.52, and the manganese was high (0.79 to 1.03), there was a distinctly brittle zone at 4.95 per cent of nickel and extending to 16 per cent. From this point there was a recovery of the ductility.

The results of the present research which are most nearly representative of the effect of nickel alone, indicate the same brittle zone with the least ductility at 11 per cent of nickel in the unannealed state and at 13 per cent after annealing.

This agrees fairly well with the observations of Hadfield and Guillet, to whose materials our compositions are most nearly similar. The location of this brittle area can hardly be definitely fixed, since differences in impurities and heat treatment no doubt affect the composition and the range.

CONCLUSIONS

As a result of this research, taken in conjunction with the other tests enumerated, and of which it is largely confirmatory, it may be said that the effect of the addition of nickel to iron is as follows:

1. To increase the strength with a slight decrease of ductility in the range of lower nickel content.

2. Beyond this range the addition of nickel causes a sudden increase of strength with a marked decrease of ductility over a zone of decided brittleness.

3. The position of the brittle zone varies with the carbon content and probably with a variation in the other impurities, such as manganese, if present in appreciable amounts. For a pure alloy, where the effect is due to nickel alone, the area may be set between 10 per cent and 16 per cent of nickel. The addition of carbon places the zone at lower percentages of nickel, beginning in the neighborhood of about 10 per cent for carbon = 0.22 per cent and at about 7 per cent nickel for carbon = 0.82 per cent, where the other impurities are small. It may commence as low as 5 per cent nickel in medium carbon steels (C = 0.44) where the manganese rises to 1 per cent. Accompanying this brittleness there is a marked hardening in the material.

4. Annealing while not greatly affecting the region of the brittle zone or the extent of the brittleness, has a tendency to confine the range to more narrow limits.

5. For percentages of nickel above those of the zone of brittleness there is a restoration of the ductility and softness.

IRON-COPPER ALLOYS

The influence of copper on iron and steel has for long been a subject of controversy and contradiction. It is only comparatively recently that the effect has been investigated systematically and some of the doubt eliminated.

The prevalence of appreciable quantities of copper in the ores of certain districts makes of value a knowledge of its effect, detrimental or otherwise, when present in small amounts. Also, the intimate relation of copper, nickel and iron, and the beneficial effect of the addition of nickel to iron and steel, creates an interest in the influence of larger percentages.

The older opinion was that copper is deleterious; that its chief effect was analogous to that of sulphur in that it rendered iron red short and destroyed its welding power. This view was taken by many eminent metallurgists, some even contending that 0.5 per cent makes steel worthless. Others claimed that the influence was greatly exaggerated and instances were cited where rolled sections of 0.50 per cent to 0.75 per cent of copper did not display red shortness.

Within recent years more systematic investigations have been made to determine the effect of additions of varying percentages of copper to iron and steel.

Ball and Wigham (Ir. & St. Inst., 1889, No. 1) tested four steels with from 0.85 per cent to 7.17 per cent copper and a carbon content varying from 0.10 per cent in the former to 0.71 per cent in the latter, and noted an increase of tensile strength and a decrease of ductility with the increase of copper. The bar with 7.17 per cent copper was red short. Their conclusions were that the principal effect of copper was to make steel hard and that copper within reasonable limits did not materially affect the mechanical properties.

A. L. Colby (Iron Age, Nov. 30, '99) says that small percentages of copper have no deleterious effect upon the physical properties of steel. He cites various steels of about 0.5 per cent copper in shafts and gun tubes, which meet the requirements of the United States Navy. It was also used in ship plates, passing the usual tests required of carbon steels. These steels welded successfully and flanged cold, and there was no red shortness in bars or rails with a copper content of 0.39 per cent to 0.49 per cent.

W. Lipin (Stahl u. Eisen, Vol. XX) states that iron with copper content up to 3 per cent is readily worked but that there is red shortness at 4.7 per cent copper. Between 7 per cent to 10 per cent copper the material cracked badly, and fell to pieces under the hammer. With an increase of copper up to 3 per cent the tensile strength increased from 26 tons per square inch to 46 tons per square inch, with a decrease of elongation from 27.8 per cent to 13.3 per cent. He also notes that with an increase of carbon in the steel the maximum percentages of copper must be decreased; also that copper does not affect the welding.

Stead (Ir. & St. Inst., '01, Vol. I.) made a very thorough investigation of the influence of copper in steel rails and plates, since, he claimed, the general belief that copper was deleterious, was an unjustified prejudice. His article goes fully into the previous work and impressions. He finds that copper between 0.5 per cent and 1.5 per cent is not detrimental either hot or cold; that 2 per cent copper makes steel more liable to be overheated; that small quantities raise the tenacity and the elastic limit and reduce the elongation, but not greatly, however, for small percentages of copper; also that there is no great liability to fracture by shock.

Papers of Stead and Wigham (Ir. & St. Inst., '09, Vol. II) and of Wigham (Ir. & St. Inst., '06, Vol. I) deal with the effect of copper in cold wire drawing and come to the conclusion that copper up to 0.25 per cent is no detriment in the manufacture of the best classes of wire.

Wigham, in the last article mentioned above, brings out the important point that Professor Turner, in the discussion of the former article of Stead and Wigham, says that where copper is present in ores it is found in association with sulphur. Consequently discrepancies in the earlier tests may be due to this sulphur and not to the direct effect of the copper. This fact, together with that of the increase of red shortness with an increase of carbon, may explain the differences of opinion resulting from the earlier observations.

All of the above research was to excuse the presence of the copper; to break down a seeming prejudice. An investigation with a different object is that of Pierre Breuil, the results of which were given in a paper before the Iron and Steel Institute (Ir. & St. Inst. '07, No. 2). His effort was to see if there was a beneficial effect due to the addition of copper and was suggested by the favorable influence of copper on steels for railway axles, which was noted on some of the French railroads. It is a very extended research, and as the author says, is a supplement (very much elaborated) to Stead's work, in that the results agree. Being the most exhaustive and latest contribution to our knowledge of copper steels, we shall have occasion to refer to the results frequently throughout this discussion.

Breuil's work was carried out on four series of steels,—mild steel ($C = 0.10$ per cent — 0.17 per cent), semi-mild steel ($C = 0.28$ per cent — 0.41 per cent), and hard steel ($C = 0.56$ per cent

—0.75 per cent) and a final series with about 1 per cent of carbon.

In view of the evidence that the influence of copper varies with the amounts of carbon and of sulphur present in the steel, and very likely also with the other impurities met with in commercial materials, it is of interest to record the results of tests where these elements are a minimum. The following discussion is of a series of iron-copper alloys made with electrolytic iron and copper as a basis. The preparation of the test samples is described in the introductory chapter of this paper.

FORGING

A wide range of alloys of varying copper content was made. The observations made in the forging are as follows: Alloys up to 2 per cent of copper forge well at low heats. Those from 2 per cent to 7 per cent will not forge at a low heat, and rather poorly at white heat, the ease of workability varying inversely as the percentage of copper. From 7 per cent to 75 per cent—80 per cent the alloys may be classed as non-forgeable. Between 80 per cent and 100 per cent they will forge at a fair red heat but not at a normal forging heat for iron.

In the earlier work alloys above 5 per cent could not be forged. However, in later tests, in the trial of many alloys of 5 per cent to 10 per cent of copper at all heats, it was found that with care and a high heat (welding) a 7 per cent bar could be forged, or even rarely, an 8 per cent.

The lower percentages (below 5) will weld easily in forging, and some bars of 7 per cent copper were welded while forging. All alloys up to 7 per cent copper could be welded in an electric welder.

SEGREGATION

The forgeable samples between 0 per cent and 8 per cent of copper and a few of the alloys of high copper and low iron were made into test bars for investigation of the tensile strength. Analyses were not made upon all bars; rather, random samples of the entire range of composition were tested, with the following results:

ANALYSES.

Mark.	Cu. added.	Analysis.
147B	0.1	0.089
158A	0.2	0.202
158B	0.4	0.422
158C	0.6	0.592
158D	0.8	0.804
147H	1.0	1.006
147J	1.5	1.510
158G	2.0	2.005
158I	4.0	3.990
158J	5.0	5.070
158K	6.0	6.160
147U	7.0	7.050
86F	95.0	94.340

As will be seen from the tables, the agreement between the added and the actual copper content is exceedingly close. The analytical samples were taken from turnings obtained in the machining of the test bars after the forging of the ingots. This close agreement after such a method of selection would indicate an entire lack of segregation; also that the materials alloy very well up to a copper content of 7 per cent. This is in accord with the work of earlier investigators. In view of this close agreement it may be assumed that the actual copper content of the alloys is the same as the added amount.

HARDNESS

The hardness tests are merely qualitative, being observations made in the machining, sawing and filing. In machining, the alloys with low percentages of copper worked very easily. The hardness increased with the increase of copper until at 5 per cent to 7 per cent, while it was still possible to turn in the lathe, this was done with some difficulty. There were the same evidences in the sawing and filing tests, the high copper being classed as extremely hard to saw.

Breuil's investigations show that the hardness does not result through a lowering of the points of transition, thus leaving the steels in a martensitic state, as might be supposed. The structure is a fine pearlite, with a fibrous cementite which is liberated with increasing copper, and this explains the increasing hardness.

Wedding (Stahl u. Eisen, December, '06) says that copper and sulphide of copper prevent the formation of pearlite and promote the formation of crystals of cementite. It is due to this fact that there is greater hardness of iron when copper and sulphur are present. These explanations seem hardly sufficient to account for the hardening which we have observed without the presence of either carbon or sulphur and we expect to make a metallographic study with the hope of throwing a little light upon the subject.

The summary of the results of these tests is given in Tables 3 and 4, and in Plates III and IV. The detailed data are not given; also no account is taken of bars which showed abnormal results, due to flaws, etc.

In conformity with those of the nickel-iron series, the results enumerated are the yield point, the ultimate strength, the percentage of elongation in 2 inches and the percentage of reduction of area at fracture. Again, in order to show the consistency of the data there is also given the maximum and minimum values for each of the above items, together with the average of these items for the number of samples enumerated in a separate column. The plates are plotted from the averages of the tables and smooth curves drawn to indicate as nearly as possible the tendency of the increasing additions of copper. The values of the yield point, ultimate strength, elongation and reduction of area are designated as points used in plotting the curves by the same symbols used for the nickel alloys.

UNANNEALED

From the curves of Plate III, it will be noted that the rise in the ultimate strength and the elastic limit is almost a linear function of the percentage of copper. The ultimate strength increases from 61,180 lbs. per sq. in. at 0.1 per cent copper to 132,400 lbs. at 7 per cent. The yield point rises from 52,580 to 122,900. There is a corresponding fall in the elongation from 28½ per cent at 0.1 per cent copper to 4 per cent at 7 per cent copper. The reduction of area increases slightly from 69.2 per cent at 0.1 per cent copper to 72.1 per cent at 0.6 per cent and 0.8 per cent copper; from this point it falls off to 7.3 per cent at 7 per cent of copper.

From Table III it will be noted that the ratio of maximum and minimum values to the average is good, except in certain instances which are no doubt somewhat abnormal, but not so much so as to be rejected from the tables. Also, where there are a sufficient number of samples for a good average, the results fall well in line. The greatest variation is at 3½ per cent of copper, but here there is only one bar, which is clearly abnormal, with its low ultimate strength and elastic limit and its high reduction of area. The greatest variations from the curve lie above 4 per cent of copper, and this is to be expected, especially in the unannealed samples, since here we are almost beyond the limit of satisfactory workability.

The results of these tests indicate a high tensile strength which increases with the percentage of copper. The alloys are rather brittle as indicated by the elongation, and the reduction of area. Also the elastic ratio (ratio of the elastic limit to the ultimate strength) is large, varying from 0.86 per cent at 0.1 per cent copper to 0.93 at 7 per cent copper.

ANNEALED

The results of annealing are given in Table 4 and Plate IV, and the effect is very marked, especially for the higher percentages of copper. There is greater consistency in the results, as is to be expected, and the points fall fairly well in line on the curves.

The greatest deviation from the curves is at 2½ per cent and 5½ per cent of copper, where there was only one bar of each for the test. The maximum and minimum figures of the table are not widely different.

Up to 1 per cent copper there is a marked increase of the elastic limit and the ultimate strength with the additions of copper and no falling off in the elongation or reduction. In fact, the latter increases to a maximum at 0.4 per cent of copper. Beyond one per cent the curves bend sharply and become more nearly horizontal, being a linear function of the percentage of copper. The elongation and reduction of area fall off in accordance.

Between 1½ and 7 per cent of copper the elastic limit increases from 51,570 to 56,950 lbs., the ultimate from 65,720 lbs. to 67,900, while there is a decrease in the elongation and the reduction of area. Between 0.1 per cent and 1.5 per cent of copper the elastic limit rises from 35,570 lbs. to 51,570 lbs; the ultimate strength from 54,050 lbs. to 65,720 lbs; while the elongation and the reduction of area remain about the same. Breuil mentions the 4 per cent alloy as worthy of further study, and this is no doubt true, since he observes very high values, as indicated by the following table, where the results are given for his mild steel series (C = 0.10 per cent to 0.17 per cent), as rolled, and as annealed at 900° C.

Tensile Strength—Breuil.

Per cent Cu.	Yield point No. 1 per in.^{2}	Ultimate stress No. 1 per in.^{2}	Elongation Per cent.	Reduction area per cent.
Unannealed:				
0	....	....	....	....
0.5	55.000	66,800	25.5	66
1.0	58.900	70.200	26.5	60
2.0	67,300	88.700	16.0	58
4.0	97,600	109,500	13.0	46
Annealed:				
0	35,800	54,900	30.3	63
0.5	38,200	59,000	28.0	60
1.0	54,800	69,500	26.0	57
2.0	58,700	70,300	25.0	58
4.0	65,000	71,500	22.0	63

A comparison of these figures with the results of our researches shows a great similarity, and in plotting the figures there is a very close agreement. In his 4 per cent alloy as unannealed, to which Breuil calls particular attention, there is an elastic limit of 97,600, an ultimate strength of 109,500, and an elongation of 13 per cent and a reduction of area of 46 per cent. In our tests the results for corresponding materials are—elastic limit, 100,560; ultimate strength, 108,640; elongation, 13½ per cent; reduction of area 45.6 per cent. For the annealed sample, Breuil gives elastic limit 65,000 lbs; ultimate strength 71,500, elongation 22 per cent, reduction of area 63 per cent. Our results are; elastic limit 53,570 lbs., ultimate strength 66,540, elongation 24.8, reduction of area 54.2. In our tests the annealing shows a greater effect, resulting in less strength with greater ductility; and the same is true in the comparison for all percentages of copper.

While Breuil lays particular stress on the 4 per cent alloy, our results would seem to indicate the greatest value between 1 per cent and 2 per cent for the annealed samples. The very sharp rise of the elastic limit and the ultimate strength at this percentage gives values but very little less than those for higher copper content, with a lessened cost for the addition element; also there is a removal from the region where the forging and welding properties are poorer with consequent uncertainties of result. Likewise, between 1 per cent and 2 per cent the elastic limit and the reduction of area are very high.

It would hardly be advisable to work with 1 per cent of copper, since this is on the edge of the incline, where there is liable to be a fall to the lower values obtained with lower percentages of copper. At 1½ per cent we are far enough removed so that the slight differences of composition to be met in practice would not bring the material into a dangerous region. Also, the elastic ratio of 0.78 is slightly lower than for the 4 per cent alloy, where it is 0.81.

We would have considerable hesitation in bringing forward this point without a vast number of tests for confirmation, but for the fact that an observation of Breuil's results indicate the same condition. He has perhaps noticed this fact, but makes no mention of it in his article. As will be seen from the table of his results which is given above, there is a sharp increase between 0.5 per cent and 1 per cent of copper, where the elastic limits are 38,200 and 54,800, respectively, and the ultimate strengths 59,000 and 69,500. Again, we note but little change in the elongation and reduction of area. Between 0 per cent and 0.5 per cent, and between 1 per cent and 2 per cent, the change is not marked. The fact that two independent investigations give this result, seems fairly conclusive of the effect.

Another fact to be noted in our tests is the marked difference between the unannealed and the annealed samples. It would indicate that there is an intermediate heat treatment which, not being so drastic in its effect as our long annealing, would give intermediate values, and thus approximate the nickel steels. This condition might be reached in commercial rolling, where heavier masses of metal leave the material in a condition analogous to that resulting from a partial anneal.

To compare the copper-iron with the nickel alloys we may again quote Waddell's extensive investigations mentioned previously under the nickel-iron series. Repeating this data, we note a steel with about 3½ per cent nickel, carbon 0.38 per cent, the values for which may be taken as: elastic limit 60,000 lbs. per sq. in., ultimate strength 105,000 lbs. per sq. in., elastic ratio from 0.55 to 0.60. These values are for sections as rolled without heat treatment and working at a somewhat low ductility, as shown by the figures 15 per cent for nickel steel, 27 per cent for carbon steel (in 8 inches) and a reduction of area of 41 per cent for nickel, compared to 55 per cent for the carbon steels. With the carbon reduced to 0.15, the values become: elastic limit 45,000, ultimate 70,000, elongation 25 per cent; and for (C = 0.45) the elastic limit is 65,000, the ultimate 115,000 and the elongation 12 per cent.

These values are higher than ours for the copper-iron alloys but are not in direct comparison. Our own figures for a nickel-iron alloy made under the same conditions as the copper-iron series, are given in the following table, where the 4 per cent nickel is compared with the 1.5 per cent and 4 per cent copper alloys. There is also given Hadfield's test of an alloy of 3.82 per cent nickel and carbon = 0.19 per cent (Inst. Civ. Engrs., March 28, '99).

	Elastic limit.	Ultimate strength.	Elastic ratio.	Elongation per cent	Reduction of area per cent.
UNANNEALED:					
4.00 Ni	67.000	76,000	0.88	28.5	68.6
1.50 Cu	73.920	77,300	0.95	23.5	66.9
4.00 Cu	105,560	108,640	0.97	13.5	45.6
3.82 Ni	62.720	82,800	0.76	30.0	54.0
ANNEALED:					
4.00 Ni	57,000	69.000	0.83	26.5	64.8
1.50 Cu	51,570	65.720	0.79	29.2	63.1
4.00 Cu	53,570	66,540	0.81	24.5	54.2
3.82 Ni	56,000	73,920	0.76	35.0	55.0

The above values for the nickel alloys are less than are commonly given to commercial materials where the carbon is a factor. The results are very comparable to those of the copper-iron series made under identical conditions.

Waddell, in his paper, shows that there is an economic advantage in the use of the nickel steels in comparison with the carbon steels ordinarily used, since their increased cost is more than offset by the less weight required, due to their increased strength. Our results and comparisons would indicate that the copper-iron alloys are also worthy of consideration and might be comparable to the nickel steels in use, even if the strength should not reach such high values as those of the nickel. A 1½ per cent copper alloy is of promise, since the smaller percentage required and the lessened cost per pound of copper as compared with nickel would result in a lessened cost of construction, even if there is some increase in tonnage required because of the slightly decreased strength per equal weight.

NICKEL-COPPER-IRON ALLOYS

In view of the well known beneficial effect of the addition of nickel to iron, and because of the very great and rather unexpected increase of tensile strength observed in our tests of the copper-iron alloys, due to the increasing percentage of copper, it was thought advisable in order to round out this research, to carry out tests on a series of alloys in which both nickel and copper were added to the electrolytic iron.

The advisability of the investigation was prompted by several considerations—first, the benefits observed by the separate additions of nickel and copper might be coupled in their joint use; second, the close relationship of copper, nickel and iron in chemical and physical properties might lead to interesting results in a ternary alloy; third, and this was the reason of greatest weight—there is on the market a nickel-copper alloy which would make a very desirable addition agent, in case the simultaneous presence of both copper and nickel give a tensile strength no less than that observed for the two separate alloys.

We refer to Monel metal, resulting from the reduction of certain ores from which, by smelting alone, this alloy is formed. The main constituents are nickel and copper, in the proportions of about three to one, respectively. The material is, for the present at least, obtainable at a price about equal to that of copper, and very much below the cost of nickel. Here, then, is a means of getting nickel additions for steel, provided the copper carried does not neutralize the effect of the nickel.

Some particulars regarding Monel metal are given below.

ANALYSIS.

Nickel	66.90	67.96
Copper	24.35	26.00
Iron	5.00	2.80
Manganese	2.18	1.62

Mechanical tests gave for cast samples ultimate strengths of 30,400 and 35,000 lbs. per sq. in. For rolled samples the figures were—elastic limit 74,400 and 79,000 lbs. per sq. in., ultimate strength 100,000 and 104,000 lbs. per sq. in.

Thus we see that the alloy has very good inherent mechanical properties, and the analyses indicate, besides the nickel and copper, only iron and manganese, and the latter is no detriment to steels.

Examining the above table of analyses, we note a nickel-copper ratio of about 2½ or 3 to 1. The desirable addition of nickel to steels is from 3 per cent to 4 per cent. Our tests give as a desirable amount in the iron-copper series about 1½ per cent of copper. Comparing these, we see here a ratio of about 2½ or 3 per cent to 1, the Monel proportions. This striking analogy prompted the following series, in which the nickel was varied from 2 per cent to 6 per cent, with the copper in each case in a 3 to 1 ratio. Unfortunately, at the time the alloys were made, no Monel metal was on hand, and they were made by adding the proper amounts of copper and nickel. The preparation of the alloys and test samples was in all respects in conformity with the methods described previously in this paper.

The series prepared was as follows—

Bar.	Per cent Nickel.	Per cent Copper.
174M	2	0.7
174N	3	1.0
174O	4	1.3
174P	5	1.6
174R	6	2.0

Analyses have not been made to confirm these percentages. However, in view of the close agreement in both the nickel and the copper series, the actual amounts are probably very close to those indicated above.

The whole series forged without difficulty at normal heats, and could be worked at low temperatures also, without indications of red shortness in any instance.

For the tests, three samples were cut from each forged rod, of which two were annealed at 900° C, and one left unannealed. Before annealing, the bars were machined to a diameter of about 3/8 inch over a free length of 2 inches. This work was done without difficulty in the lathe, except for Bars 174P. One of these samples was finished in the lathe, showing no material hardness; the other two, however, had a hard seam which made it necessary to grind them to size. Subsequently, both bars showed flaws in the tests.

TENSILE STRENGTH TESTS

The results of the tests are indicated in Table 5 and in Plates V and VI. In the plates the yield point, maximum stress, elongation and reduction of area are plotted for each composition, the same designating symbols being used as noted in the previous tests. No effort has been made to draw a smooth curve following the points; instead, they have been connected by straight lines. This was done because of the small number of tests of each alloy.

UNANNEALED

But one bar of each composition was available for test in the unannealed condition. The results, however, are fairly consistent, and reference to Plate V shows an increase of ultimate stress and elastic limit with increase of the alloying elements, from 79,200 to 99,700 and from 62,100 to 81,700, respectively, for the limiting alloys of 2Ni 0.7Cu and 6Ni 2Cu. The elongation and reduction of area fall off in accordance, but the extreme values of 22 per cent to 28 per cent, and 39.3 per cent to 58.8 per cent, respectively, are very good, and indicate an absence of any brittleness. Even the minimum addition of 2Ni 0.7Cu gives the very high values of elastic limit 62,100, ultimate stress 79,200, elastic ratio 0.78, elongation in 2 inches 26 per cent and reduction of area 56.7 per cent.

ANNEALED

Two bars of each composition were annealed at 900° C for several hours. The tabulated results show very great consistency, not only for the two bars of each alloy, but also in the tendency of the increasing additions, best shown in Plate VI.

Annealing has had but little effect on the bars, except at the lower end of the series, where it has decreased the elastic limit and ultimate strength, with a corresponding increase of elongation and reduction of area. The values for the composition 2Ni 0.7Cu are, however, still good, averaging in ultimate stress 68,700, yield point 48,600, elongation 30 per cent, reduction of area 61.5 per cent. For the higher percentage additions the strengths have been increased by annealing and, as is shown most clearly by the curves of Plate VI, the increase of strength has been uniform with increase of nickel and copper, and rises to the extremely high values of elastic limit 83,200 and maximum stress 103,700, for the highest percentage of 6Ni 2Cu. Elongation and reduction fall off with the increase, but here again we have values denoting good ductility, with no sign of brittleness.

COMPARISONS

In order to make direct comparison of the effects of the addition of nickel and copper to iron in the ternary alloys, with those of the separate additions of the two elements in the binary series, Table 6 has been prepared. In this table the average yield points, ultimate stresses, elongations and reductions of area are noted in columns for the nickel-copper-iron, nickel-iron and copper-iron groups. The percentage compositions are given in the order of the ternary series; for the binary series the values are those from Tables 1, 2, 3 and 4, corresponding to the percentages of the separate elements in the two columns of compositions of Table 6.

The comparability of the copper-iron and nickel-iron groups is clearly shown, indicating the facts noted in the previous discussion of the separate groups. The values, line for line, are not markedly different, even though we are comparing directly proportions of nickel to copper in the ratio of 3 to 1.

The effect of the double addition of nickel and copper is most striking. In all but one or two instances the values for the ternary alloy exceed those for either binary series, at both the elastic limit and the ultimate stress. And the effect is even more marked after the annealing of the bars. In fact, one might almost treat the copper as so much added nickel, and compare bars on the basis of a certain nickel content on the one hand, against an equal quantity of nickel plus copper in the ternary alloy. And even this will not do for the higher percentages of the nickel-copper-iron alloys, since, especially for the annealed bars, the strength of the ternary alloys is very markedly greater than any of the binary series. These high strength values are reached with practically no diminution of ductility, as is indicated by comparison of the figures for elongation and reduction of area.

For those annealed bars corresponding to a composition of 4 per cent nickel plus 1.3 per cent copper, which in the discussion of the binary series we mentioned as the percentages in each case of greatest utility, we find the same relation holding as in the rest of the alloys. The values for the ternary alloy are: elastic limit 67,600, maximum stress 84,700, elongation 28 per cent and reduction of area 55.6 per cent. For a binary 4 per cent nickel alloy the figures are: elastic limit 52,100, ultimate stress 70,100, elongation 28 per cent and reduction of area 66.7 per cent; and for a 1½ copper alloy: yield point 51,600, ultimate 65,700, elongation 29 per cent, reduction of area 63.1 per cent.

These tests, therefore, while not as extensive as would be desirable to draw absolute conclusions, are sufficiently consistent to warrant the statement that for carbon-free materials, at least, the simultaneous presence in iron of nickel and copper does not destroy the good effects of the separate percentages of each element in a binary series. On the contrary, there is apparently an increase of tensile strength without loss of ductility, and no evidence of a brittle zone throughout the series.

How far the percentages may be increased before there is evidence of brittleness or red shortness, is problematical.

While all of our alloys have been made of iron practically free from carbon, the combined influence of nickel and copper has appeared so marked as to warrant further investigation. It also appears highly important that similar study be made of the effect of these alloying agents upon commercial steels with their customary percentages of impurity.

TABLE 1.

TENSILE STRENGTH—IRON NICKEL ALLOYS—UNANNEALED

		YIELD POINT lbs/sq. in.			ULTIMATE STRESS lbs/sq. in.			ELONGATION PER CT′2″			REDUCTION OF AREA PER CT.		
% Ni	No.	Max.	Min.	Av.	Max.	Min.	Av.	Max	Min.	Av.	Max	Min.	Av.
0.25	3	66.600	48,250	57,180	75,600	55,600	66,730	28.5	22.0	25.2	65.7	50.0	57.6
0.50	2	58.200	55,400	56,800	70,700	67,700	69,200	21.5	19.0	20.2	62.8	58.8	60.8
1.00	4	58.200	51.300	56,820	72,300	66,200	68,320	32.5	18.5	25.2	68.5	53.0	60.9
2.00	4	63.000	55,000	58,520	74.700	67,300	70,350	33.5	17.0	26.0	65.5	43.7	60.7
3.00	5	67,500	56.700	62,920	79.500	67,900	74,860	29.0	17.5	24.5	65.0	62.4	63.9
4.00	3	68,900	61,000	65,000	75.500	75,200	75,670	29.5	26.5	27.8	68.8	62.2	66.5
5.00	4	71.800	59.700	64.030	78,700	73,800	77,150	32.0	26.5	29.2	69.7	59.7	65.3
6.00	1	....	....	67,300	....	....	77,300	.....	.....	29.0	.....	.....	68.5
7.00	4	82,750	63,200	70,080	93,100	79,800	85,500	28.0	18.5	25.2	66.1	57.3	61.0
8.00	4	78,300	65,700	70,770	98,000	83,900	91,740	31.0	14.5	22.6	64.8	49.1	56.1
9.00	5	113,800	69,800	80,760	136.000	97,700	112,700	24.0	8.0	18.4	63.4	45.4	57.7
10.0	5	141,400	89,400	118,960	172,900	120,500	153,360	15.5	2.0	9.1	53.7	2.9	29.0
10.5	3	150,000	104,900	134,470	188,800	123,600	166,200	10.0	4.0	6.2	48.5	3.5	24.9
11.0	7	166.800	123,400	148,240	205,000	165,200	181,230	9.5	2.0	7.0	55.7	1.4	25.5
12.0	5	168,800	116,700	141.140	195.800	150,000	178,420	13.0	2.5	7.1	43.5	2.2	23.8
13.0	4	152,000	116,500	130,620	203,500	129,500	171,500	10.5	3.0	6.9	70.5	9.8	43.4
15.0	2	133.500	116,500	125,800	211.000	167,000	184,730	14.5	4.5	8.2	58.7	12.1	34.1
18.0	3	148,800	126,800	138,170	183.500	176,000	178,500	13.5	10.5	12.3	60.0	38.5	49.3
19.0	3	149.000	126,200	134,730	189.500	175,800	183,270	15.0	13.5	14.2	59.6	46.8	53.1
20.0	1	....	....	115,500	....	....	186,100	.....	.....	16.0	.....	.....	59.5
21.0	2	143,700	131,700	137.700	181,700	180,000	180.850	14.0	8.0	11.0	48.9	25.1	37.0
25.0	1	....	....	56,400	....	....	104,200	.....	.....	45.0	.....	.....	68.3
45.0	1	....	....	54,600	....	....	82,000	.....	.....	32.5	.....	....	65.0
50.0	1	....	....	75,700	....	....	108,500	.....	.....	26.5	.....	.....	63.7

TABLE 2.

TENSILE STRENGTH—IRON NICKEL ALLOYS—UNANNEALED

		Yield Point lbs/sq. in.			Ultimate Stress lbs/sq. in.			Elongation Per Ct 2".			Reduction of Area Per Ct.		
%Ni.	No.	Max.	Min.	Av.	Max.	Min.	Av.	Max	Min.	Av.	Max	Min.	Av.
0.25	2	40,200	35,900	38,050	51,100	49.600	50,350	36.5	33.5	35.0	74.4	65.0	69.7
0.50	2	43.300	37,700	40,500	59,200	51,700	55,450	37.5	35.5	36.5	70.3	70.1	70.2
1.00	3	44,400	43,300	43,830	63,500	57,200	59.830	36.0	29.5	33.7	73.5	66.5	68.8
2.00	3	51,200	46,400	48,770	66,600	63,100	64,400	39.0	31.0	34.2	69.1	58.6	65.4
3.00	3	61,100	44,400	55,400	74,900	65.700	70,670	30.5	23.5	27.5	73.3	62.8	67.6
4.00	3	57,800	47,400	52,100	73,300	67,900	70,070	32.0	22.5	28.4	70.3	63.4	66.7
5.00	4	62,800	52,800	58,320	77,100	70,750	73,230	32.5	29.0	31.4	70.3	64.5	68.1
6.00	1	....	....	56,400	....	....	75,300	.....	.....	29.5	.....	.....	64.2
7.00	3	58,600	57,300	58,130	79,100	74,000	72,970	31.0	19.0	26.7	67.5	43.2	58.5
8.00	3	67,250	61,300	63,320	82,600	73,400	78,470	33.0	27.0	30.2	68.8	58.3	64.4
9.00	3	74.700	70.750	72,180	96,300	84,300	89,970	29.0	21.0	25.1	65.4	50.3	60.2
10.0	3	81,200	66,400	71,800	98,400	80,700	89,300	24.0	20.5	22.3	61.5	54.7	50.1
10.5	3	71,900	67,500	69,100	92,750	86,200	89,450	25.5	22.0	24.2	61.0	50.5	57.2
11.0	5	127,200	69,800	99,160	167,500	95,750	121,810	23.0	0.0	11.6	65.2	0.0	34.6
12.0	3	111.300	80,000	97.730	142,400	118,200	121,800	16.5	11.0	14.5	47.2	8.3	35.8
13.0	4	140,700	114.800	128,050	177,000	148,500	161,370	14.0	2.0	6.8	51.6	1.4	18.2
15.0	2	142,000	110,600	126,300	180,500	126,100	153,300	16.5	4.5	10.5	66.5	4.8	35.8
18.0	2	146.900	128.500	137,700	182,100	181,000	181,550	15.0	5.0	10.0	58.9	9.9	34.4
19.0	2	120,800	111.500	116,150	185,200	176,200	180,700	11.5	10.0	10.7	45.6	31.9	38.7
20.0	1	....	....	111,000	....	....	124,900	.....	.....	21.5	.....	.....	62.5
21.0	1	....	....	122,600	....	....	193,700	.....	.....	4.0	.....	.....	4.2

TABLE 3.

TENSILE STRENGTH—IRON COPPER ALLOYS—UNANNEALED

Per cent Cu.	No. of samples.	Yield Point lbs/sq. in.			Ultimate Stress lbs/sq. in.			Elongation Per Ct., 2"			Reduction of Area Per Ct.		
		Max.	Min.	Av.	Max.	Min.	Av.	Max.	Min.	Av.	Max.	Min.	Av.
0.1	3	56,600	45,400	52,580	64.000	60,300	61,180	30.0	27.5	28.5	70.9	66.7	69.2
0.2	5	55,600	50,700	52,930	63,750	60.000	61,470	36.5	24.5	28.7	73.2	56.9	67.2
0.4	2	59,700	56,100	57,900	67,250	66.400	66,825	29.5	23.0	26.2	72.1	69.4	70.7
0.45	2	54,800	53,100	53,950	62,700	58,800	60,750	27.0	24.0	25.5	67.7	47.2	57.4
0.6	2	57,750	51,600	56,170	67,000	64,750	65,875	27.5	27.5	27.5	72.8	71.5	72.1
0.8	2	55,500	53,800	54,650	67.500	65,100	66,300	27.0	25.5	26.2	72.5	71.7	72.1
1.0	5	63,300	56,800	61,290	70,800	68,300	71,380	28.0	21.0	25.8	72.0	66.8	70.3
1.2	2	66,300	59,500	62,900	74,600	72,300	73,400	28.0	24.5	26.2	68.8	64.5	66.6
1.4	2	68,100	67,300	67,700	78,600	76,600	77.600	25.0	18.0	21.5	70.0	64.5	67.2
1.5	3	77,250	70,000	73,920	78,800	76,200	77,300	26.5	20.5	23.5	68.6	64.5	66.9
1.6	2	75,100	73,300	74,200	82,750	79,100	80,925	25.0	25.0	25.0	66.6	66.2	66.4
1.8	2	81.100	77,600	79,350	91,000	86,000	88,500	27.0	24.0	25.5	66.3	61.8	64.0
2.0	3	78,800	73,400	76,930	89,500	82,250	87,010	23.5	17.5	21.2	59.5	54.5	57.0
2.5	2	81,100	81,000	81,050	89,400	86,400	87,900	19.0	18.5	18.7	62.8	59.1	61.0
3.0	6	95,100	73,750	86,810	105,000	83,750	99.640	24.0	12.5	16.7	65.6	36.1	53.8
3.5	1	....	....	75,400	....	....	82,300	.....	.....	16.0	.....	.....	72.7
4.0	5	111,100	92,700	100,560	120,700	99,200	108,640	17.5	8.0	13.5	52.3	24.8	45.6
4.5	2	117,800	106,400	112,100	127,900	115,500	121,700	17.0	4.5	10.8	47.2	3.9	25.1
5.0	2	119,700	113,000	116,350	123.800	120,000	121,900	15.0	14.0	14.5	50.1	48.3	49.2
5.5	2	114,700	102,300	108,500	122,800	105,200	114,000	15.5	15.0	15.2	53.8	46.6	49.7
6.0	3	124,200	103,000	113,400	136,700	107.500	122,900	9.0	4.5	6.5	36.5	4.1	20.3
7.0	2	128,800	117,000	122,900	133,800	131,000	132,400	6.0	2.0	4.0	12.9	1.7	7.3
8.0	2	146,800	133,500	135,100	160,000	137,500	148,750	.....	.....	0	.....	.....	0

TABLE 4.

TENSILE STRENGTH—IRON COPPER ALLOYS—ANNEALED

Per cent cu.	No. of samples.	Yield Point lbs/sq. in.			Ultimate Stress lbs/sq. in.			Elongation Per Ct/2″			Reduction of Area Per Ct		
		Max.	Min.	Av.	Max.	Min.	Av.	Max	Min.	Av.	Max	Min.	Av.
0.1	2	40,750	34,400	35,570	56,500	51,600	54,050	35.0	28.0	31.5	62.2	58.3	60.2
0.2	4	36,850	31,800	33,660	56,000	52,700	54,020	36.0	29.5	34.1	67.7	62.7	65.6
0.4	2	35,700	35,100	35,400	53,700	53,100	53,400	37.5	34.0	35.7	72.1	69.3	70.7
0.45	2	42,250	41,250	41,750	66,300	64,600	65,400	26.0	25.0	25.5	53.6	46.0	49.8
0.6	2	38,400	38,300	38,850	56.600	56,500	56,550	36.5	32.0	34.2	69.0	68.8	68.9
0.8	2	41,300	40,700	41,000	59,750	57,400	58,570	34.0	32.0	33.0	65.1	64.7	64.9
1.0	4	54.800	46,400	48,920	63,900	61,100	62,350	34.0	30.0	32.6	68.3	66.1	66.8
1.5	6	58,100	48,100	51,570	70,500	62,100	65,720	33.5	22.0	29.2	65.1	59.3	63.1
2.0	6	54,300	49,400	51.620	65,100	60,500	63,200	31.5	27.5	29.5	68.1	62.0	64.4
2.5	1	....	....	56,200	....	....	73,500	.....	.....	23.0	.....	.....	47.3
3.0	7	53,800	48,700	50,890	67,600	59,000	62,360	33.5	10.5	26.9	69.1	46.3	61.7
4.0	5	57,100	49,600	53,570	70,900	62,700	66,540	28.0	22.0	24.8	65.3	39,3	54.2
5.0	4	57,600	50,700	54.460	74,700	62,400	68,250	29.0	20.5	24.2	62.9	29.1	50.3
5.5	1	....	....	54,800	....	....	69,300	.....	.....	25.0	.....	.....	60.4
6.0	3	56,500	52,750	54,420	65,600	62,500	64,530	24.5	18.0	20.8	59.0	39.2	49.9
6.45	2	55,750	54,600	55,170	69,700	69,200	69.450	25.5	16.5	21.0	48.1	30.5	39.3
7.0	2	57,150	56,750	56,950	70,900	67,900	69,400	20.0	14.0	17.0	40.8	21.7	31.2
7.5	1	....	....	60,200	....	....	79,000	.....	.....	16.5	.....	.....	34.2

TABLE 5.

NICKEL-COPPER-IRON ALLOYS—TENSILE STRENGTH

Bar.	% Ni.	% Cu.	Diam.	Area.	Stress/in². Yield point.	Stress/in². Maximum.	Elong. %/2''	Reduced Area.	Reduction of Area %	Remarks.
UNANNEALED.										
174M	2	0.7	0.372	0.1087	62,100	79,200	0.26	0.0471	56.7	....
N	3	1.0	0.374	0.1099	67,400	80,100	0.28	0.0452	58.8	....
O	4	1.3	0.373	0.1093	63,300	85,700	0.23	0.0594	45.7	....
P	5	1.6	0.375	0.1104	66,500	89,400	0.22	0.0670	39.3	....
R	6	2.0	0.374	0.1099	81,700	99,700	0.25	0.0511	53.5	....
ANNEALED.										
174M	2	0.7	0.371	0.1081	48,400	68,200	0.30	0.0391	63.8	....
	.....	.....	0.373	0.1093	48,900	69,200	0.30	0.0445	59.3	....
N	3	1.0	0.375	0.1104	56,400	73,600	0.35	0.0337	69.5	....
	.....	.....	0.375	0.1104	57,100	71,400	0.28	0.0456	58.7	Slight flaw.
O	4	1.3	0.373	0.1093	68,600	84,700	0.28	0.0437	59.9	....
	.....	.....	0.373	0.1093	66,600	84,700	0.27	0.0531	51.3	....
P	5	1.6	0.375	0.1104	73,100	83,400	....	....	....	(Machined hard.) Broke at flaw } Split.
	.....	.....	0.200	0.0314	75,200	89,500	....	....	....	Broke at flaw
R	6	2.0	0.374	0.1099	83,900	104,800	0.22	0.0452	58.8	....
	.....	.....	0.375	0.1104	82,500	102,700	0.21	0.0511	53.7	Short longitudinal split.

TABLE 6.

COMPARISON—IRON-NICKEL-COPPER ALLOYS

% Ni.	% Cu.	Yield Point.			Max. Stress.			% Elong.			% Red.		
		Ni-Cu	Ni.	Cu.	Ni-Cu.	Ni.	Cu.	Ni-Cu.	Ni.	Cu.	Ni-Cu.	Ni.	Cu.
UNANNEALED.													
2	0.7	62,100	58,500	54,700	79,200	70,300	66,300	26	26	26	56.7	60.7	72.1
3	1.0	67,400	62,900	61,300	80,100	74,800	71,400	28	24	26	58.8	63.9	70.3
4	1.3	63,300	65,000	67,700	85,700	75,670	77,600	23	28	21	45.7	66.5	67.2
5	1.6	66,500	64,000	74,200	89,400	77,150	80,900	22	29	25	39.3	65.3	66.4
6	2.0	81,700	67,300	76,900	99,700	77,300	87,000	25	29	21	53.5	68.5	57.0
ANNEALED.													
2	0.7	48,600	48,800	41,000	68,700	64,400	58,600	30	34	33	61.5	65.4	64.9
3	1.0	56,800	55,400	48,900	72,500	70,670	62,300	31	27	33	59.1	67.6	66.8
4	1.3	67,600	52,100	51,600	84,700	70,100	65,700	28	28	29	55.6	66.7	63.1
5	1.6	74,200	58,300	....	86,400	73,200	....	.....	31	.....	.....	68.1	.....
6	2.0	63,200	56,400	51,600	103,700	75,300	63,200	22	29	29	56.2	64.2	64.4

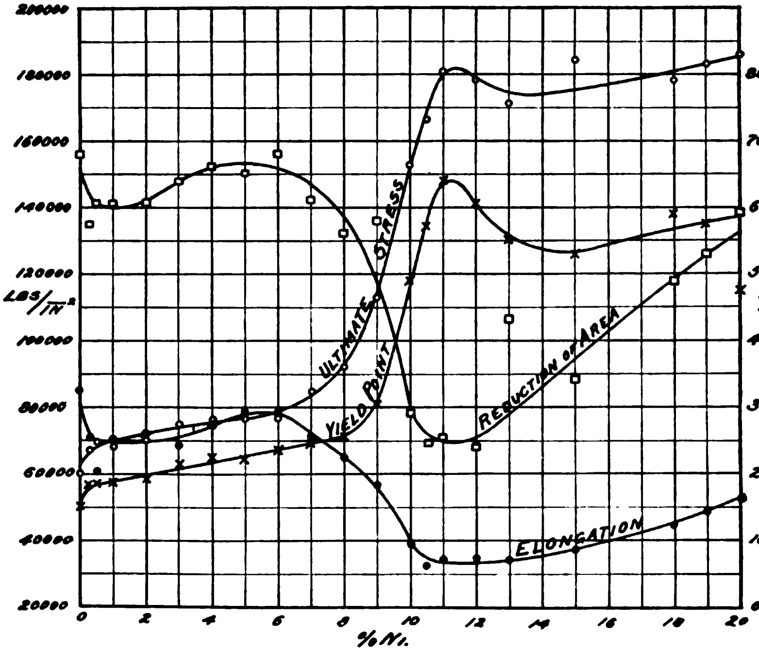

PLATE I

TENSILE STRENGTH IRON-NICKEL ALLOYS UNANNEALED

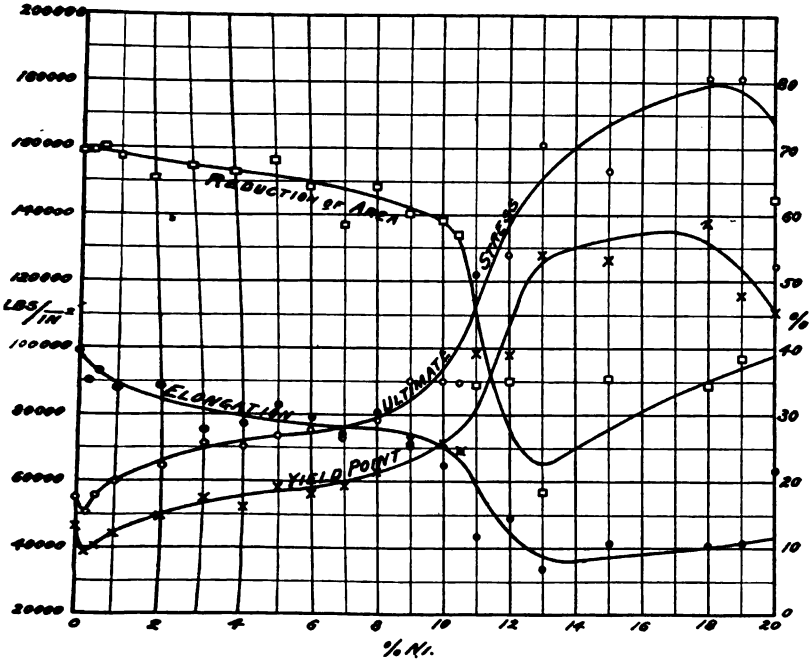

PLATE II

TENSILE STRENGTH IRON-NICKEL ALLOYS ANNEALED

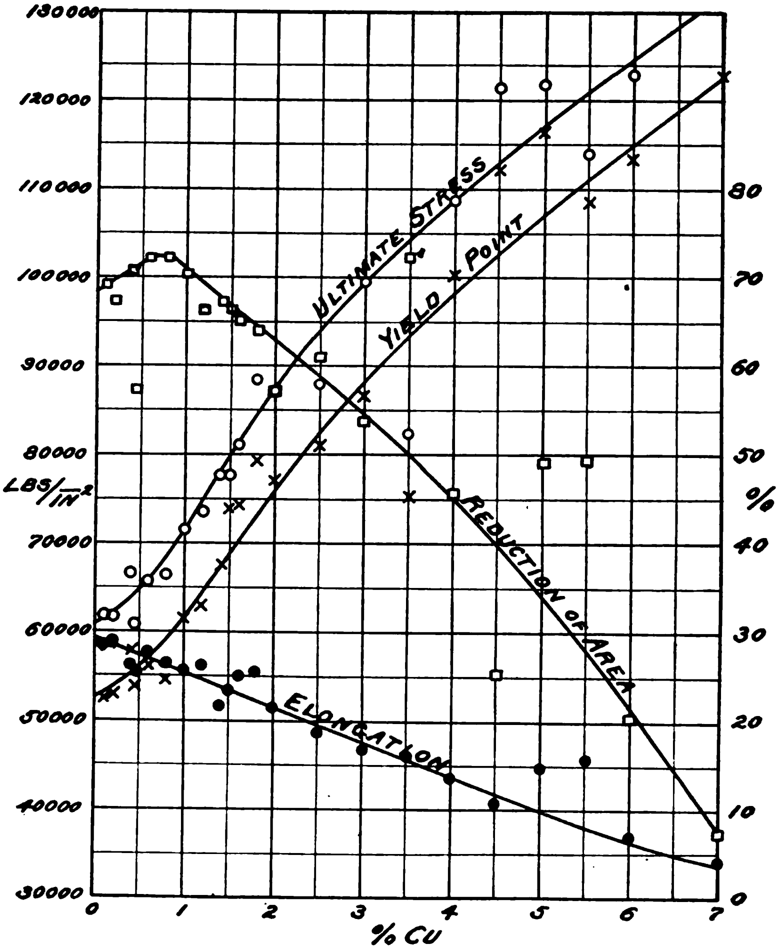

PLATE III

TENSILE STRENGTH IRON-COPPER ALLOYS UNANNEALED

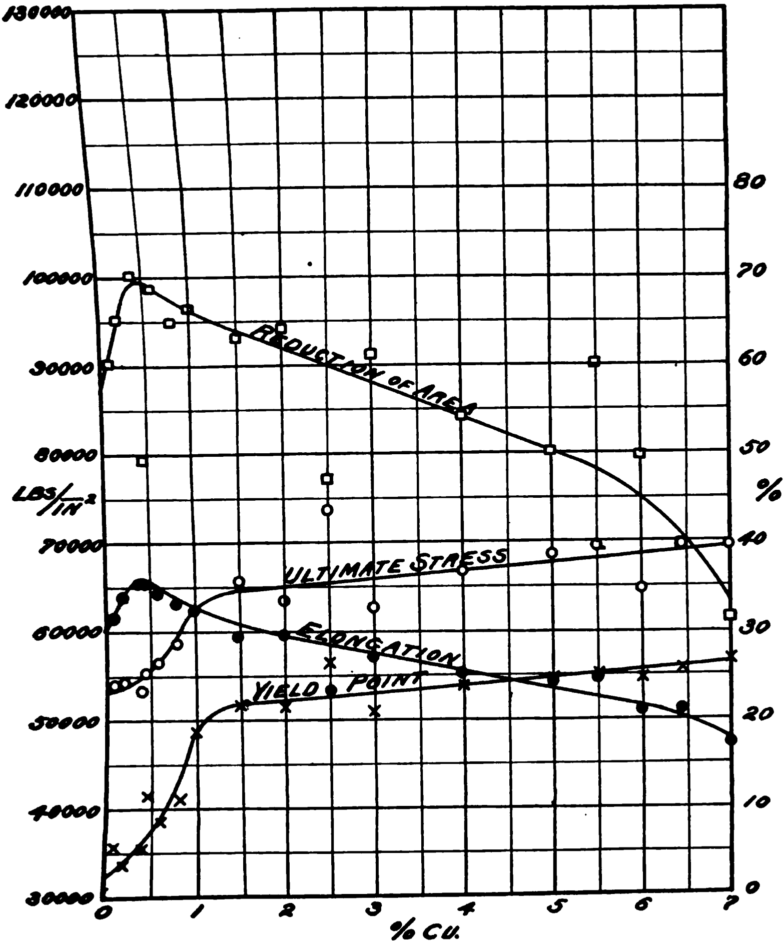

PLATE IV

TENSILE STRENGTH IRON-COPPER ALLOYS ANNEALED

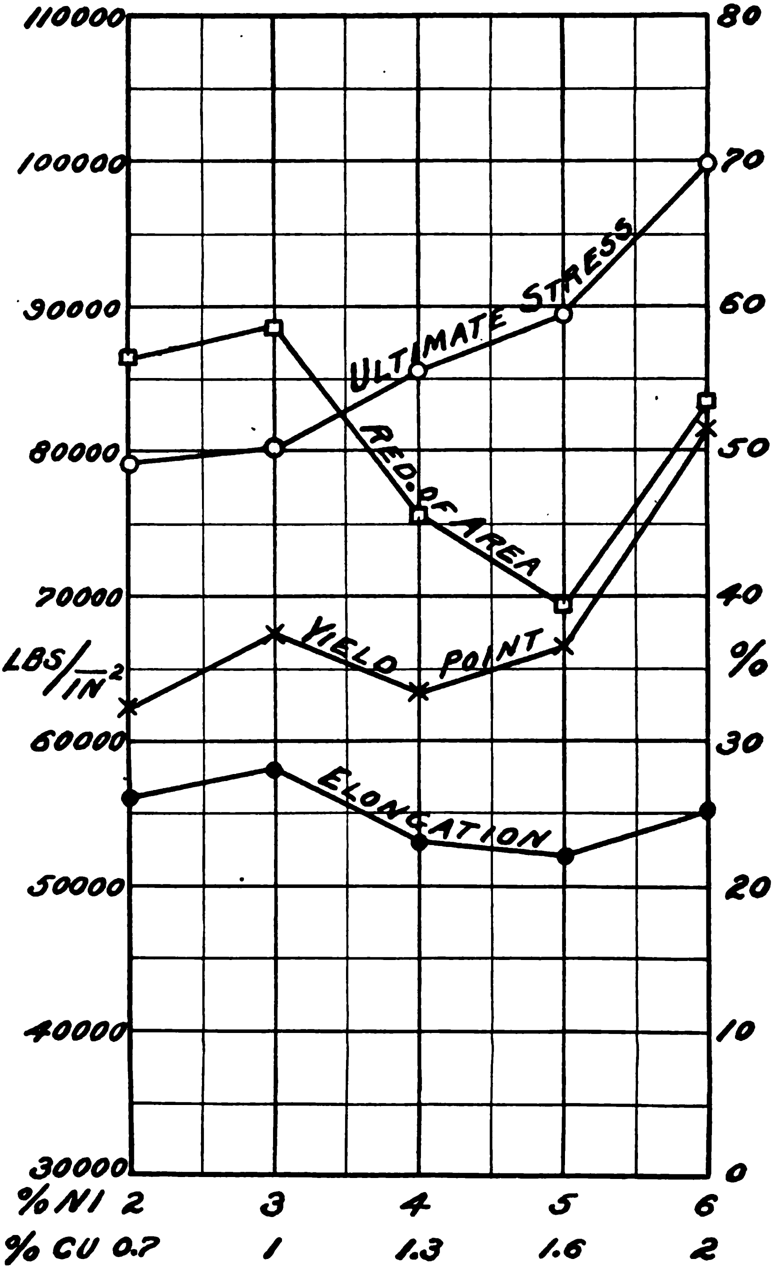

Plate V

Tensile Strength Iron-Nickel-Copper Alloys Unannealed

[78]

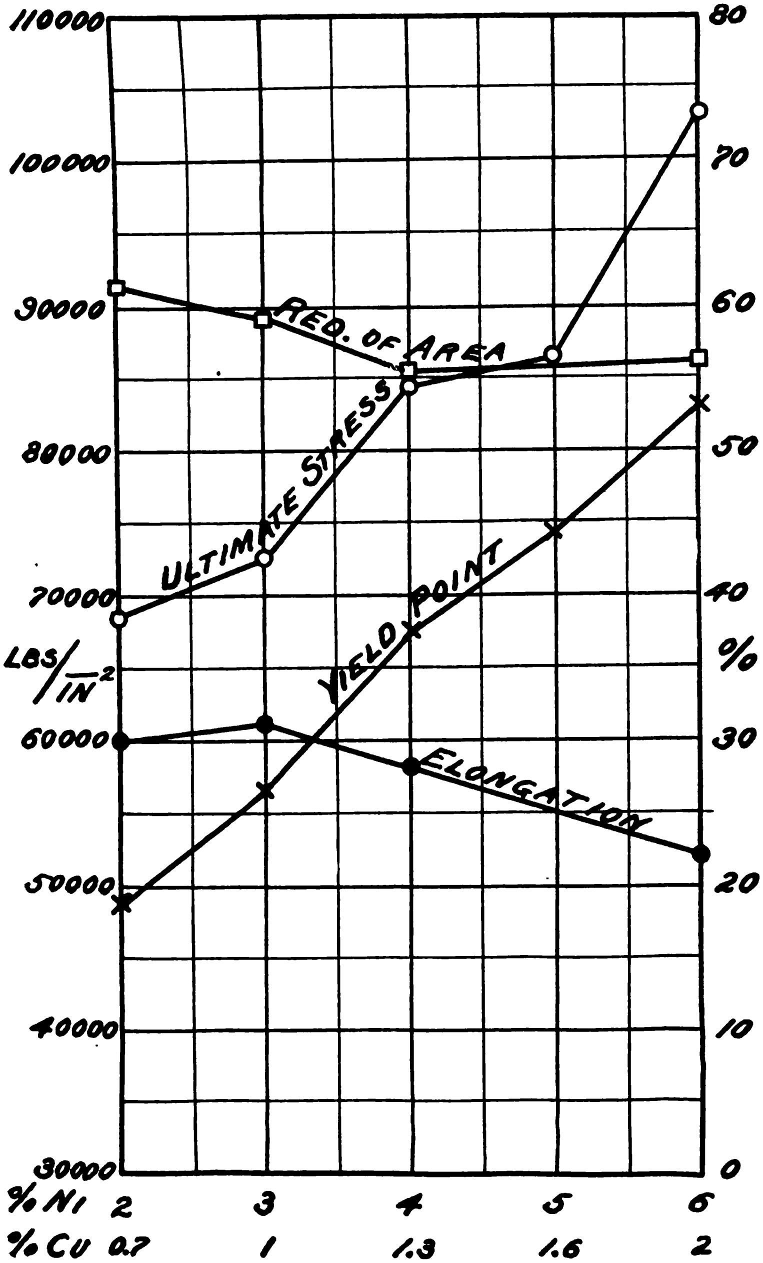

PLATE VI

TENSILE STRENGTH IRON-NICKEL-COPPER ALLOYS ANNEALED

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LONG DISTANCE TRANSMISSION OF STEAM AND ITS EFFECT ON POWER PLANT ECONOMICS.

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MADISON, WISCONSIN
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INTRODUCTION

The accompanying bulletin has been prepared for the purpose of calling the attention of engineers to the high efficiencies which may be procured in transmitting steam over considerable distances.

The results were compiled by Messrs F. R. Brownlee and W. C. Lindemann, in the preparation of their thesis for the degree of Bachelor of Science in the year 1908, and include a detailed statement of their test of a superheated steam pipe line 1,200 feet in length, designed and installed at the Schlitz Brewing plant of Milwaukee, Wisconsin, by Mr. J. C. White, Mechanical Engineer for D. W. Mead, Consulting Engineer at Madison, Wisconsin.

LONG DISTANCE TRANSMISSION OF STEAM AND ITS EFFECT ON POWER PLANT ECONOMICS.

Although it is impossible to destroy energy absolutely, still it seems equally impossible to change a given amount of energy in one form to an equal amount of some other form of energy, the difference being due to apparently unavoidable dissipation of some of the energy into forms of energy other than that desired.

In comparing energy transformations, it is customary to divide the output of energy by the input, and the resulting quotient is referred to as the efficiency of the transfer.

Although the sun is the source of all energy, still the greater part of the energy used in doing such useful work as operating steam ships, railroads, factories, street railways, public utilities and industries of all kinds, comes from coal and other fuel which represents energy received from the sun years and ages ago. It is the province of the engineer to make the transformation from the energy contained in this fuel to some desired form of energy in as economical a manner as possible. In order to increase the economy of this change, engineers have attempted from time to time to increase the efficiency of each transfer of energy and to reduce as far as possible the number of transformations made in reaching the desired form of energy. That there is considerable room for improvement is apparent when attention is called to the number of changes now used in modern electric lighting plants in transforming the energy contained in coal or other fuel into the desired form of artificial light.

The energy contained in the coal is rendered available by the chemical union of its carbon and hydrogen with the oxygen of the air, and the heat generated is given in part to the boiler, part being lost through incomplete combustion and a failure to utilize this heat as fast as generated. Of the energy given to the boiler, part is lost by radiation in turning the water in the boiler into steam. As the steam is transmitted through the pipe line, further loss takes place because of radiation and friction, and at the engine itself it is impossible to convert more than 20 or 25 per cent of the energy of the steam into mechanical energy of the moving piston. A further loss occurs in overcoming the friction of the engine, and at the electric generator other losses take place as well as losses of the electric transmission line in delivering this energy in the form of electricity to the motor or lights. If the energy is used for lighting, only a small fraction is turned into the desired form of energy. The efficiency of each step depends upon the losses taking place in that step.

EFFICIENCY OF STEAM PLANTS

FURNACE EFFICIENCY

"The grate design and size must be accurately worked out to suit the draft, fuel and other conditions of each furnace in order to give the best results. The largest loss is probably in the flue gas, depending upon whether the heat had been properly extracted therefrom by the boiler. The total furnace efficiency is generally between 70 and 85 per cent for ordinary furnaces.

BOILER EFFICIENCY

"The boiler proper has a very high efficiency. Its only loss is radiation, and this factor has in most cases been properly taken care of. Radiation for the most part can be overcome by insulation of the boiler. The efficiency can be estimated at about 85 to 95 per cent. The total efficiency of the boiler and furnace, combining the values of each given above, is (from 70 to 85) times (85 to 95) or from 56 to 80 per cent.

STEAM PIPE EFFICIENCY

"The pipe line losses are radiation, friction and condensation. The radiation depends largely upon the difference of temperatures between the pipe and the surrounding air, and can be reduced considerably by good insulation, such as magnesia covering or some similar insulating substance. The friction is an uncertain quantity and depends largely upon the pressure of the steam in the pipe, its velocity, and the size of the pipe. It can be reduced by using a pipe which is considerably larger than necessary, but, by increasing the pipe the radiation is also increased, and an economic mean must be used in order to secure the best results. The condensation in most cases is considerable and is an important item. There is some condensation even with superheated steam. It depends on friction and the difference of the temperature of the pipe and the steam itself.

Engine Efficiency

"The efficiency of the engine can be divided into two partial efficiencies, thermal and mechanical efficiencies. The thermal efficiency of a steam engine is the ratio of the B. T. U. doing work as shown by the indicated horse power to the B. T. U. supplied in the steam. Its value is very low and depends largely on the type of engine and the running conditions. The principal losses are imperfect expansion and compression of the steam in the cylinder, and cylinder-condensation and radiation. The most efficient steam engines are probably the triple expansion Corliss condensing engines and the large power steam turbines. The thermal efficiency at a maximum, can be estimated at from 15 to 20 per cent. The mechanical efficiency is the ratio of the brake horse power or power output to the indicated horse power. It depends solely on the friction loss of the engine and can be estimated about 85 to 95 per cent. The total efficiency or the product of the two is about 12 to 17 per cent for the best type of engines.

Generator Efficiency

"As in the case of the engine, there are losses in the generator which decrease its efficiency. The most important of these are losses due to field resistance, armature resistance, friction in the bearing of the machine, and those due to eddy currents and hysteresis. The field loss is due to the heat generated in the field windings by the field current and represents the power required to excite the field magnets. The armature loss is due to the heat generated by the armature current in the brushes, in the brush contacts and in the armature windings. Hysteresis and eddy currents losses chiefly in the armature core are due to the reversals of magnetization as the armature rotates, and appear as heat. Losses due to friction occur in the bearings and as air friction or windage due to the fanlike action of the rotating armature. The losses all add up and lower the total efficiency of the machine. There are several different efficiencies for the electric generator such as electrical, mechanical, and others. The one most generally used is the commercial efficiency which represents the ratio of the useful energy or output to the power actually absorbed by the machine in being driven. The efficiency of a generator should be between 85 and 96 per cent at normal load.

Efficiency of Electric Line

"The power from the generator is conducted to the lamps and motors using it, through wires, and here again is a loss which affects the combined efficiency of the power plant. The chief losses in the line are due to resistance and leakage. The heat loss or resistance depends on the diameter, length, and material of the conductor, and the leakage is due to improper or poor insulation, especially noticeable in high tension lines. The usual efficiency for a conductor in lines of medium length is above 90 per cent.

Efficiency of Motors or Lamps

"The last loss and drop in efficiency is brought in the lamps or motors or other uses the power may be put to. In the case of motors the losses are the same as those of the generator and the efficiency is the ratio of the power received by the motor in horse power to the actual horse power delivered by it to a line shaft or other transmission. Motor efficiencies vary from 85 to 95 per cent at normal load. The lamp efficiencies are very low, ranging from 3 per cent for incandescent lamps to 19 per cent for mercury vapor lamps. The excessive losses in the lamps are due to the heat caused by the extreme resistance necessary in the lamp."

EFFICIENCIES AT VARIOUS LOADS

The efficiencies of these various transformations have different values for different loads, and as the selection of power plant apparatus depends not only on the maximum load but the variations of load which occur from time to time, or the so called load curve, it is essential that the designing engineer know the efficiency of each element of the plant under these different loads in order that these elements or units may be selected of such size or type as to give the maximum combined efficiencies for the particular conditions of service in order to obtain a solution that is commercially and economically correct. Data has been gathered from various sources giving the efficiency under different loads of the essential parts of a steam power plant enumerated above, and this data is indicated graphically in charts, Figs. 1 to 9.

FURNACE AND BOILER EFFICIENCY AT VARIOUS LOADS

"In gathering information on boiler efficiency, Fig. 1, it was found that probably the most valuable work on this subject has been done by Prof. Breckenridge in a series of tests made for the United States Geographical Survey at St. Louis. However, in his tests, several hundred in number, the efficiency of a boiler was found for varous kinds of coal at normal load only. No tests were made for efficiencies under various loads. The curve used herein for boiler efficiency from no load to 50 per cent overload was obtained principally from a number of tests made at a large Chicago power plant. This curve shows gradual increasing efficiency from 100 to 150 per cent load. In replotting the curve, values below normal load were assumed as near actual conditions as possible. The efficiency in this case seems rather low, but this can be accounted for by the fact that the power output at this plant varies considerably and cannot be absolutely regulated. The efficiency curve for the boiler and furnace was plotted from the following data:

Per cent. Load	Per cent. Efficiency
0	00.0
25	50.0
50	53.3
75	56.6
100	60.0
125	62.5
150	65.8

STEAM PIPE LINE EFFICIENCIES AT VARIOUS LOADS

"Practical information in regard to steam pipe line efficiency, especially where superheated steam is used, is lacking. Otto Berner of Berlin, Germany, gives some valuable information on the subject in regard to heat losses. He experimented with short pipes using both steam and hot water and determined heat losses. From his results a formula was derived for heat loss by using a combination of formulae for radiation as calculated by DuLong and Petit, and Peclet. However, in using any information given by him it must be remembered that the results were taken from a pipe only about six feet long and superheated steam was not used.

"The superheated steam pipe-line at the Jos. Schlitz Brewing Company of Milwaukee, designed by Mr. J. C. White for D. W. Mead, Consulting Engineer, afforded excellent opportunities for obtaining information on superheated steam pipe-line efficiency. It was designed to carry steam at 120 pounds gage pressure and 250 degrees superheat, a distance of about 1,170 feet (Fig. 2). The steam is used for operating two pumps, a De Laval steam turbine with centrifugal pumps of six million gallons capacity, and an Allis-Chalmers cross compound heavy duty pump of the same capacity The steam is taken from a battery of Babcock and Wilcox boilers, with superheaters, at about 100 degrees superheat. It is then arranged to pass through a Foster separately fired superheater and the temperature is raised to the desired degree of superheat. The pipe line which was tested starts at the superheater and delivers the steam to a small receiver in the pumping station. The pipe is so connected that steam can be taken either through the superheater or directly from the boilers. It is made of steel pipe four inches diameter and ¼ inch thick with welded flanged joints. The joints were closed by means of special steel gaskets, patented by Mr. White, the engineer in charge. All turns in the pipe were made by easy bends of large radius so as to avoid any extra losses due to elbows. There were two bends, one of 51 degrees and one of 90 degrees. The entire pipe had a total drop in elevation of about 15 feet. The pipe line was covered with 2½ inch 85 per cent magnesia covering and enclosed in a concrete tunnel 3 feet high and 4 feet wide. A manhole was situated at each bend.

"The main object of testing the pipe line was to determine the efficiency, the pressure loss and the general performance of the line. The heat energy at several points along the line and especially at the ends was determined by taking readings of pressure and corresponding temperatures at the points. Throttling calorimeters were used at the ends of the line, but it was found that the steam was superheated in all cases and the calorimeters were not necessary.

"The instruments used in the test were thermometers, pressure gauges, calorimeters and apparatus for weighing the condensed water. The instruments in which any inaccuracy was liable to be present were carefully calibrated before the test. In selecting a method for obtaining the temperature of the steam, two ways presented themselves. The first was to use electric pyrometers with a common connection so that all readings could be taken simultaneously at some common point. The second was to use a high reading mercury thermometer with a socket at each point of reading. Information was obtained from the Bureau of Standards at Washington in regard to the first method and it was found that a complicated amount of calibration for the pyrometers and wiring might impair the accuracy of the readings. It was decided to use mercury thermometers as a simple and, in this case, probably more accurate method for taking the temperatures.

"The thermometers were made by Taglibue and read from 200° to 800° F. The instruments screwed in a brass cup made especially for them, which was threaded into the steam pipe and extended to about the middle of the pipe, thus giving an accurate reading of the temperature of the steam. These thermometers were calibrated with apparatus as shown in Fig 3. The standard thermometer was one which had recently been calibrated at the Bureau of Standards at Washington, and for which the correction was known. A stem connection was also applied which accounted for the temperature of the air surrounding the stem. Both thermometers, the standard and the Taglibue, were immersed in an oil bath and various readings taken up to 550° F. Above this melted lead was used as high as 700° F. Both thermometers were exposed as nearly alike as possible in the melted substance so an exact comparison could be obtained. After the apparatus had been set up and a full set of readings taken, the amount the Taglibue thermometer was in error was computed for approximately every 40° and a table made showing the corrections on that instrument. The standard thermometer corrections were as follows:

Reading	Correction°	Reading	Correction°
0	0.0	250	3.5
100	0.0	300	4.5
150	1.0	445	6.0
200	1.5		

The stem correction formula,—

$$0.00016 \times n\ (T^\circ - t^\circ),$$

Where:

n=number of degrees emergent from bath,
T=temperature of the bath,
t=mean temperature of emergent stem.

"The pressure gauges were all calibrated with an instrument which consisted of a small cylinder filled with oil and a piston on which was a table. The gauge was connected by a pipe to the cylinder. To obtain a given pressure on gauge a given weight was placed on the table and the piston caused the dedesired pressure in the cylinder and therefore on the gauge. Knowing the diameter of piston and weight upon it, the exact pressure on gauge was determined.

"The instruments for the test were placed at four different points, (Fig. 2). At the boiler house, at the first bend (51°) 406 feet from the boiler house, at the second bend (90°), 640 feet from 51° bend and at the pumping station, 114 feet from 90° bend. Those used at the boiler house were two calorimeters, two pressure gauges, two thermometers for steam pipe and one for room temperature. A thermometer, pressure gauge, and calorimeter were placed on the steam pipe before entering the superheater and also a set on the pipe leaving the superheater.

In this way the work done by superheater could be exactly determined. While the superheater was in operation the coal used and the ash remaining from it were weighed. At the 51° bend a thermometer and pressure gauge were tapped into the pipe and a thermometer was also used to obtain the tunnel temperature. The same apparatus was used at the 90° bend. A pressure gauge and thermometer were tapped into the steam pipe as it entered the pumping station and just ahead of the separator. Suitable apparatus was supplied here for weighing all water from trap in connection with separator which represented the entire amount of water discharged from the pipe line. Similar apparatus was also applied for weighing the condensed steam from condenser in connection with the turbine.

"The tests, four in number, were run from two to four hours and readings were taken simultaneously at all four points every fifteen minutes. Before these tests one was made to determine the condensation in the line at no load. In this case the water in the trap connected before the receiver at the pumping station was weighed for two hours. As the line has a drop throughout its length the amount of condensation was accurately measured at this point. The first two tests were conducted with the steam direct from the boilers without flowing through the separately fired superheater. In these tests the steam passed through the superheaters in the boilers and was heated to about 100° superheat. All tests were run with the turbine alone in operation, its output being varied from 2.5 to 4.0 million gallons.

"With the data taken it was possible to obtain the number of B. T. U. per hour contained in the steam passing any of the four points where instruments were used. By multiplying the number of pounds available steam passing any point per hour by the sum of the total heat of steam at that pressure and the specific heat at that temperature times the number of degrees superheated, the actual number of B. T. U. delivered by the steam past this point per hour was found. This value of B. T. U. was found at all four points and the drop due to condensation, friction, and radiation was found for each length of straight pipe. The efficiency of the entire line was determined by dividing the actual number of B. T. U. delivered at the pumping station per hour by the actual number of B. T. U. delivered to pipe at the boiler house. The pressure loss was also determined for the line from pressure readings taken at the four points.

Formulae:

B. T. U. at any point=$[H+c\ (T_1—T)]$ x pounds steam passing per hour,

Where H =total heat of saturated steam,
T_1 =temperature of the steam,
T =temperature of saturated steam at that pressure,
c =specific heat of superheated steam.

"Values of specific heat of superheated steam were taken from an article in Power by Prof. Sydney A. Reeve, and a curve plotted for 99 lbs. gauge pressure (Fig. 4). As all pressure values averaged about 100 lbs., this is thought justifiable for lack of better information on the subject.

TABLE 1.

Average Data of Four Tests.

Reading point	*Temperature*	*Pressure*	*Degrees Superheat*	*Specific Heat*
Entering superheater	447.2	108	104.6	.496
Leaving superheater	421.4	104.5	80.9	.510
51° Bend	382.1	106.3	40.5	.538
90° Bend	363.8	101.2	25.5	.556
Pumping Station	362.7	101.1	24.5	.558

Condensed steam 7085.2 lbs. Total per hour 3603.6 lbs.
Condensation in pipe 122.0 lbs. Barometer 14.5.
Total 7207.2 lbs. Average temperatura of tunnel 72 ° F.

Calculations:—

Available B. T. U., at any point equals $[H + c\,(T_1 —T)] \times$ lbs. steam at that point.

Available B. T. U. entering superheater 4,459,237.8 per hour.

Available B. T. U. leaving superheater 4,421,980 per hour.

Available B. T. U. at 51° bend 4,331,408.6 per hour.

Available B. T. U. at 90° bend 4,254,636.6 per hour.

Available B. T. U. at pumping station 4,246,749 per hour.

Equivalent horse power delivered equals. $\frac{778 \times 4,246,749}{33000 \times 60}$

equals 1,668.

B. T. U. total loss in pipe equals 4421980 — 4246749 equals 175,228.5 per hour.

B. T. U. condensation loss equals $\frac{122}{2}$ (1185.5 + .534 × 50) equals 73,932 per hour.

B. T. U. radiation and friction loss equals 175,228.5 — 73,932 equals 101,296.5 per hour.

Total efficiency of pipe equals $\frac{4,246,749}{4,421,980}$ equals 96.0 per cent.

Three hour test (without superheater).

Total efficiency of pipe equals 99.52 per cent.

Four hour test (with superheater).

Total efficiency of pipe equals 94.5 per cent.

Five hour test (with superheater).

Total efficiency of pipe equals 95.51 per cent.

"The results of the test on the superheated pipe-line (Table 1) were very satisfactory. The efficiency seems a trifle high for a line of this length, but this result can be accounted for by the unusually good construction of the line. The tests show that a moderate degree of superheat as used in the first two tests gave the best results since the radiation and friction are not large and the condensation is moderate. When high superheat was used, the condensation was reduced to a minimum and the radiation and friction became extremely large giving a correspondingly greater total loss. The average per cent drop in pressure of the steam in the line was about five per cent. This factor is low for the reason that the pipe-line was designed somewhat larger than absolutely necessary and the drop in pressure is necessarily reduced. The pressure reading of the steam leaving the superheater is low due to a sudden turn in the pipe increasing the velocity and decreasing the pressure of the steam slightly. The average total loss in B. T. U. of the four tests is about 194,000 B. T. U. per hour or about 162 B. T. U. per square foot of pipe surface per hour. The average drop in superheat for the first two tests when the Foster superheater was not used was about 57 degrees or about one degree F. for every twenty feet length of pipe. This is considerably better than was estimated in the design. On the the other hand, the drop in degree superheat in the last two runs averaged 133 degrees, showing a loss of one degree for every nine feet length of pipe. This value is about what was expected with the high superheat.

"The normal load on the pipe line was determined by using a velocity of steam of 6,000 feet per minute and multiplying by the area of the pipe in square feet. This value gives the amount of steam delivered by the pipe in cubic feet per minute and by multiplying by 60 and dividing by 3.5 (specific volume of steam under the given conditions) the number of pounds of steam passing through the pipe per hour is determined:

$$\frac{6{,}000 \times (4)^2 \times .7854 \times 60}{144 \times 3.5} = 9000 \text{ pounds per hour.}$$

Using the average number of pounds of steam used in the four tests—3,600 pounds per hour—and knowing the average efficiency, one point on an efficiency curve is found. Since this point gives the efficiency at 40 per cent load as about 95 per cent, the efficiency at full load is estimated to be about 96 or 97 per cent. With these two points, an approximate curve was drawn showing the pipe-line efficiency for various loads on the pipe (fig. 5)."

Efficiency curves of this kind for steam pipe lines are highly desired, and the engineering department of The University of Wisconsin is planning to conduct other experiments on steam pipe lines of various designs, using saturated steam in order to gather other data from which to construct similar curves. The remarkable results given above show that it is possible to get very economical transmission of energy by means of properly designed and insulated steam pipes, and to use this method for transmitting energy for considerable distances.

Steam Engine Efficiencies at Various Loads.

"The engine efficiency curve (Fig. 6) was calculated from data in Meyer's Steam Power Plants, page 59, curve 3. In this book Mr. Henry C. Meyer, Jr., M. E., shows the results of a large number of engine tests for different types of engines under various loads. His results are shown in a series of economy curves giving pounds of steam consumed by the engines per indicated horse power per hour under loads from zero to 50 per cent overload. The curves given were constructed from the economy curves by the following formula:

$$\text{Efficiency} = \frac{1}{\dfrac{\text{Lbs. steam} \times 1200 \times 778}{33000 \times 60}}$$

The following table shows these results for a cross compound Corliss condensing engine:

Per cent. Load	Per cent. Efficiency
0	00.00
50	8.16
75	9.24
100	9.74
125	9.44
137.5	8.85

"In calculating these values the normal load was taken at 39 pounds mean effective pressure on the curve. The value 1200 was used as the B. T. U. per pound of steam.

778 = foot pounds in 1 B. T. U.

33000 = foot pounds per minute in 1 horse power.

GENERATOR EFFICIENCIES AT VARIOUS LOADS

"S. P. Thompson in his book 'Dynamo Electric Machinery' gave the best available information on generator efficiency. On page 657 he shows an efficiency load curve for a 550 Kilowatt, Oerlikon Co. generator which has been used in this investigation in connection with the other curves plotted. This curve (Fig. 7) was chosen as a fair average for generators and shows efficiencies above 90 per cent. for loads from 150 K. W. to 700 K. W. or from 30 per cent. load to 40 per cent. overload with efficiencies at other loads as indicated below.

Load	Per cent. Load	Per cent. Efficiency
0	0	0
50	10	77
100	20	85
150	30	90
200	40	91
250	50	92.5
300	60	93
500	100	94
600	120	94
700	140	93"

EFFICIENCIES OF ELECTRIC LINE AT VARIOUS LOADS

The efficiency of an electric line for transmitting energy to some distant point (Fig. 8) will necessarily depend on the design of that line. In this comparison such a line was assumed of length equal to the steam pipe line described above in order to make further comparisons later. "The case assumed is for transmission of 250 K. W. normal load at 550 volts with a 5 per cent. drop in voltage at normal load.

$.05 \times 550 = 27.5$ volts drop $= R1$.

$R = 27.5/455 = 0.0605$ ohms resistance in line.

$R = 10.8 \times 2L/d^2$. $d^2 = \dfrac{10.8 \times 2 \times 1200}{0.0605} = 429000.$

$d = 429000 = 655$ mills or ⅝ inch cable.

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{Loss}}{\text{Input}}.$$

Loss $= 1^2R$.

For normal load, eff. $= \dfrac{250 - .0605 \times (455)\ 2/1000}{250} = 95$ per cent.

"In this manner the efficiency at any load can be found since the resistance of the line is a constant. The following calculated values give the efficiency for various loads:

Per cent. Load	Per cent. Efficiency
0	100.00
25	98.8
50	97.5
75	96.4
100	95.0
125	93.7
150	92.6

"The efficiency curve for a motor (Fig. 9) closely resembles that for a generator. The motor efficiency is a trifle lower and in the curve taken, it is 84 per cent. at full load. The curve used is taken from Franklin and Esty, "Direct Current Machinery," page 142, for a compound motor, and is constructed from the following data:

Per cent. Load	Per cent. Efficiency
0	00
25	58
50	77
75	83.5
87.5	84
100	83.5
125	80 "

TOTAL EFFICIENCIES.

A total efficiency curve may be calculated by combining efficiencies of the separate curves shown together in Fig. 10. In this combination an equipment consisting of boiler, engine, generator, electric line and motor are considered.

Very instructive comparisons may be made from the data given above in comparing a system of steam transmission with an electric system for transmitting energy. For example, the selection of a steam pipe line in the case of the pumping plant for the Schlitz Brewing Company in preference to an electric system is amply justified by drawing a combined efficiency curve for the steam system, assuming a separate boiler plant with a pipe line connecting two steam driven pumps at the pumping station, and comparing this with a combined efficiency curve for a plant having a boiler, engine and electric generator at the power plant and electric transmission from there to the station and motor driven pumps at the pumping station. This comparison is made in Fig. 11, which shows remarkable advantages of the steam equipment particularly at full loads and loads above.

A comparison which would more nearly represent the actual conditions of the problem met by Messrs. Mead and White would be to assume two possible installations, one an electric installation taking current from a generator and transmitting through a line to a motor. Under the probable conditions the motor load would be only part of the generator load, and if it is assumed that the boiler efficiency under the various loads at the pump station remains at a constant quantity, say 65 per cent., (this assumption is justified because of the large steam boiler plant installed) the engine efficiency at a constant quantity of say 7 per cent and a generator at a constant efficiency of 83 per cent, while the efficiency in the line and motor will vary according to charts given above, the total efficiency of this system would be as indicated in Fig. 12. This is to be compared with a steam pump of high economy connected by a steam pipe line to this boiler plant operating at a constant efficiency of 65 per cent and giving a total efficiency as indicated in Fig 12. The difference between these two systems is seen to be very great due to the fewer changes of energy required and to the high pipe line efficiency possible. The comparisons given above illustrate the value of drawing efficiency curves for the separate elements of a power plant and a combined efficiency curve in order that the engineer may be able to select proper apparatus to fit the actual conditions of his problem.

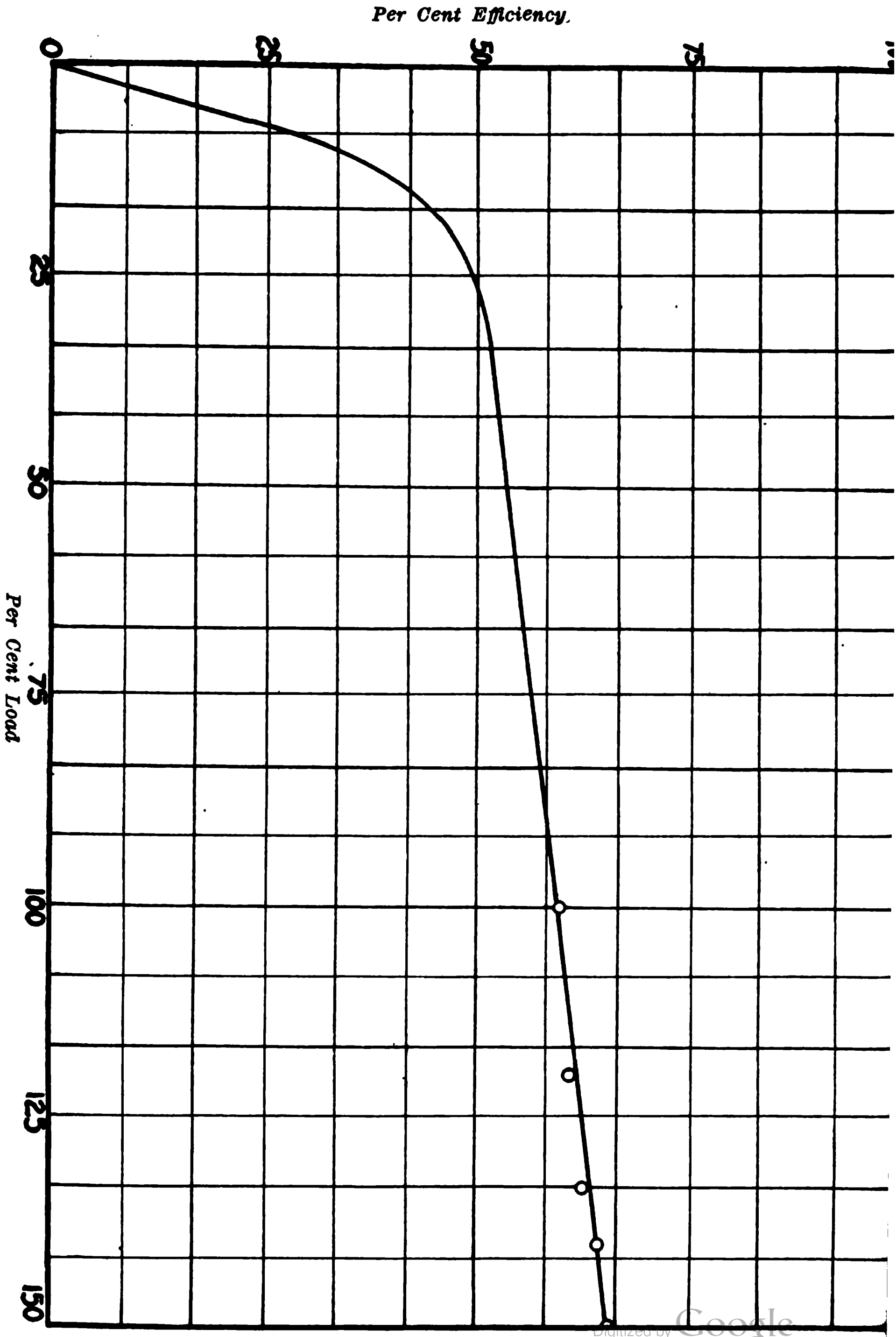


FIG. 1. BOILER AND FURNACE EFFICIENCY.

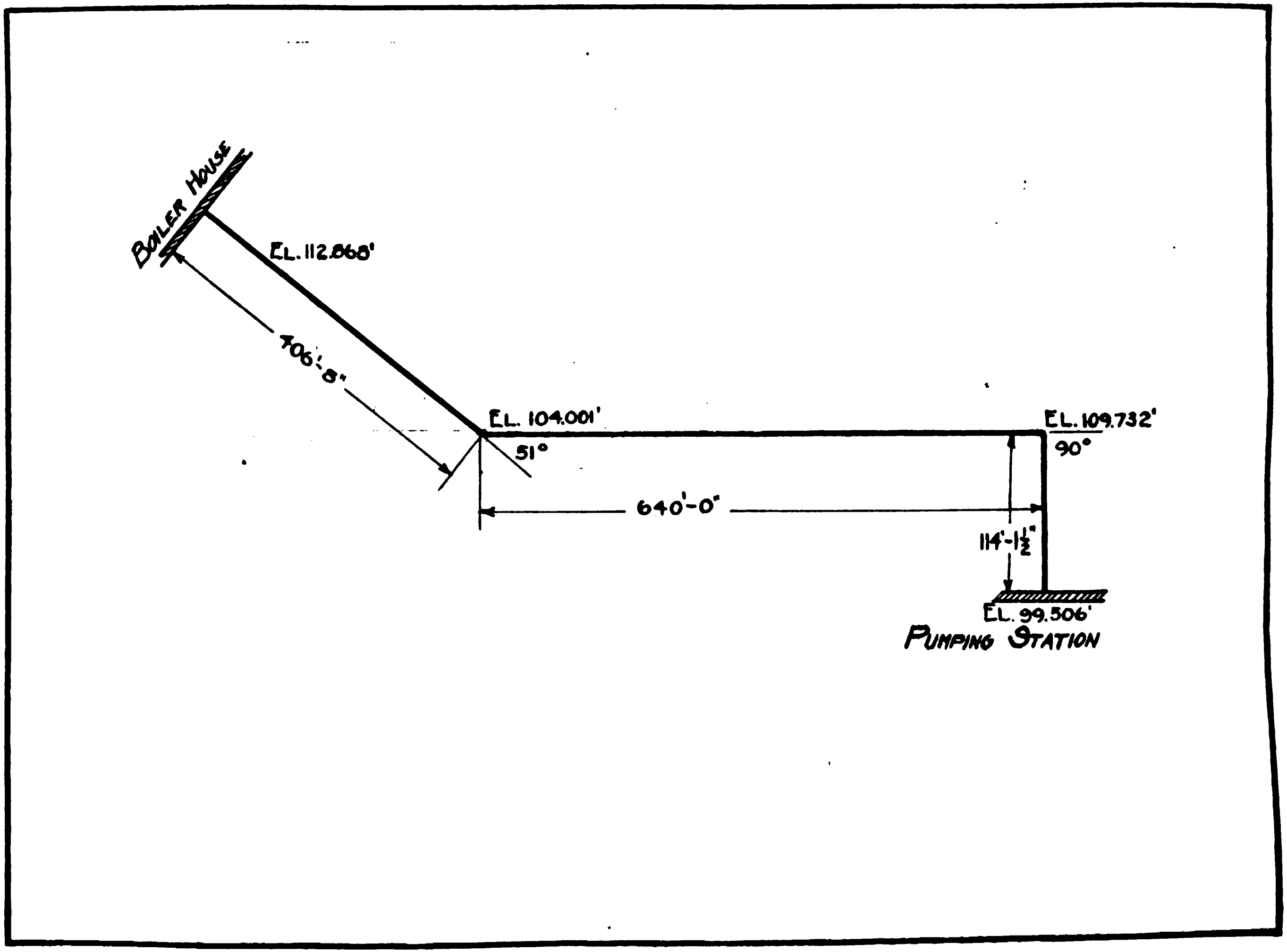
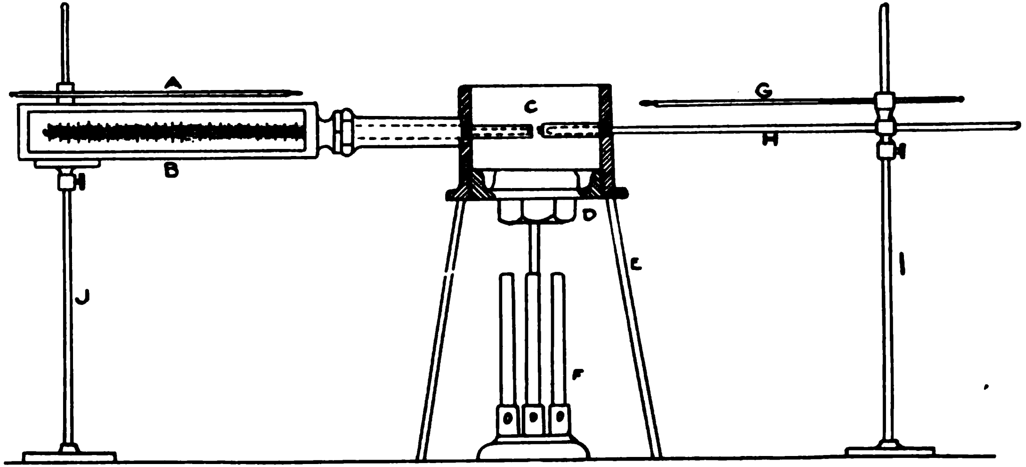


FIG. 2. PLAN OF PIPE LINE TESTED

A-TEMP. OF STEM OF TAGLIBUE THERMOMETER.
G- " " " " STANDARD "
B- TAGLIABUE THERMOMETER
H- STANDARD.
D- CUP FOR OIL OR LEAD.
C- OIL OR LEAD.
F- BUNSEN BURNER.
J, E, I,- SUPPORTS FOR APPARATUS.



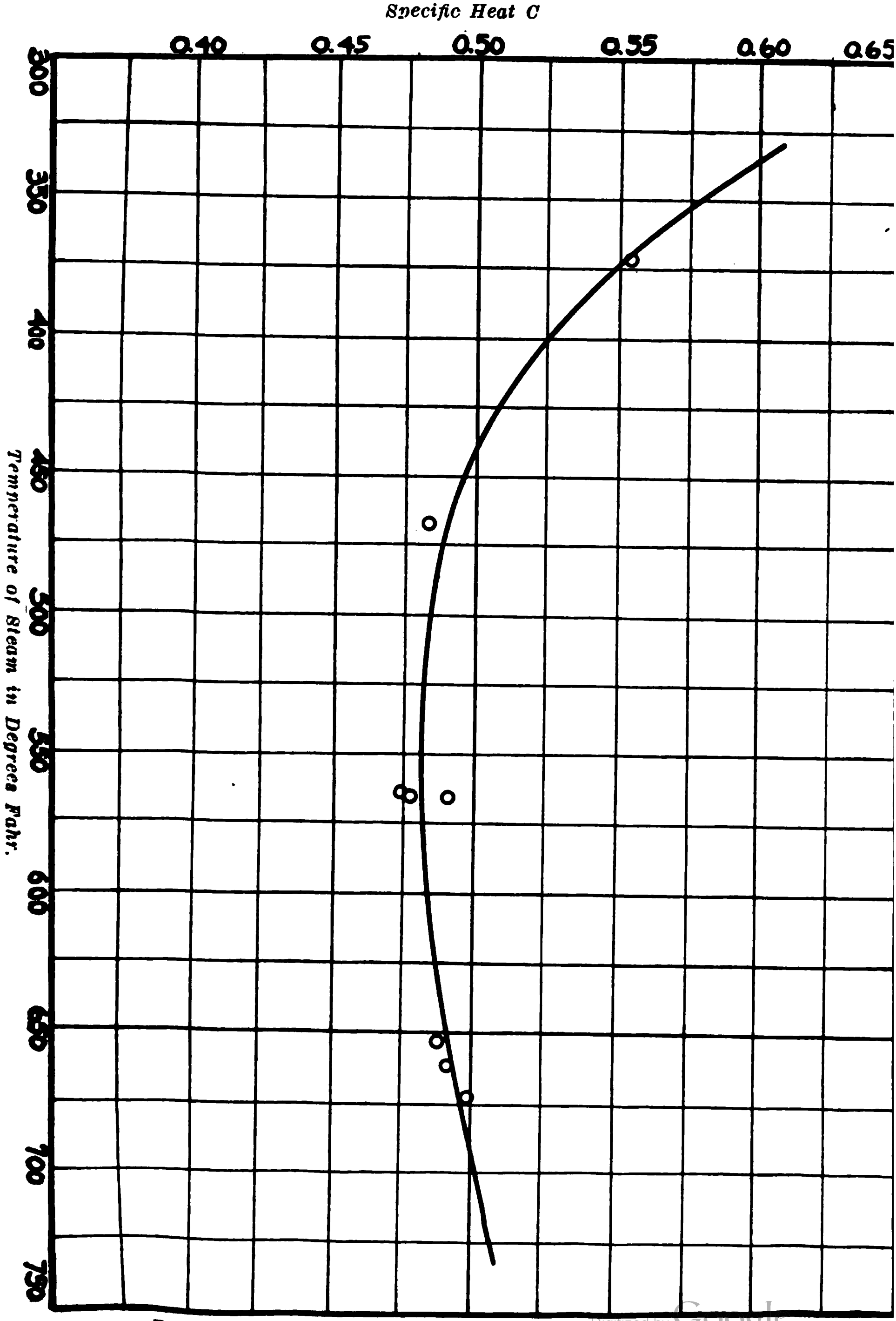


FIG. 4. SPECIFIC HEAT OF SUPERHEATED STEAM AT 99 LBS. GAGE.

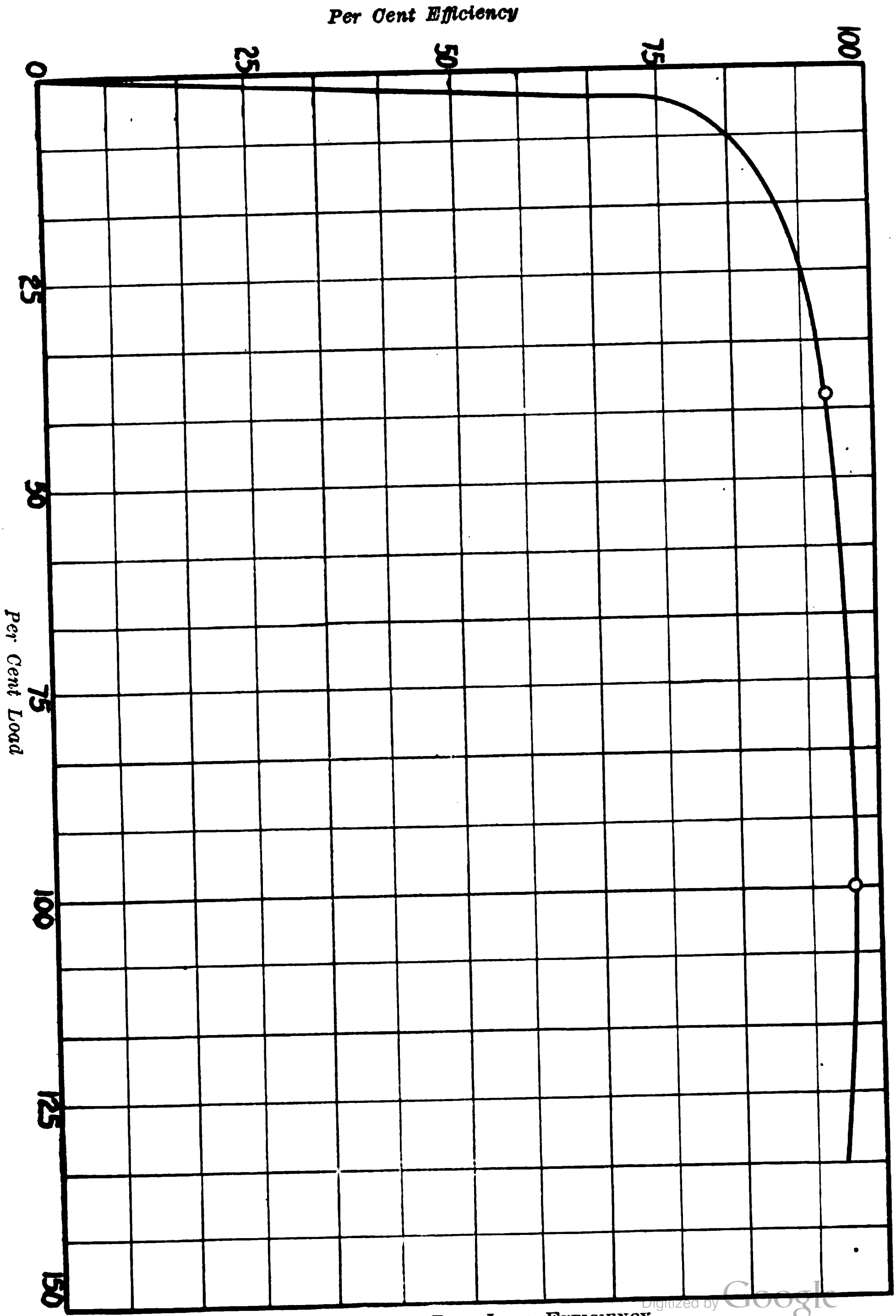


FIG. 5. STEAM PIPE LINE EFFICIENCY.

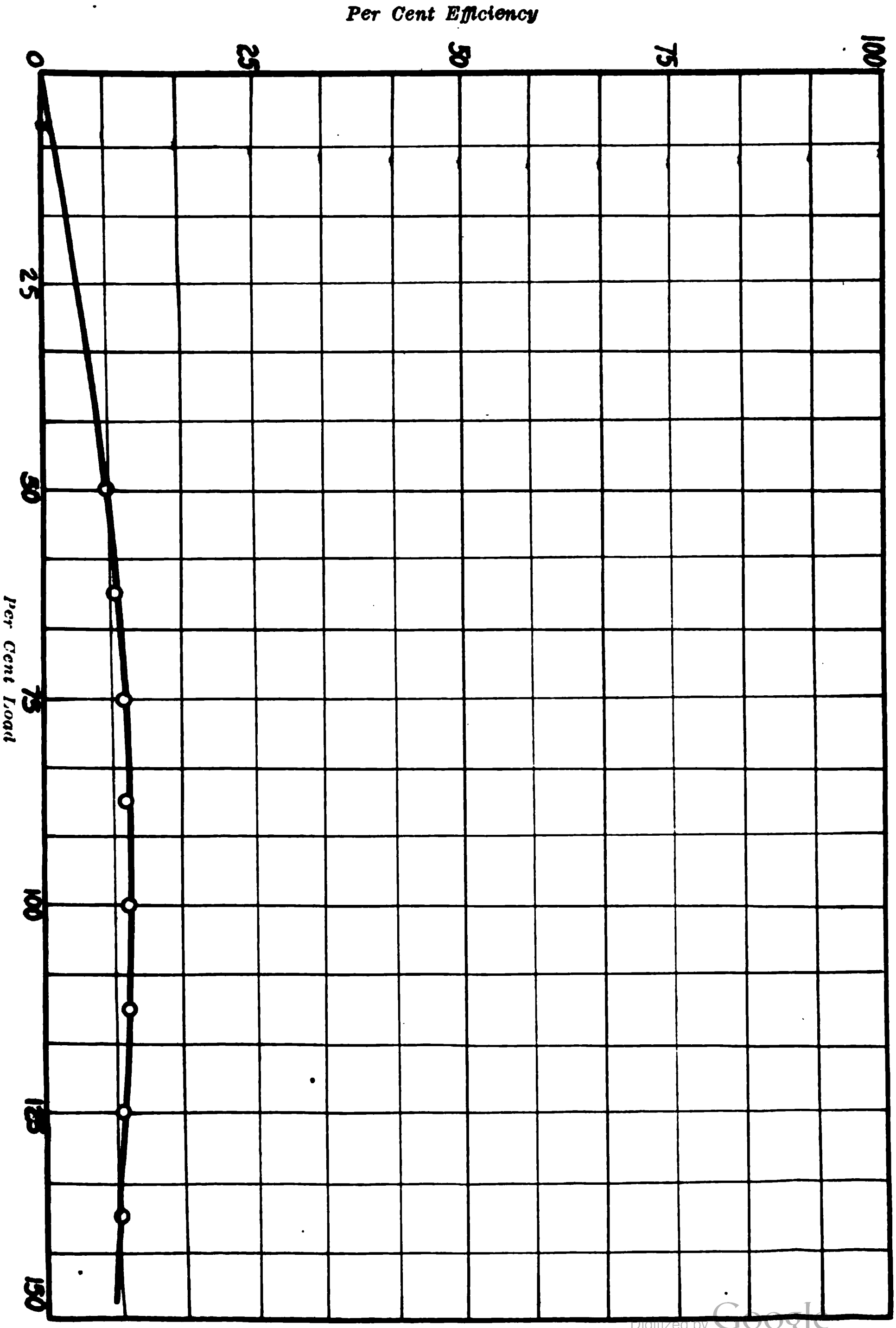


FIG. 6. STEAM ENGINE EFFICIENCY.

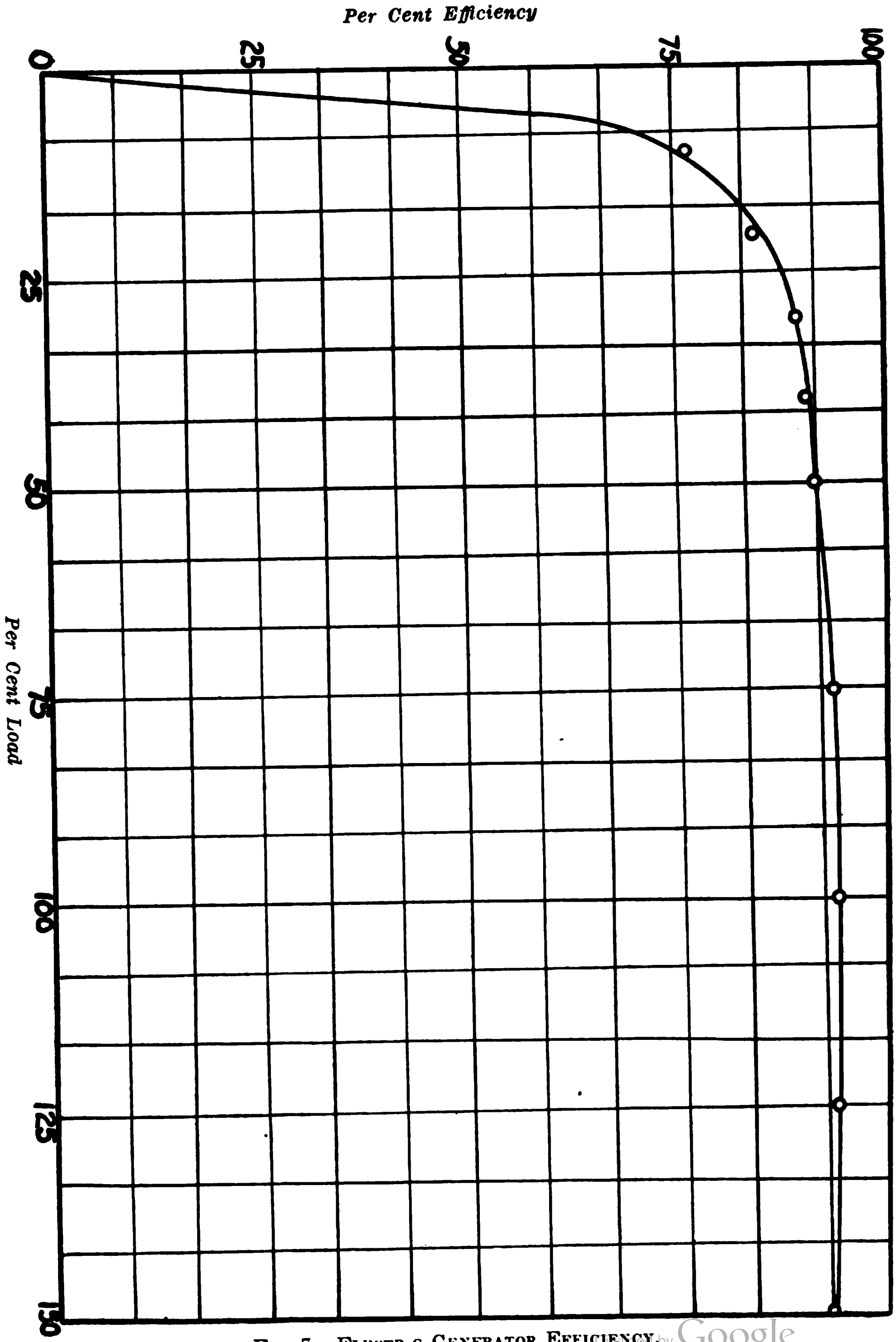


FIG. 7. ELECTRIC GENERATOR EFFICIENCY.

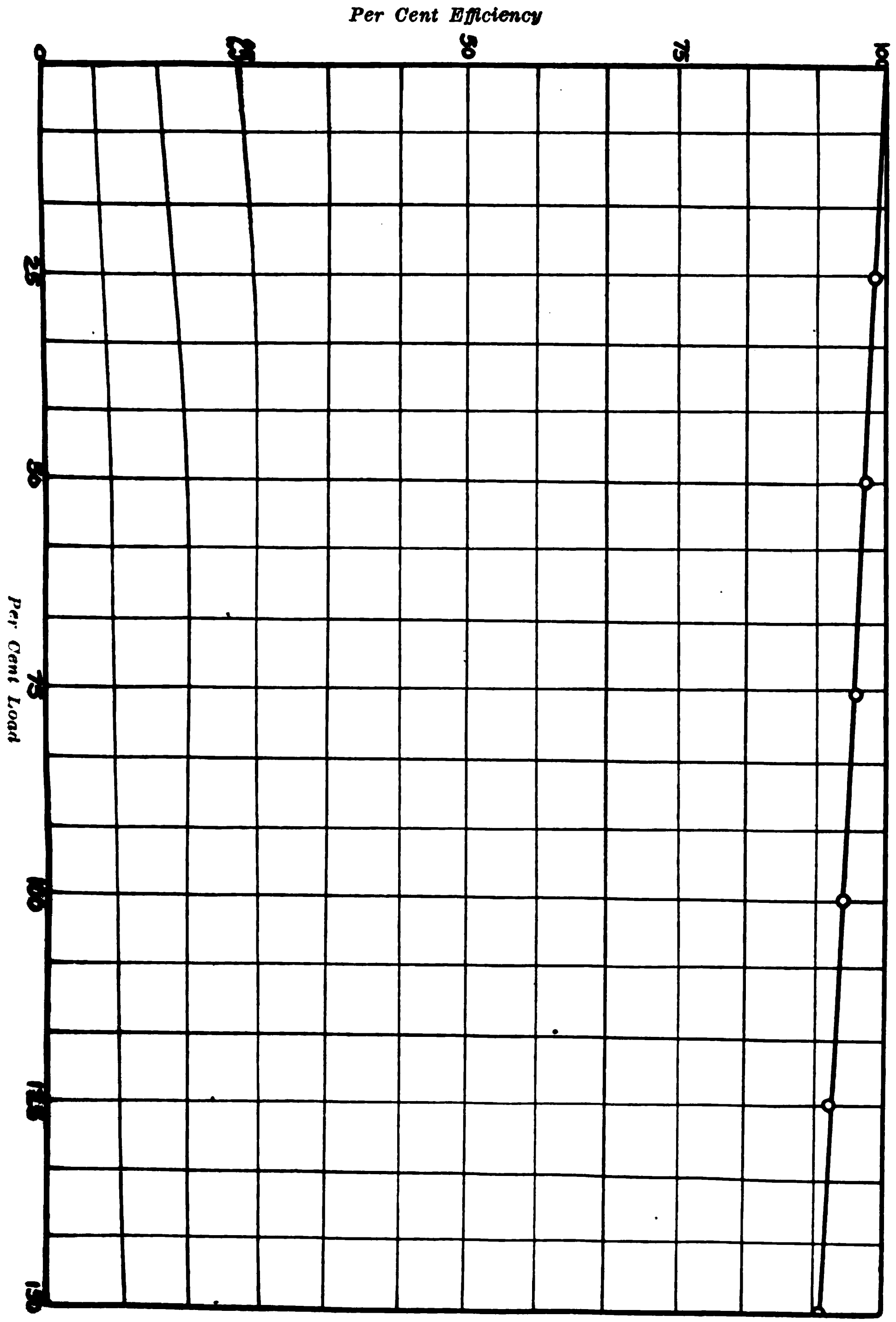


FIG. 8. ELECTRIC LINE EFFICIENCY.

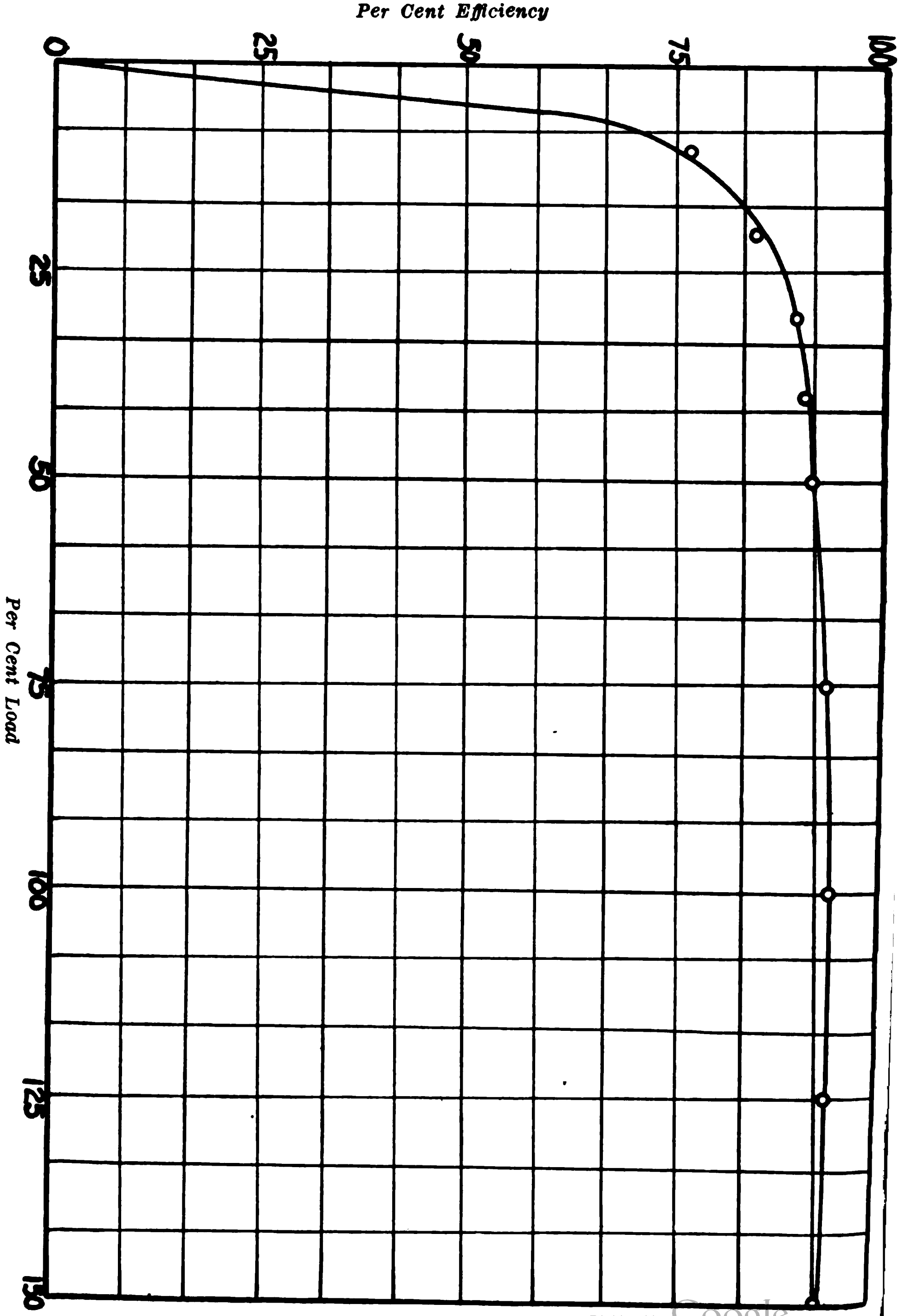


FIG. 7. ELECTRIC GENERATOR EFFICIENCY.

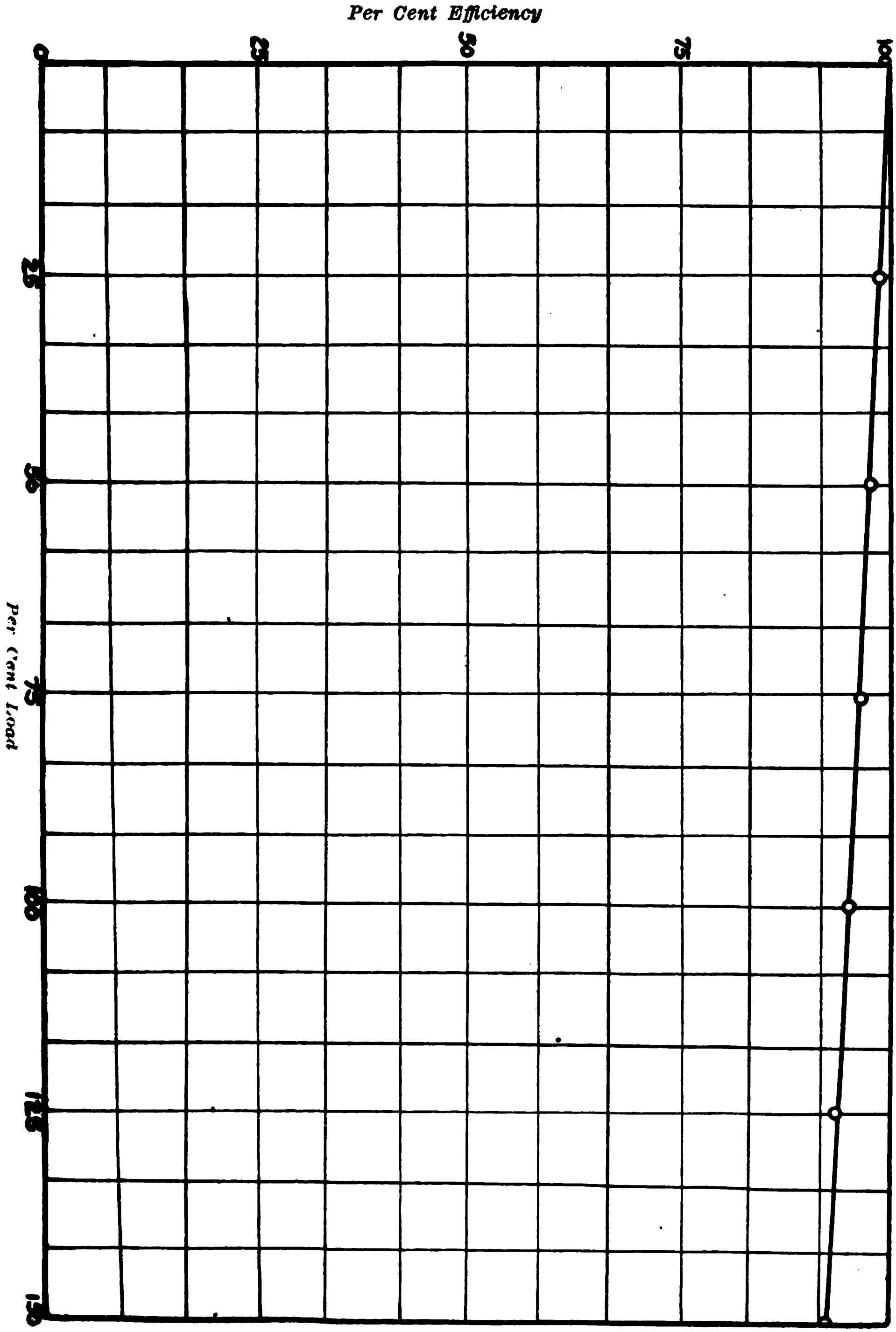


FIG. 8. ELECTRIC LINE EFFICIENCY.

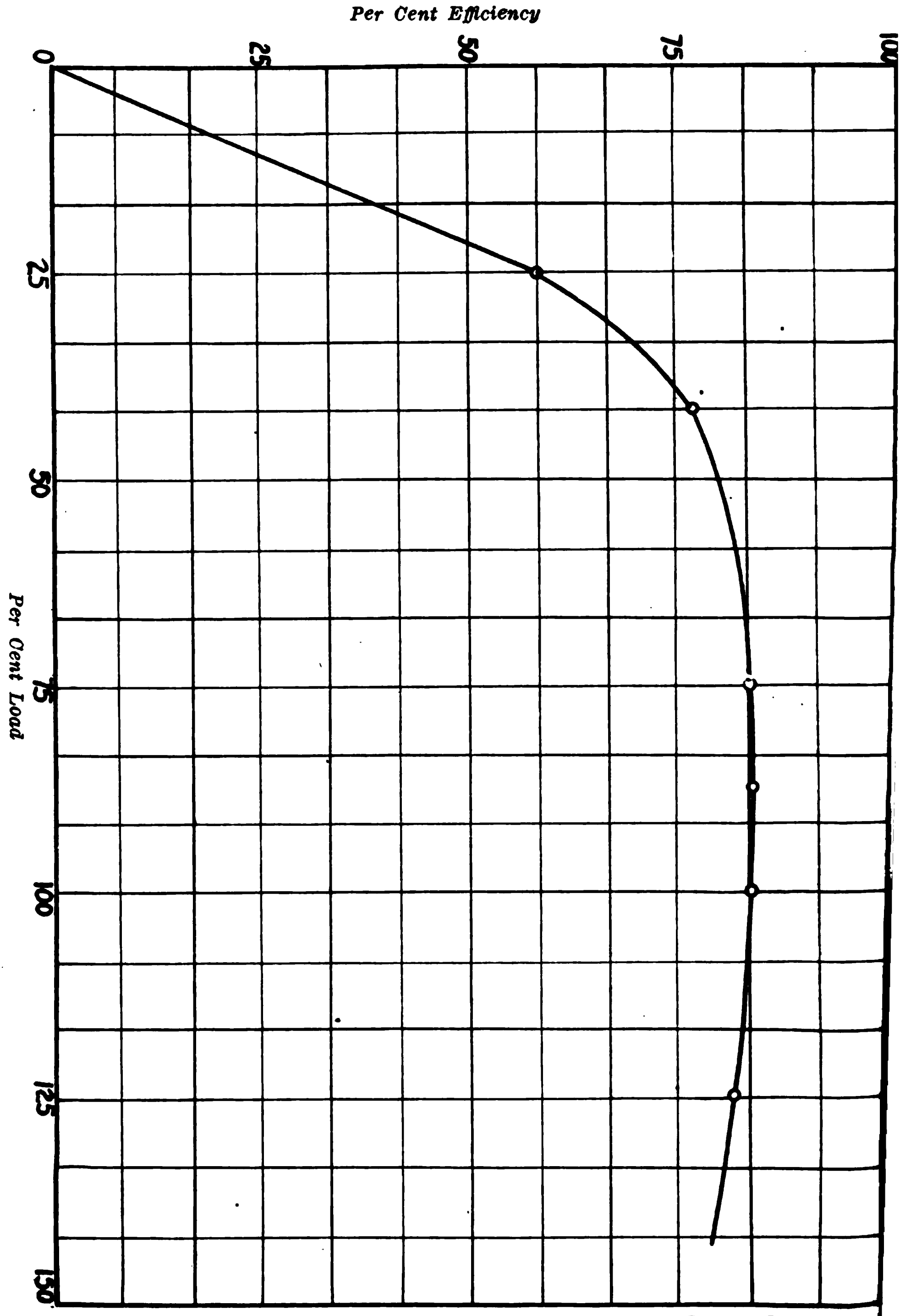


FIG. 9. ELECTRIC MOTOR EFFICIENCY.

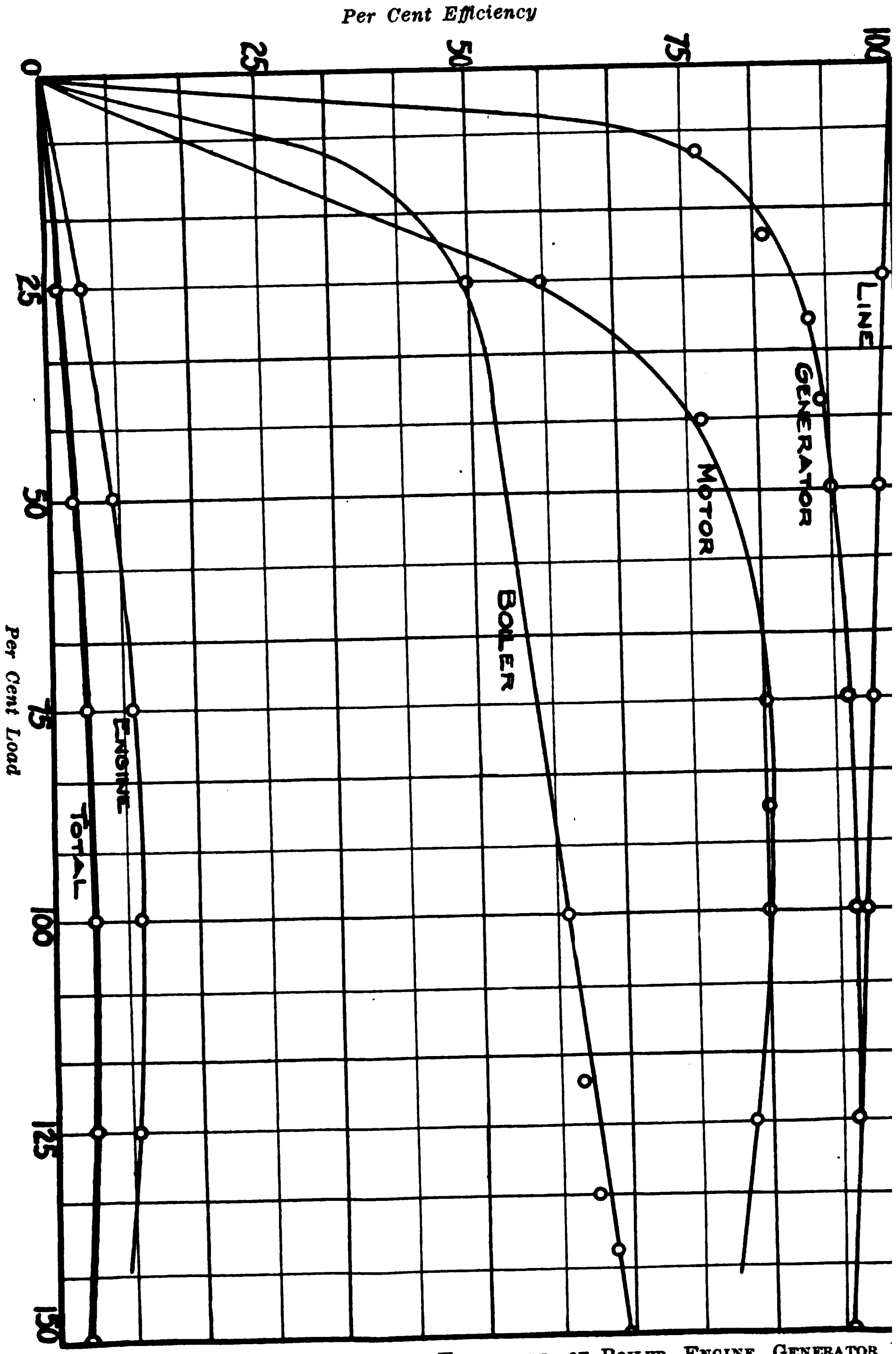


FIG. 10. INDIVIDUAL AND TOTAL EFFICIENCY OF BOILER, ENGINE, GENERATOR AND MOTOR.

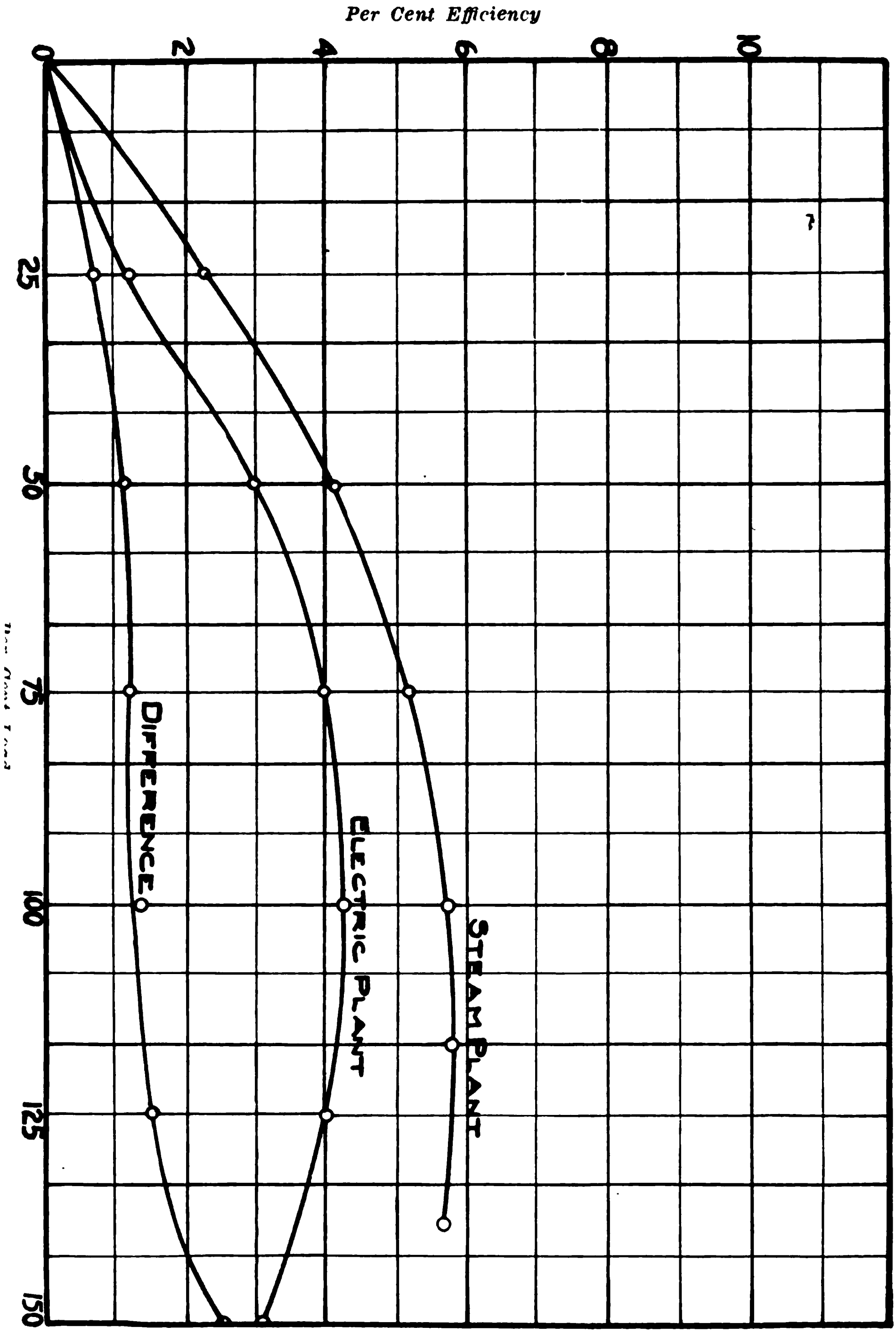


FIG. 11. COMPARATIVE TOTAL EFFICIENCIES OF STEAM AND ELECTRIC SYSTEMS OF TRANSMITTING ENERGY FOR ONE SET OF CONDITIONS.

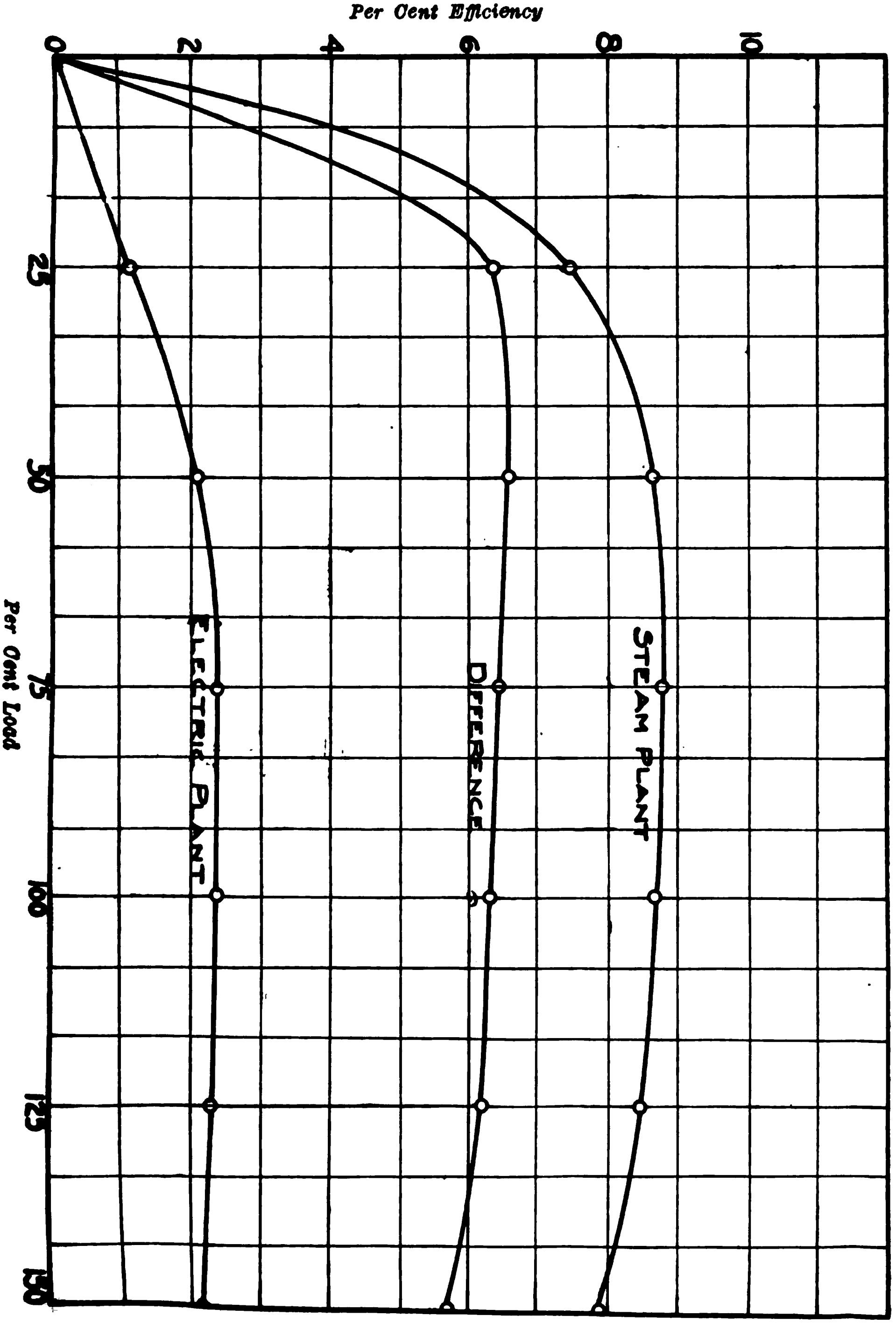


FIG. 12. COMPARATIVE TOTAL EFFICIENCIES OF STEAM AND ELECTRIC SYSTEM OF TRANSMITTING ENERGY UNDER SECOND SET OF CONDITIONS.

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INVESTIGATION OF HYDRAULIC CURVE RESISTANCE

EXPERIMENTS WITH TWO-INCH PIPE

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INVESTIGATION OF HYDRAULIC CURVE RESISTANCE

INTRODUCTION

In this bulletin are presented the results of some experiments made in the Hydraulic Laboratory of the University of Wisconsin. These experiments were made with the purpose in view of determining the relation existing between hydraulic curve resistance and the length of the radius of the curve. For the purposes of comparison there are also included in this bulletin a discussion of some experiments made with the same purpose by other experimenters working in other laboratories. The experiments to be discussed were limited to pipes of circular cross-section having elbows or bends with central angles of 90 degrees.

Several types of 90 degree elbows are suggested by the sketches in Fig. 1. In this figure (a) and (b) represent pipes having cross-sections of uniform size and shape throughout the turn, such as drainage fittings, flanged elbows, bell and spigot pipe fittings, and pipe bends. The type of elbow shown in Fig. 1 (c) is now seldom used where large quantities of liquid are to pass through the pipe. Fig. 1 (d) represents a cast iron screw elbow, and Fig. 1 (e) and (f) represent cast iron screw tees such as are commonly used on the smaller sizes of piping.

Various forms of commercial fittings used in these experiments are shown in section in Figs. 2 to 7.

THE CAUSE OF CURVE RESISTANCE

Experiments by various observers, made by injecting coloring matter into water flowing in transparent pipes, have shown that the particles of a liquid when flowing very slowly travel in

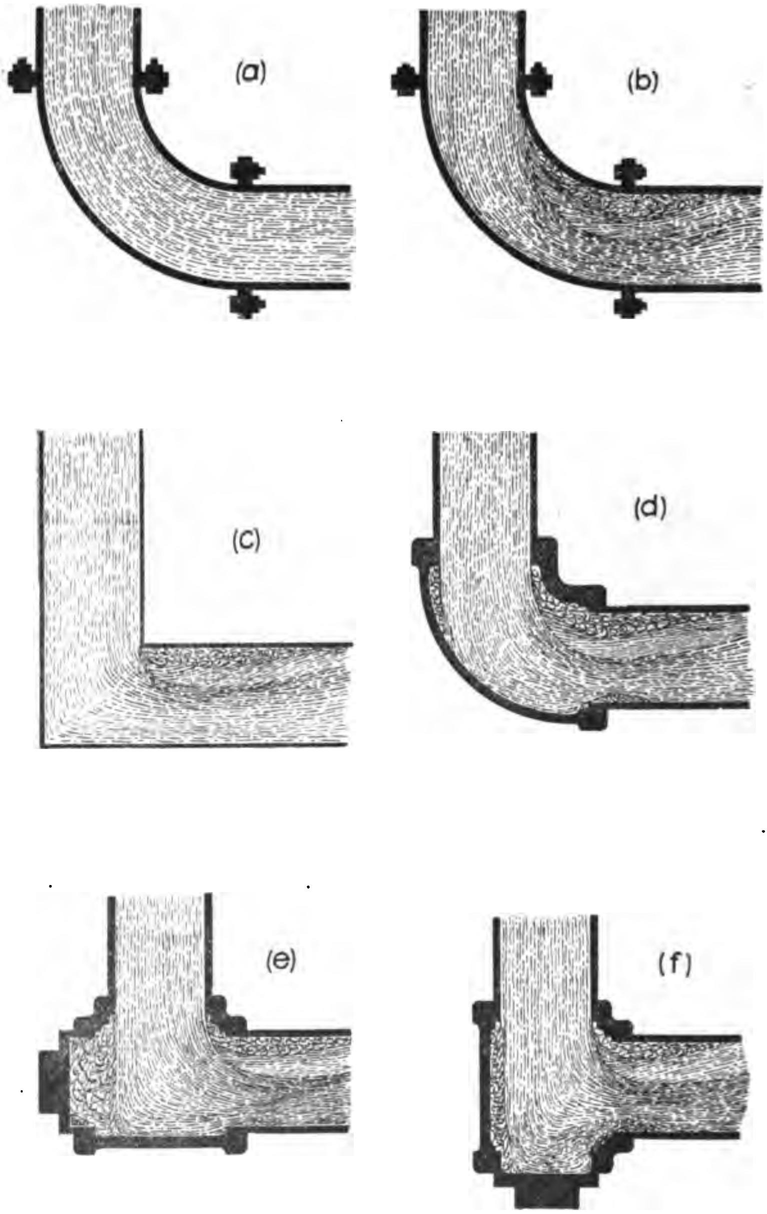


Figure 1.—Types of Pipe Elbows.

definite parallel paths similar to the lines shown in Fig. 1 (a). At velocities as great as exist in pipes conducting water under usual conditions of practice the particles of water do not move in parallel paths, but have a sinuous motion. Such motion of the particles is irregular and causes a disturbed condition of flow. Nevertheless, if the flow is in a straight pipe of circular cross-section, the maximum axial component of the velocity is at the center and the minimum at the sides of the pipe, with the intermediate velocities symmetrically distributed about the center-line, as illustrated by the Pitot tube traverses shown in Fig. 8. This condition of flow produces less loss of head than does any other arrangement of the velocities.

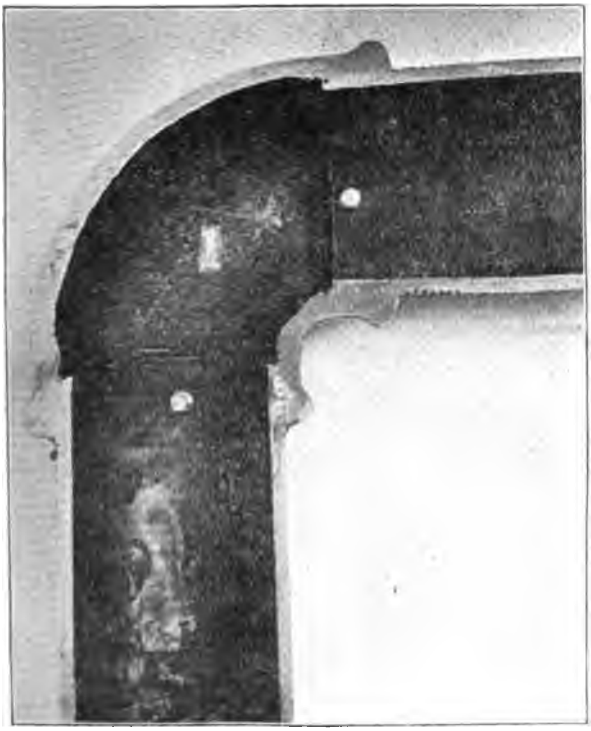


Figure 2.—Malleable Iron Elbow.

When the fluid is made to pass rapidly through a curved pipe the particles at the center, which are moving with the highest velocities, exert a greater centrifugal force than the more slowly moving ones. This causes a rearrangement of the relative positions of the moving particles; those at the center being projected to the outside of the curve and those originally on that side being thereby crowded around to the inside of the curve, as indicated in the sketches Fig. 1 (b) and (c). When the fluid leaves the elbow or curved pipe, the highest velocities are therefore near one side of the pipe as illustrated by the Pitot tube traverses shown in Fig. 9. Greater difference between the velocities of adjacent particles of fluid and between the fluid and the pipe exist under the conditions of flow shown in Fig. 9 than under the conditions of Fig. 8. The resistance to

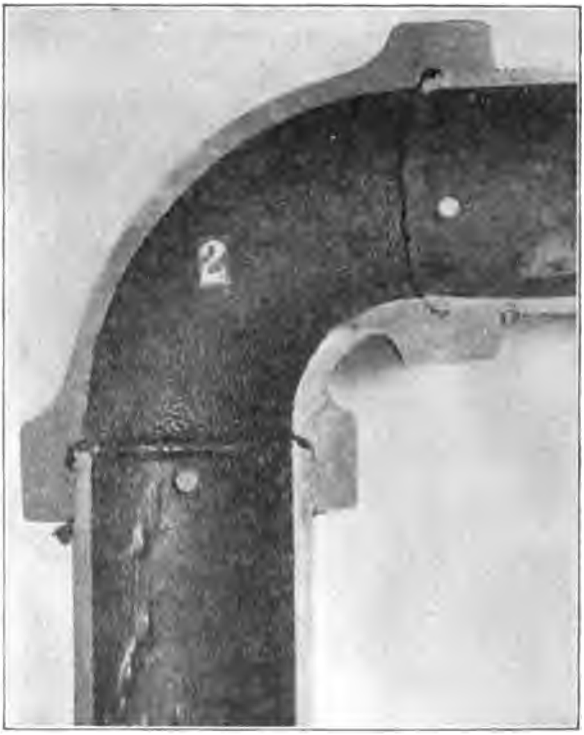


Figure 8.—Cast Iron Long Turn Drainage Elbow.

flow of a fluid increases as the relative velocities of the particles increase. Hence the conditions of Fig. 9 are accompanied by a greater loss of head than are the normal flow conditions.

If the velocity of flow is high and the radius of curvature of the pipe bend is short, the centrifugal force exerted by the moving column of fluid is sufficiently great to cause the fluid to leave the surface of the pipe at the inner side of the curve. The space between the pipe wall and the flowing water may be vacuous, but generally it is filled with water moving in an eddy. This eddy causes a loss of head in addition to that already mentioned as caused by the abnormal arrangement of the velocities.

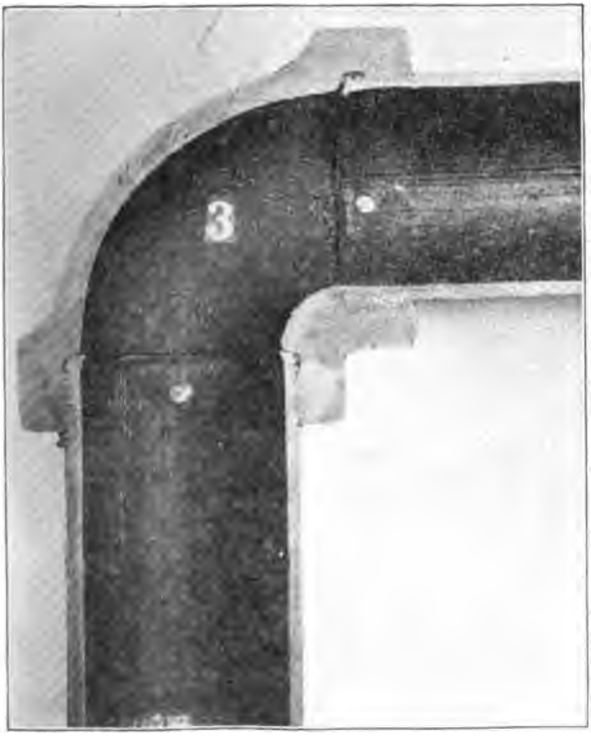


Figure 4.—Cast Iron Drainage Elbow.

Commercial pipe fittings of the types in most common use, such as screw tees and screw elbows, are of larger section than the pipes which they connect. Fluid passing through them is therefore subjected to a sudden enlargement of section at entrance and a sudden contraction of section on leaving them. The eddies produced by such enlargements and contractions cause losses of head in addition to the other losses of head above mentioned. The conditions of flow in fittings of these types are indicated in Fig. 1 (d), (e) and (f).

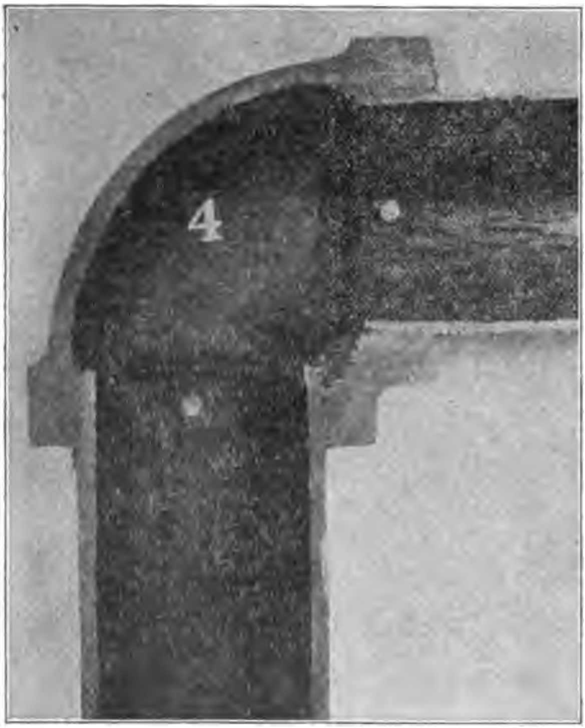


Figure 5.—Cast Iron Short Turn Elbow.

THE MEASUREMENT OF CURVE RESISTANCE

Since the eddies caused by bends occur partly in the straight pipe following the fitting it is evident that the entire loss of head due to it cannot be measured by piezometers placed at the points of tangency of the curve. The piezometers must be placed beyond the region of disturbed flow in order to include all curve losses. Owing to the fact that a number of investigators have failed to appreciate this point, a considerable part of their experiments have little value. It is not known just how far down stream from the curve it is necessary to place the piezometer to include all the loss due to the distorted arrangement of the velocities.

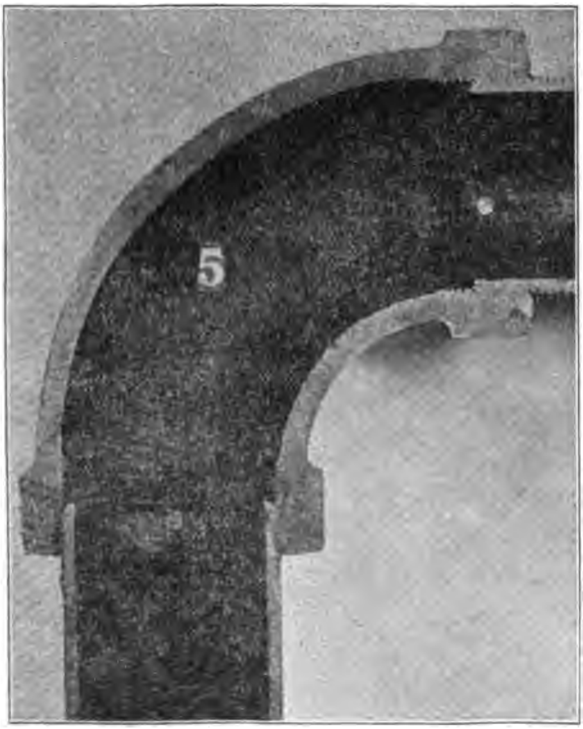


Figure 6.—Cast Iron Long Sweep Elbow

As a result of the Detroit experiments Williams, Hubbel and Fenkell concluded that the distortion of the velocity curves caused by a 10 ft. radius curve in a 30 in. diameter pipe did not extend 295 diameters except at very low velocities, and that it was doubtful if that caused by a 6 ft. radius curve in a 30 in. diameter pipe was apparent 95 diameters down stream.

From the Cornell experiments of 1907 Schoder concluded that at 76 diameters downstream from a screw elbow normal flow existed in a 6 in. pipe.

In order to be satisfied that all of the loss properly chargeable to the curve was being measured in the Wisconsin experiments, piezometers, designated C, D and E, were placed at points 1 foot, 7 feet, and 17 feet, or 5.82, 40.7 and 98.9 diameters, downstream from the turn, and the excess loss due to the eddies caused by the curve was measured in each section.

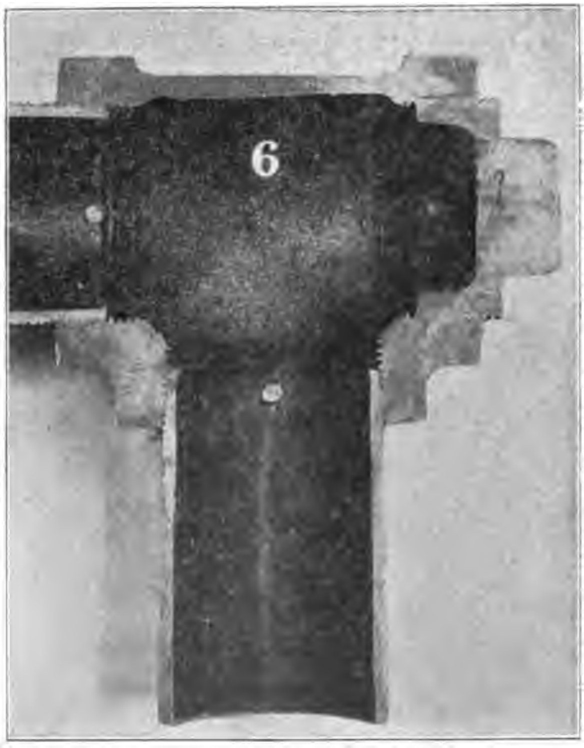


Figure 7.—Cast Iron Tee.

Figs. 10 and 11 are based on these measurements. In Fig. 10, which is plotted on logarithmic cross-section paper, the abscissas are the mean velocities and the ordinates are the quotients obtained by dividing the total loss of head between piezom-

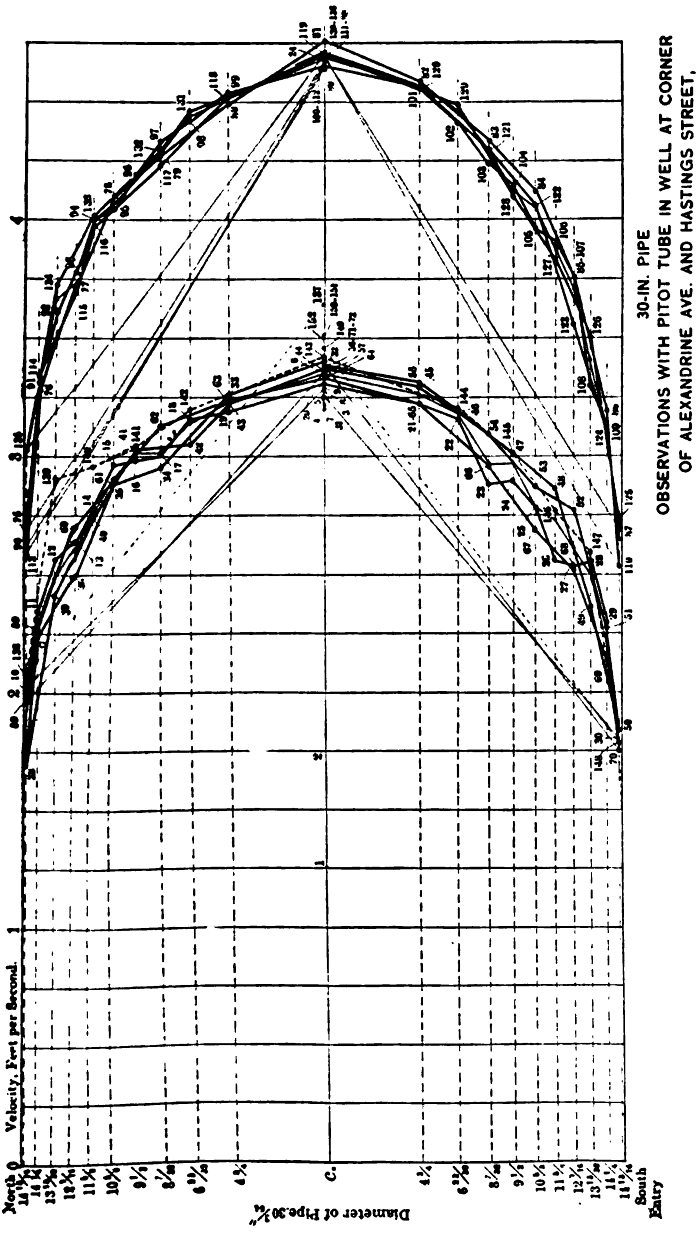


Figure 8.—Normal Velocity Curves for Flow in a Straight Pipe.

(From Experiments by Williams, Hubbell and Fenkell, Trans. Am. Soc. C. E. Vol. 47.

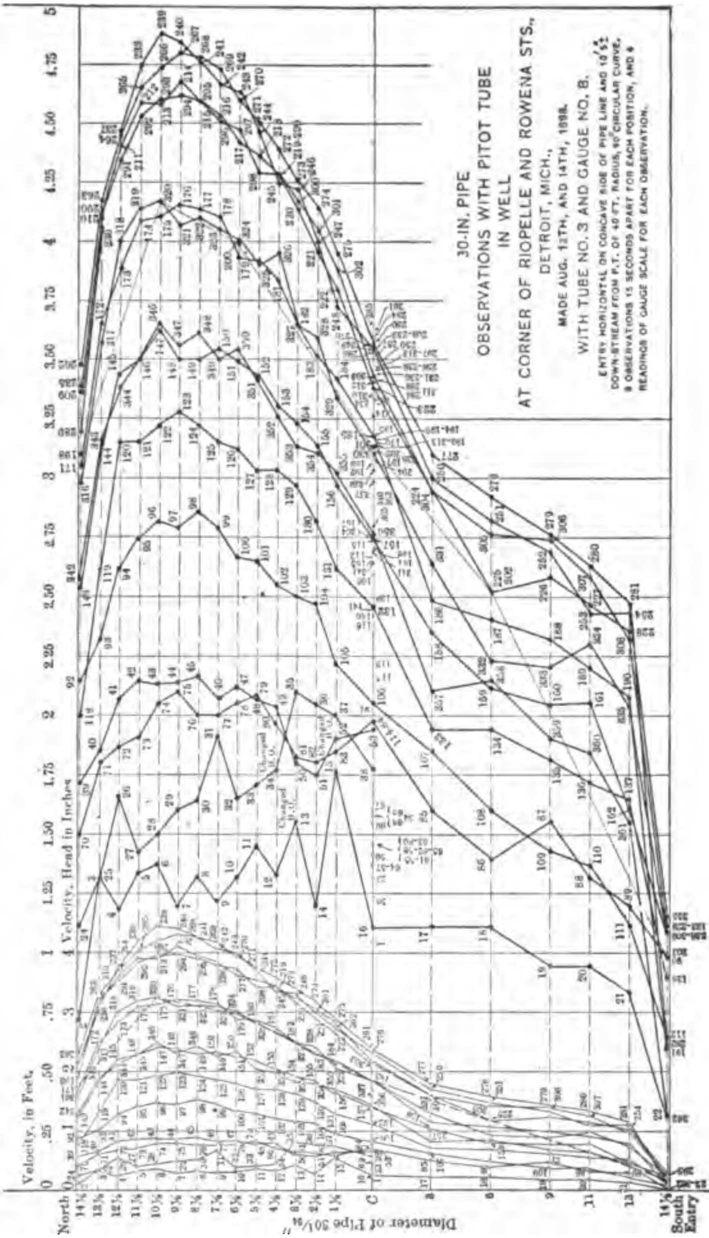


Figure 9.—Horizontal Velocity Curves in Straight Pipe Downstream from Curved Pipe.

From Experiments by Williams, Hubbell and Fenkell, Trans. Am. Soc. C. F. Vol. 47 1902.)

eters C and D by the distance between piezometers. The solid line represents the average of the points for the normal straight pipe friction per foot of pipe, and the dash line represents the average of the points determined for loss per foot in this section when the flow was disturbed by a tee with the water leaving through the branch. The squares indicate the loss when the water was entering the tee through the branch. The difference between the ordinates of the two lines, at any velocity, is the average loss per foot of pipe between C and D, caused by the distortion of the velocity curves or by the eddies due to the tee. This loss is in addition to the normal pipe friction loss. At a velocity of 10 feet per second the vertical intercept between the two lines in Fig. 10 shows that the average excess loss of head, between C and D, due to the tee, was about 0.02 foot per foot of pipe. In a similar manner the loss of head per foot of pipe between piezometers B and C was plotted and the normal pipe friction in the 2 feet of straight pipe between these piezometers was deducted from it. At a velocity of 10 feet per second the loss between B and C due to the tee was 2.34 feet. Piezometer C was located 1 foot down stream from the elbow. The average length of path of the particles of water passing through the tee was estimated to be 0.2 foot. The total excess loss between B and C occurred after the water entered the tee and was therefore distributed over 1.2 feet length. The average loss of head per foot of pipe between the entrance to the tee and piezometer C due to the tee was therefore 1.95 feet. Between piezometers D and E no measurable excess loss of head occurred.

$\frac{2.34}{1.2} = 1.95$

The excess losses of head due to a velocity of ten feet per second in the tee are shown graphically in Fig. 11. The rectangular areas in this figure represent to scale the total excess loss between the piezometers, and the altitude of the rectangles represent the average excess loss of head per foot of pipe. Although the shaded areas represent the total loss accurately they do not show the true distribution of the losses along the pipe.

As soon as the fluid enters the tee eddies are set up due to the enlargement of section; as it passes on through the tee the violence of the eddies is greatly increased by the sudden change of direction of flow. It is probable that the greatest violence of eddy motion occurs in the tee and consequently that the

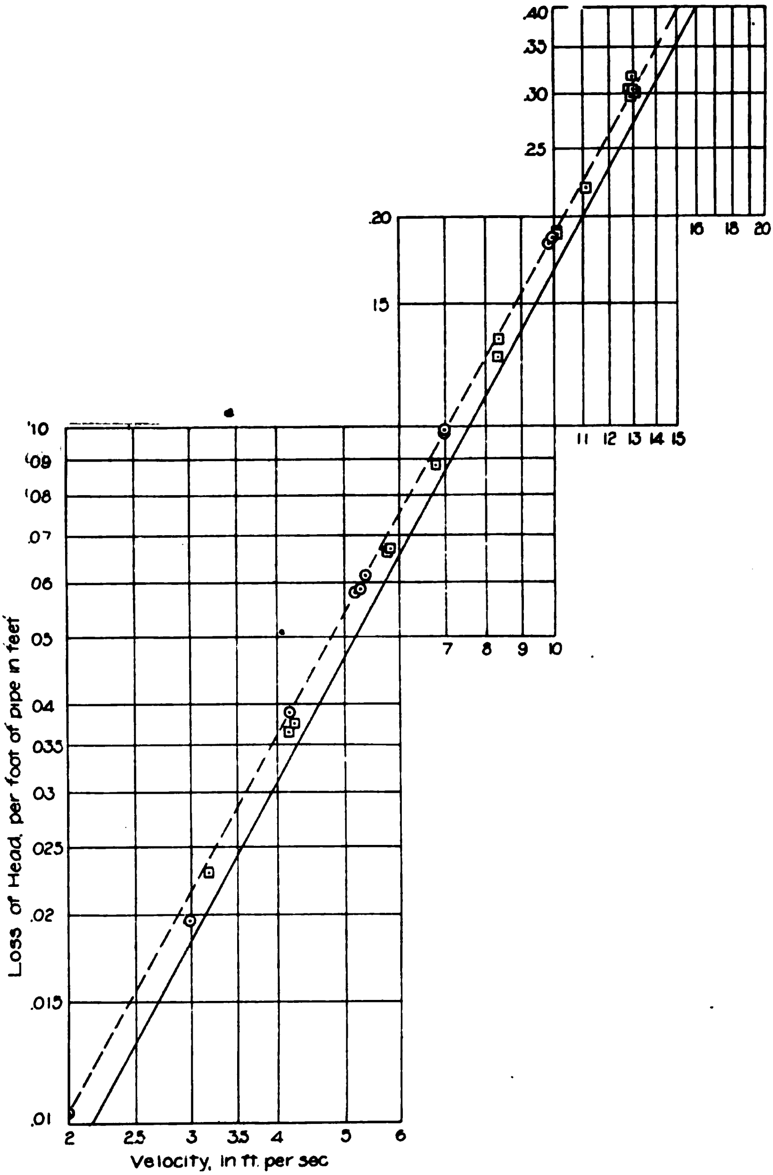


Figure 10.—Logarithmic Plotting of Loss of Head due to a Tee.

greatest rate of loss of head occurs there also. As the water leaves the tee and enters the pipe sudden contraction maintains the violent eddy motion for a short distance, after which the fluid gradually assumes its normal condition of flow. Assuming that the loss of head per foot of pipe is proportional to the violence of the eddies in the pipe it seems probable that the loss of head per unit length of pipe varies approximately as the ordinates to the curved line in Fig. 11. The total area under this curved line is equal to the areas of the rectangles, hence the total excess loss, due to the tee, occurring between the various piezometers is represented by the curved line as well as by the rectangles. If the curved line is even approximately correct it seems reasonable to assume that the effect of the tee did not cause any measurable loss of head beyond 20 diameters down stream from the tee, and therefore that the entire loss due to the elbows was measured in the Wisconsin experiments.

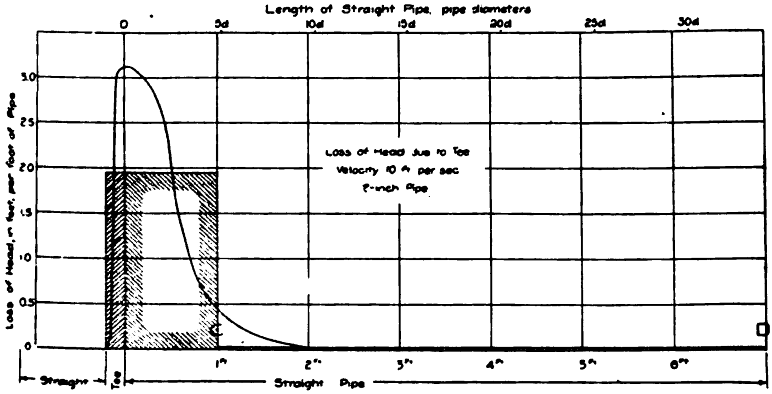


Figure 11.—Distribution of Losses in a Tee and the Following Straight Pipe

In making this study the results of the experiments on the tee were used because the tee caused a greater loss than any of the other elbows used and also because the loss due to the tee was apparent further down stream than was that from any other elbow.

STUDY OF CURVE RESISTANCE

The results of experiments on curve resistance, may be considered in many ways, only two of which will be discussed in this bulletin.

Case 1. The curve resistance may be considered to be the difference in the loss of head occurring in a straight pipe and a similar pipe curved through an angle of 90 degrees; all other conditions of center-line length, diameter, shape of cross-section, roughness of inner surface, velocity of flow, etc. being the same in the two pipes. These net losses being due to the effects of curvature alone, it is probable that they are practically the same for old and new pipe.

Case 2. The curve resistance may be considered to be the total loss of head due to an elbow or curve. The magnitude of the loss determined on this basis for a given velocity will differ from that determined on the basis of Case 1 by the amount of the normal friction loss in a straight pipe of length equal to the center-line length of the elbow. Since the total loss includes that caused by the roughness of the inner surface of the pipe, it will not be constant, but will vary with the roughness of the pipe, which varies with the age of the pipe.

The curve resistance determined on the basis of either of the two above cases may be expressed in the following ways.

(a) The loss of head for the entire 90° turn, either net (Case 1) or total (Case 2), expressed in linear units, as feet, inches or meters. Sometimes the curve resistance is expressed as the loss of pressure instead of the loss of head.

(b) The loss of head, or of pressure, per unit length of pipe. The unit of length generally chosen has been the foot. Such a unit of fixed length is convenient for studying the relative losses in bends of different radii in a given size of pipe, but for more general use in studying the losses due to curves in different sized pipes, it is more instructive to use as a unit the radius of curvature expressed in pipe diameters.

(c) The loss of head or pressure equal to that caused by a given length of straight pipe.

(d) The loss of head expressed in terms of the velocity head.

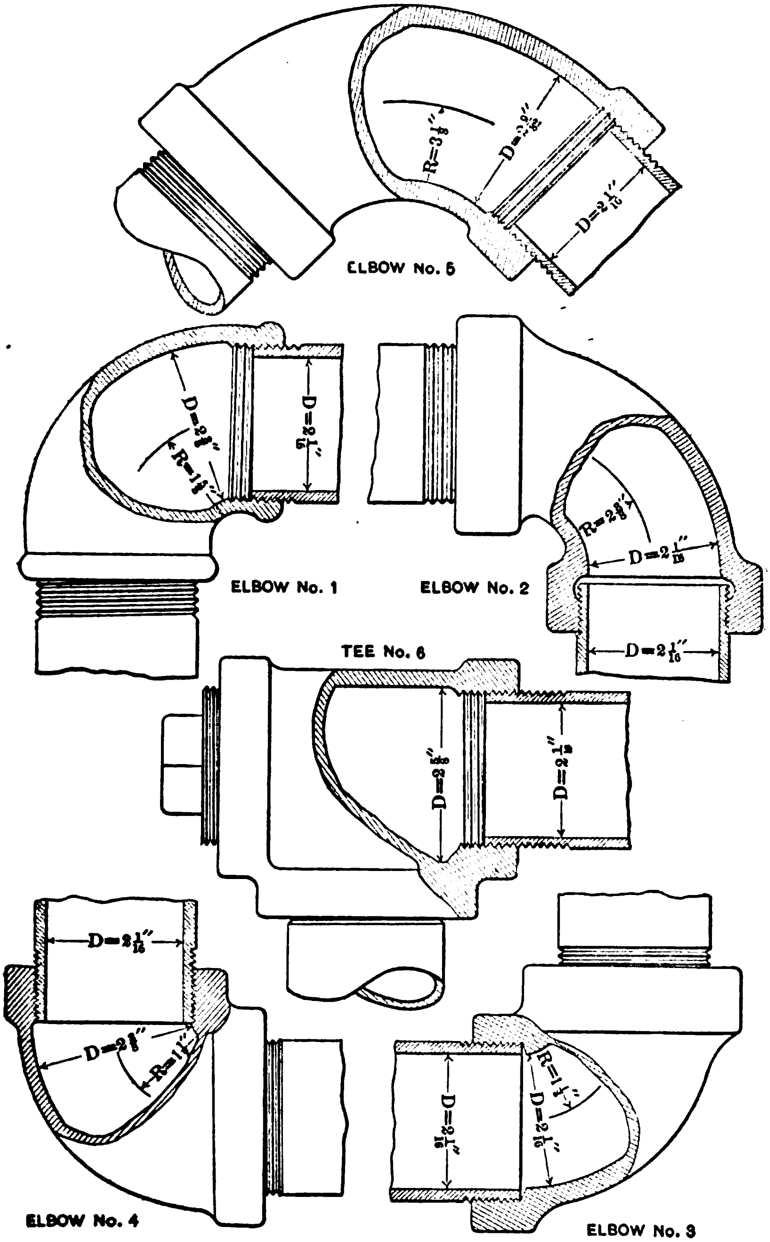


Figure 12.—Dimensions of the Commercial Fittings Used in the Wisconsin Experiments.

UNIVERSITY OF WISCONSIN EXPERIMENTS

Experiments on two-inch elbows and bends were made in the Hydraulic Laboratory of the University of Wisconsin by students working under the direction of the writer.

The first of these series of experiments were made by H. Hosler, C. M. Kehr and R. K. McComb during the spring of 1908, and their results were embodied in an unpublished thesis presented for the degree of Bachelor of Science in Engineering. Their work included the tests of the elbows numbered 1, 2, 3, 4, 5 and 6 in Table 1 and Figs. 2, 3, 4, 5, 6, 7 and 12. The test of number 6, which is a cast iron tee, was made with the water leaving by the branch of the tee. The results obtained in the spring of 1908 with elbow number 5 were not satisfactory and are therefore not presented in this bulletin.

During the summer of 1908 the work was continued by A. F. Coleman, C. M. Kehr and B. M. Reynolds. They repeated the work on elbow number 5 with satisfactory results, tested the tee with the direction of flow reversed, the water entering the branch of the tee, and experimented with curves numbered 7, 8 and 9.

Further work was done during the school year of 1908 and 1909 by F. W. Greve, Jr. and F. M. Graham on long radius bends, numbered 10, 11 and 12 in Table I. In August, 1909, the writer made a number of runs to determine the amount of the change that had occurred in the pipe friction in two sections of straight pipe and also some runs to check the losses in curve 12.

DESCRIPTION OF APPARATUS

All the curves experimented on made a turn of 90° and all were left uncoated. Numbers 1 to 6, inclusive were commercial fittings purchased from the Crane Company. Number 7 was cast from a pattern made in the shop of the mechanician of The Engineering College. It was similar to numbers 2 and 3, the only difference being in the longer radius of curvature. Numbers 8, 9, 10, 11 and 12 were bent from 2 inch pipe by the

University blacksmith. For convenience in bending and threading the ends of the bent pipes they were made a foot longer than the 90° arc. The latter five were fitted with companion flanges screwed on as in ordinary practice. The joint, therefore, was not flush, but caused a slight enlargement of section for a fraction of an inch. Referring to Table 1 and the views of the curves it may be seen that curves 2, 3, 7, 8, 9 and 10 had actual internal diameters of 2$\frac{1}{16}$ inches. They, and curves 11 and 12, were equal in diameter, at the joint, to the pipes with which they connected, while curves 1, 4 and 5 and the tee were of larger diameter than the pipe. The diameters of the cast and

TABLE 1

DESCRIPTION OF CURVES

1.	2.	3.	4.	5.	6.	7.	8.
No. of curve	Internal diameter in inches	Radius in inches. R.	Radius in feet.	$\frac{R^*}{d_p}$	Length, in feet.	Material.	Description.
1	2⅜	1⅝	.135	.788	.21	Malleable iron	Elbow.
2	2$\frac{1}{16}$	2⅜	.198	1.15	.31	Cast iron	Long turn, drainage elbow.
3	2$\frac{1}{16}$	1½	.125	.728	.20	Cast iron	Drainage elbow.
4	2⅜	1½	.125	.728	.20	Cast iron	Short turn.
5	2$\frac{9}{32}$	3⅛	.260	1.52	.41	Cast iron	Long sweep.
6	....	0	0	0	0	?Cast iron	Tee.
7	2$\frac{1}{16}$	5$\frac{5}{32}$	.430	2.5	.68	Cast iron	Special casting.
8	2$\frac{1}{16}$	10$\frac{5}{16}$	.861	5.0	1.35	Wrought iron or steel	Bent pipe.
9	2$\frac{1}{16}$	20⅝	1.720	10.0	2.71	Wrought iron or steel..............	Bent pipe.
10	2.06	31	2.580	15.0	4.06	Wrought iron or steel..............	Bent pipe.
11	2.03	42½	3.540	20.6	5.56	Wrought iron or steel..............	Bent pipe.
12	2.01	85	7.080	41.2	11.12	Wrought iron or steel..............	Bent pipe.

* $\frac{R}{d_p}$ = Ratio of radius of curve to diameter of straight pipe, d_p = 2$\frac{1}{16}$ inches.

malleable iron elbows and tees were determined, after the completion of the experiments, by cutting them, as shown in the figures, and then calipering them. In the case of the bent pipe curves the mean diameter was found by volumetric measurements. A cap was screwed on one end and the curved pipe was filled with water. From the weight of water contained by the pipe, deducting the contents of the cap, the mean diameter was readily computed. The results of two measurements for curve number 12 are given in Table 2. The diameter of this pipe was determined both before and after bending. A comparison of the two results indicated that the mean diameter was decreased one per cent by bending.

TABLE 2

THE UNIVERSITY OF WISCONSIN, HYDRAULIC LABORATORY

Experiment on Determination of diameter of curve No. 12.
Data by Greve and Graham. Computed by Davis.
Date of experiment, May 20, 1909.
General data, Cubical contents of cap=2.74 cubic inches.

Length of curved pipe, in inches.	Weight of water contained in pipe and cap, in ounces.	Volume of water in pipe and cap, in cubic inches.	Net contents of pipe, in cubic inches.	Mean area of section, in square inches.	Mean diameter, in inches.
133.5	245.5				
	246.5				
	246	425	422.26	3.16	2.005

A general plan of the apparatus is shown in Fig. 13.

The water used in the experiments was drawn from the University mains through a two-inch pipe; the rate of flow being controlled by a gate valve placed a few inches from the entry to a drum. The purpose of this drum, which was 8 inches in diameter and 4 feet long, was to free the water from the violent eddies caused by the gate valve and other fittings in the supply pipe. While testing the two straight pipes for friction loss it was concluded, from the violent oscilations of the fluid in the gages, that the drum was not satisfactorily serving its purpose. Therefore, two baffles were placed in the drum in about the position shown in the figure. They were made of wood one inch thick, each containing 36 ¾ inch holes. The baffles were spaced about 2 inches apart and so arranged that the holes in one were opposite solid spaces in the other. After the installation of these baffles the flow was much steadier. The water left the drum and entered the pipe through a bell-mouth or rounded entrance.

The straight pipes, which were new at the beginning of the experiments, were made of iron or steel with lap-welded longitudinal joints. The nominal size of the pipes was 2 inches, but the actual internal diameter, as determined by calipering the ends was, as above noted, $2\frac{1}{16}$ inches. The two lengths of pipe immediately above and below the curve are designated in the tables and illustrations as pipes number 1 and 2. Pipe number 1 was always placed down stream and pipe number 2 up stream from the curve, as shown in Fig. 13. Pipe number 1 was provided with three piezometers spaced 1, 7 and 17 feet respectively from its upstream end. Pipe number 2 was provided with two piezometers spaced 1 and 11 feet respectively from its downstream end. The five piezometers are designated A, B, C, D and E, beginning with the one on pipe number 2 farthest upstream. During the first series of experiments the distance "l" between the drum and piezometer A was only 9 feet. This distance was changed during the following experiments to 62 feet

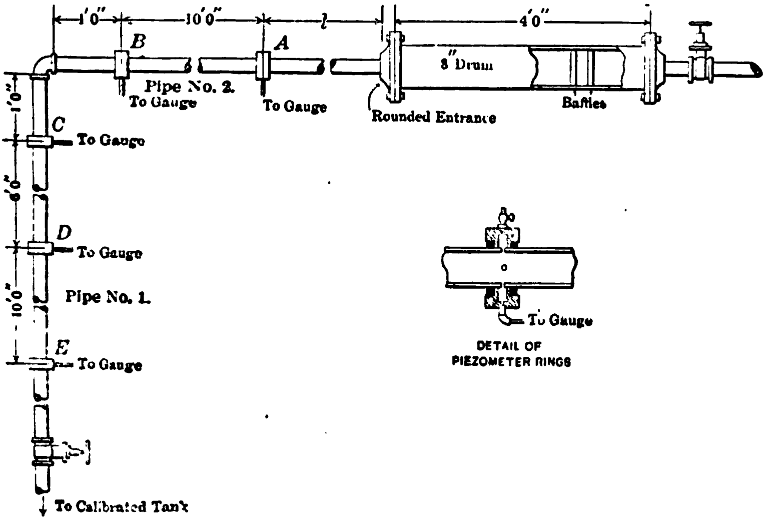


Figure 13.—General Plan of the Apparatus Used in the Wisconsin Experiments.

by inserting several lengths of pipe. The pipe joints in this length "1" were made with couplings.

There was a gate valve at the down-stream end of the pipe the purpose of which was to enable sufficient pressure to be put on the apparatus to effectively flush out the gages, piezometers and connecting tubes.

The pipes were used full length, as received from the maker. The nipples shown in Figs. 2 to 7 inclusive were not used in the experiment, but were inserted afterwards to show the exact distance that the pipes were introduced into the fittings. There was no bur on the ends of the pipes used in the experiment.

The piezometers consisted of four ¼ inch holes drilled through the pipe walls, 90° apart, and communicating with an equalizing chamber intended to average the pressure in case any difference existed at the different points on the circumference of the pipe, and also to act as a reservoir to trap all air bubbles passing through the piezometer openings. A cross-section of one of the piezometers is shown in Fig. 13. The piezometers were connected to the gages by ¼ inch iron pipes. No attempt was made to have these pipes leading to the various piezometers of equal length or capacity.

Fluid differential gages, containing air as the gage fluid, were used to measure the loss of head. A drawing of one of the gages is shown in Fig. 14. The gage scales were graduated to read in feet, tenths and hundredths.

During the first two sets of experiments the quantity of water discharged was measured in a cylindrical, galvanized iron tank, of 50 cubic feet capacity, on which a scale had previously been calibrated to give readings to ½ cubic foot. During the third set of experiments the quantity of water discharged was determined by weighing it on platform scales.

The duration of runs was determined by stop-watches regulated to keep practically correct time.

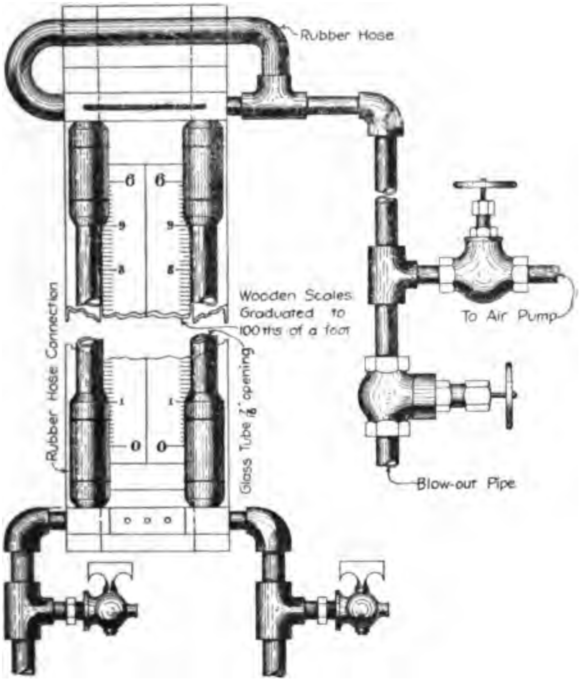


Figure 14.—Details of Fluid Differential Gage.

METHOD OF EXPERIMENTATION

The friction loss in the straight pipe included between piezometers A and B, and C and E, was determined a number of times during the series of experiments. Each pipe was tested separately. During the first series the determination was made by screwing the pipes directly into the drum. With this arrangement piezometer A was 9 feet from the drum and with pipe number 1 piezometer E was three feet from the drum. During the later series several lengths of straight pipe intervened between the drum and the pipe experimented upon as in the case with the elbows.

After determining the friction loss in the straight pipes they were connected to the curves as shown in Fig. 13. During the first two series of experiments on curves and elbows three gages were used to measure separately the loss of head between piezometers B and C, C and D, and D and E, but during the last series only one gage was used, connected between piezometers B and E.

The gage readings were recorded to thousandths of a foot, but owing to the oscillations of the water columns the last figure is only approximate as the gage height for any reading was merely estimated by eye during a few oscillations. During the first series of observations the recorded readings are the estimated average lower range of the oscillating water surface. The writer was not aware that this method of reading the gage was being used until the first series of experiments was nearly completed. This method of reading a differential gage would give correct differences of head provided the range of the oscil-

TABLE 3

THE UNIVERSITY OF WISCONSIN, HYDRAULIC LABORATORY

Experiment on Friction in straight pipe No. 1.
Data by F. W. Greve, Jr. and C. G. Burritt. Computed by F. W. G., Jr.
Date of experiment. Nov. 23, 1908.
General data, Nominal diameter = 2 inches.
Actual diameter = 2$\frac{1}{16}$ inches.
Actual area = 0.0232 square feet.

Run No.	Weight, in Pounds.			Discharge, in cubic feet per second.	Velocity in feet per second.	Duration of run.	Gage Readings.			
							10′ length.		6′ length.	
	Initial.	Final.	Net.				Right.	Left.	Right.	Left.
15	1000	4000	3000	.0790	3.40	10 m-8 s	1.705	1.450	1.795	1.662
....	....	....	....	....	....	....	1.700	1.452	1.797	1.665
....	....	....	....	....	....	....	1.711	1.447	1.803	1.666
....	....	....	....	....	....	....	1.695	1.448	1.808	1.666
....	....	....	....	....	....	....	1.699	1.459	1.805	1.667
....	....	....	....	....	....	....	1.697	1.458	1.804	1.665
....	....	....	....	....	....	....	1.699	1.505	1.807	1.665
....	....	....	....	....	....	....	1.699	1.456	1.800	1.664
....	....	....	....	....	....	....	1.694	1.453	1.801	1.665
....	....	....	....	....	....	....	15.299	13.128	16.220	14.985
....	....	....	....	....	....	....	9)	2.171	9)	1.235
....	....	....	....	....	....	....	....	.241	....	.137

TABLE 4

The University of Wisconsin, Hydraulic Laboratory

Experiment on Curve No. 9.
Data by A. F. Coleman, C. M. Kehr, and B. S. Reynolds.
Date of experiment, July 8, 1908.

General data $\frac{R}{d_p} = 10$. Length of curve, 2.71 ft. Distance between piezometers: B—C=5. 71 ft. Area of cross-section of pipe=0.0232 sq. ft.

Time in seconds.	Gage, B—C, in Feet.		Gage, C—D, in Feet.		Gage, D—E, in Feet.		Total loss of head, B-E in feet.	Total discharge in cu. feet.	Velocity in feet per second.
	Right.	Left.	Right.	Left.	Right.	Left.			
904	2.515	2.468	2.528	2.485	1.656	1.589	.155	35	1.67
....	2.509	2.466	2.535	2.500	1.658	1.590	....	....	....
....	2.514	2.464	2.535	2.495	1.658	1.590	....	....	....
....	2.513	2.461	2.534	2.492	1.658	1.590	....	....	....
....	2.512	2.465	2.540	2.500	1.657	1.589	....	....	....
....	2.517	2.470	2.540	2.502	1.658	1.590	....	... .	....
....	2.517	2.470	2.537	2.497	1.658	1.590	....	....	....
....	2.517	2.470	2.532	2.495	1.657	1.589	....	....	....
.. ..	2.511	2.458	2.532	2.495	1.657	1.589	....	....	....
... .	2.513	2.460	2.535	2.498	1.657	1.589	....	....	....
....	25.138	24.655	25.348	24.959	16.574	15.895	....	....	....
....	10)	.483	10)	.389	10)	.679	....	....	....
....	....	.0483	....	.0389	....	.0679	....	....	....

lations in the tubes of the gage were the same. They were not the same in this experiment on account of the tubes leading from the piezometer to the gages being of different length. A slight error of a few thousandths of a foot was, therefore, introduced into the work by the method of reading, but it affects only the results for the short radius' curves, which produce large losses of head. Hence, the general results are not much affected. In the later experiments the average height of the water surface in the gage tubes was read.

In Tables 3 and 4 are presented samples of data taken on one of the straight pipes and with one of the curves.

The computed results of the observations are presented in

TABLE 5

THE UNIVERSITY OF WISCONSIN, HYDRAULIC LABORATORY

Experiment on Friction loss in straight pipe.
General data. Length of section A — B = 10 feet.
Length of section C — D = 6 feet.
Length of section D — E = 10 feet. Wooden baffles put in before Mar. 12, 1908. Direction of flow on Mar. 30, 1908, was E to C.

PIPE No. 1

Date 1908.	LOSS OF HEAD IN FEET.		Mean velocity, in feet per second.	LOSS OF HEAD PER FOOT OF PIPE, IN FEET.	
	Section C—D.	Section D—E.		Section C—D.	Section D—E.
Feb. 29	0.570	0.930	7.24	0.0950	0.0930
	0.585	0.973	7.30	0.0975	0.0973
	0.586	0.960	7.33	0.0976	0.0960
	0.596	0.969	7.33	0.0995	0.0969
	0.600	0.948	7.32	0.1000	0.0948
	0.605	0.972	7.43	0.1010	0.0972
	1.045	1.629	9.88	0.1740	0.1630
	1.030	1.640	9.90	0.1720	0.1640
	1.021	1.632	9.83	0.1700	0.1630
	1.013	1.626	9.88	0.1690	0.1630
	1.067	1.615	10.02	0.1780	0.1620
	1.063	1.649	10.01	0.1770	0.1650
	1.365	2.201	11.50	0.2280	0.2200
	1.417	2.206	11.60	0.2360	0.2210
	1.432	2.212	11.60	0.2380	0.2210
	1.371	2.210	11.60	0.2280	0.2210
	1.372	2.220	11.60	0.2290	0.2220
	1.346	2.226	11.60	0.2410	0.2230
Mar. 2	1.838	2.897	13.40	0.3060	0.2900
	1.825	2.842	13.30	0.304	0.284
	1.838	2.878	13.40	0.306	0.2880
	1.840	2.802	13.30	0.307	0.2800
	1.769	2.930	13.40	0.294	0.2930
	1.839	2.915	13.40	0.306	0.2920
Mar. 30	0.235	0.354	4.35	0.0392	0.0354
	0.114	0.188	3.02	0.0190	0.0188
	1.281	1.978	11.0	0.214	0.1978
	1.249	1.935	10.7	0.208	0.1935

TABLE 5—Continued

PIPE No. 1

Date 1908.	Loss of Head, in Feet.		Mean velocity, in feet per second.	Loss of Head per Foot of pipe, in Feet.	
	Section C—D.	Section D—E.		Section C—D.	Section D—E.
	1.291	1.992	11.0	0.215	0.1992
	1.458	2.301	11.8	0.243	0.230
	0.802	1.242	8.53	0.134	0.124
	0.803	1.260	8.59	0.134	0.126
	0.792	1.222	8.42	0.132	0.122
Mar. 30	0.545	0.844	6.86	0.0910	0.0844
	0.543	0.833	6.84	0.0906	0.0833
July 29	0.067	0.114	2.25	0.0112	0.0114
	0.103	0.179	2.95	0.0172	0.0179
	0.177	0.315	4.04	0.0295	0.0315
	0.289	0.501	5.07	0.0482	0.0501
	0.409	0.741	6.33	0.0681	0.0741
	0.513	0.889	7.06	0.0855	0.0889
	0.717	1.259	8.53	0.120	0.126
	1.016	1.800	10.28	0.169	0.180
	1.512	2.623	12.47	0.252	0.262
	1.966	3.454	14.73	0.327	0.345
Nov. 20	0.470	0.840	6.57	0.0784	0.0840
	0.464	0.831	6.48	0.0774	0.0831
	0.469	0.824	6.52	0.0781	0.0824
	0.471	0.823	6.50	0.0785	0.0823
	0.669	1.214	8.02	0.111	0.121
	0.664	1.211	8.07	0.111	0.121
	0.692	1.226	8.02	0.115	0.123
	0.688	1.194	7.95	0.115	0.119
	0.768	1.340	8.43	0.128	0.134
	0.747	1.354	8.40	0.124	0.135
	0.760	1.367	8.55	0.127	0.137
	1.290	2.341	11.2	0.215	0.234
	1.297	2.336	11.4	0.216	0.234
	1.392	2.503	11.8	0.232	0.250
Nov. 23	0.137	0.241	3.40	0.0228	0.0241
	0.141	0.269	3.43	0.0235	0.0269

TABLE 5—Continued

PIPE No. 1

Date 1908.	LOSS OF HEAD, IN FEET.		Mean velocity, in feet per second.	LOSS OF HEAD PER FOOT OF PIPE, IN FEET.	
	Section C—D.	Section D—E.		Section C—D.	Section D—E.
	0.132	0.250	3.41	0.0220	0.0250
	0.142	0.243	3.39	0.0236	0.0243
	0.131	0.231	3.28	0.0218	0.0231
	0.133	0.240	3.30	0.0222	0.0240
Nov. 24	0.131	.234	3.28	0.0218	0.0234
	0.133	0.237	3.28	0.0222	0.0237
1909. Aug. 20	....	.083	1.83	....	0.0083
	....	0.129	2.29	....	0.0129
	....	0.183	2.79	....	0.0183
	....	0.244	3.28	....	0.0244
	....	0.432	4.45	....	0.0432
	....	0.500	4.80	....	0.0500
	....	0.652	5.56	....	0.0652
Aug. 23	....	1.514	8.57	....	0.1514
	....	2.429	10.90	....	0.243
	....	1.092	7.26	....	0.1092
	....	1.154	7.40	....	0.1154
	....	0.777	6.03	....	0.0777
Aug. 4	0.291	....	4.51	0.0485	....
	0.572	....	6.38	0.0955	....
	0.830	....	7.81	0.138	....
	1.271	....	9.66	0.212	....
	0.192	....	3.62	0.0320	....
	0.506	....	6.00	0.0845	....
Aug. 25	0.109	....	2.66	0.0182	....
	0.339	....	4.90	0.0565	....
	1.450	....	10.35	0.0242	....

TABLE 5—Continued

PIPE No. 2

Date 1908.	Loss of head, in feet. Section A—B.	Mean velocity, in feet per second.	Date 1908.	Loss of head, in feet. Section A—B.	Mean velocity, in feet per second.
Mar. 4	0.182	2.75	June 25	0.476	5.04
	0.442	4.62		0.458	5.04
	0.440	4.62	June 26	0.753	6.65
	1.118	8.11		0.757	6.65
	1.115	8.17		0.775	6.73
	1.036	7.84		0. 74	6.73
	1.025	7.75		0.933	7.23
	1.081	7.98		0.935	7.19
	1.680	10.13		0.916	7.13
	1.878	10.77		0.951	7.23
	1.796	10.54		0.943	7.02
	1.791	10.42		1.604	9.43
	2.620	12.90		1.550	9.32
	2.500	12.60		1.534	9.27
Mar. 12	0.190	2.83		1.563	9.33
	0.490	5.14	June 27	1.859	10.60
	1.065	7.83		1.776	10.40
	1.052	7.83		1.858	10.62
	1.791	11.00		1.870	10.68
	1.805	11.10		2.221	11.80
June 25	0.123	2.38		2.351	12.09
	0.049	1.41		2,399	12.21
	0.316	3.94		2,325	12.05
	0.318	4.03		3.388	14.70
	0.318	4.02		3.326	14.58
	0.488	5.11		3.581	15.04
	0.463	5.02		3.526	14.94

TABLE 6

WISCONSIN EXPERIMENTS
FRICTION LOSS IN CURVED PIPE

Curve No. 1.		Curve No. 2.		Curve No. 3.		Curve No. 4.		Curve No. .	
March, 1908.		March, 1908.		March, 1908.		March, 1908.		July, 1908.	
Mean velocity, in feet per second.	Loss between B & E, in feet.	Velocity.	Loss, B—E.	Velocity.	Loss, B—E.	Velocity.	Loss, B—E.	Velocity.	Loss, B—E.
1.80	0.177	1.86	.189	2.72	.368	1.93	.234	1.66	.136
2.46	0.300	3.75	.609	3.70	.650	1.98	.242	2.30	.252
2.67	0.375	3.77	.583	4.87	1.05	2.97	.505	3.12	.434
3.79	0.682	4.98	1.190	4.88	1.04	5.28	1.66	4.29	.792
4.35	0.843	5.08	1.210	5.03	1.15	6.50	2.34	4.35	.815
4.35	0.846	6.62	2.040	5.04	1.15	10.22	5.21	6.17	1.57
4.86	1.098	6.83	2.080	5.08	1.15	10.24	5.28	6.47	1.71
6.26	1.65	6.84	2.080	6.48	1.80	10.29	5.31	6.49	1.74
6.26	1.65	8.42	2.820	6.72	1.99	10.31	5.30	7.52	2.29
5.94	1.60	8.48	2.860	6.73	1.94	11.52	6.54	7.58	2.29
5.97	1.62	8.53	2.940	6.92	2.06	11.52	6.54	8.68	3.01
7.16	2.27	9.78	3.840	8.67	2.99	11.52	6.55	9.05	3.24
7.30	2.34	9.87	3.920	8.74	3.14	11.54	6.56	10.24	4.10
7.31	2.29	10.00	3.910	8.76	3.20	13.12	8.38	10.34	4.30
7.95	2.75	10.10	4.040	8.86	3.21	13.25	8.54	10.44	4.28
7.98	2.76	11.70	5.260	9.93	4.05	13.25	8.60	11.93	5.58
7.98	2.75	11.70	5.460	9.95	4.03	13.27	8.61	11.95	5.61
8.20	2.76	11.70	5.460	10.00	4.04	....	....	12.16	5.74
8.23	2.79	14.2	7.590	10.90	4.38	....	....	13.52	7.06
8.31	2.81	14.2	7.610	11.00	4.92	....	....	13.61	7.13
8.36	2.84	14.2	7.620	11.00	4.93	....	....	13.70	7.17
8.48	2.93	14.4	7.860	11.00	4.94	....	....	13.70	7.18
11.34	5.07	....	....	13.50	7.09	....	....	....	....
11.46	5.19	....	....	13.50	7.20	....	....	....	....
11.52	5.25	....	....	18.50	7.24	....	....	....	....
11.65	5.18	....	....	13.50	7.33	....	....	....	....
....	....	....	....	13.70	7.34	....	....	....	....

TABLE NO. 6—Continued

CURVE NO. 6.				CURVE NO. 7.		CURVE NO. 8.		CURVE NO. 9.	
Tee. (Water leaving branch.)		Tee. (Water entering branch.)							
March, 1908.		July, 1908.		July, 1908.		July, 1908.		July, 1908.	
Velocity.	Loss, B—E.	Velocity.	Loss, B—E.	Velocity.	Loss, B—E.	Velocity.	Loss, B—E.	Velocity.	Loss, B—E.
1.50	.201	1.40	.185	2.19	.201	1.75	.151	1.67	.155
1.99	.274	1.96	.245	3.00	.373	2.34	.248	2.14	.233
2.98	.530	3.18	.624	3.60	.520	3.31	.479	3.27	.510
4.16	1.10	4.17	1.08	5.53	1.18	5.22	1.13	4.37	.921
5.17	1.75	4.22	1.05	6.59	1.62	6.70	1.80	4.58	.958
5.27	1.79	5.74	1.90	7.57	2.12	6.73	1.84	6.45	1.84
5.37	1.86	5.78	1.91	8.86	2.87	7.28	2.12	6.53	1.85
6.98	3.09	5.79	1.90	8.88	2.89	7.45	2.26	6.55	1.89
6.98	3.09	5.81	1.92	10.68	4.18	8.58	2.96	7.40	2.38
9.84	5.70	6.79	2.63	10.70	4.23	8.93	3.12	7.40	2.39
9.97	5.87	6.98	2.79	12.09	5.21	9.12	3.26	7.55	2.39
10.03	5.89	8.32	3.76	12.20	5.31	10.40	4.21	8.82	3.30
10.07	5.94	8.39	3.88	12.37	5.47	10.71	4.40	8.82	3.30
....	....	10.09	5.56	14.70	7.56	10.76	4.50	8.92	3.37
....	....	10.11	5.58	14.86	7.72	12.30	5.78	8.93	3.41
....	....	10.11	5.64	15.05	7.90	12.37	5.90	10.55	4.67
....	....	11.10	6.75	15.05	7.91	12.42	5.88	10.55	4.72
....	....	11.10	6.75	....	....	14.74	8.01	10.69	4.79
....	....	11.10	6.72	....	....	14.80	8.01	10.71	4.78
....	....	11.20	6.73	....	....	14.80	8.05	12.17	6.06
....	....	11.20	6.68	....	....	....	....	12.44	6.38
....	....	12.90	9.06	....	....	....	....	12.44	6.44
....	....	12.90	8.93	....	....	....	....	12.60	6.52
....	....	12.90	9.10	....	....	....	....	14.30	8.38
....	....	13.00	9.04	....	....	....	....	14.33	8.42
....	....	13.10	9.10	....	....	....	....	14.33	8.46
....	....	13.10	9.14	....	....	....	....	14.43	8.47
....	....	....	....	....	....	....	....	14.43	8.48

TABLE 6.—Continued

Curve No. 10.		Curve No. 11.		Curve No. 12, Before Bending.		Curve No. 12.		Curve No. 12.	
Dec., 1908.		Jan., 1909.		March, 1909.		May, 1909.		Aug., 1909.	
Velocity.	Loss, B—E.	Velocity.	Loss, B—E.	Velocity.	Loss, B—E.	Velocity.	Loss, B—E.	Velocity.	Loss, B—E.
1.95	.236	1.84	.232	7.47	3.243	1.58	.210	1.88	.283
1.96	.234	1.93	.264	7.62	3.373	1.59	.210	2.09	.343
1.96	.237	1.94	.238	7.28	3.139	1.59	.208	2.31	.410
1.98	.239	1.94	.244	7.25	3.037	1.59	.206	2.88	.613
1.96	.235	1.94	.245	7.03	2.928	1.59	.215	3.65	.954
1.98	.237	1.95	.247	6.90	2.723	1.60	.210	4.23	1.276
1.99	.239	1.95	.240	7.03	2.882	1.61	.213	4.63	1.517
5.60	1.61	5.53	1.61	6.90	2.723	4.61	1.37	6.05	2.643
5.62	1.63	5.54	1.59	7.03	2.882	4.64	1.40	2.81	.583
5.68	1.66	5.55	1.63	7.42	3.131	4.67	1.42	4.39	1.351
5.68	1.67	5.55	1.63	7.21	3.066	4.67	1.42	5.05	1.782
5.72	1.66	5.60	1.64	1.562	.201	4.67	1.41	5.18	1.844
5.77	1.71	5.61	1.65	1.585	.200	4.69	1.41	6.10	2.517
5.88	1.76	5.67	1.68	1.552	.198	4.71	1.43	6.02	2.444
5.90	1.78	5.74	1.72	1.518	.195	7.04	3.07	6.33	2.828
7.83	3.03	7.16	2.92	1.532	.197	7.05	3.06	6.58	2.891
7.85	3.02	7.36	2.82	1.508	.195	7.06	3.06	2.18	.367
7.88	3.05	7.39	2.83	1.526	.198	7.08	3.07	....	....
7.88	3.06	7.41	2.86	4.65	1.353	7.10	3.10	....	....
7.88	3.06	7.41	2.85	4.64	1.335	7.12	3.08	....	....
7.88	3.08	7.43	2.88	4.68	1.325	7.13	3.06	....	....
7.90	3.06	7.45	2.87	4.74	1.389	7.16	3.03	....	....
7.90	3.08	7.48	2.99	4.67	1.349	7.20	3.08	....	....
7.96	3.10	7.54	2.91	4.76	1.392	....	....	....	....
....	....	7.72	3.08	4.73	1.383	....	....	....	....
....	....	....	....	4.70	1.375	....	....	....	....

Tables 5 and 6. Each value of loss of head given in these tables is the average difference of a large number of gage readings.

METHOD OF COMPUTATION

Most of the multiplication and division was done with a 10-inch slide rule.

In all computations of velocities the discharge in cubic feet per second was divided by the actual area of the straight pipe. This gave the mean velocity in the straight pipe, but not always the mean velocity in the curved part owing to variations in the dimensions of the diameters of the various curves. Also in computing the ratio of the radius of curvature, R, to the diameter, d_p, the diameter of the straight pipe was used. A comparison of the losses in curves of the same radius but of different diameters, connected to the same pipe is made in another paragraph.

The loss of head per foot of straight pipe was computed for the various sections of the pipes by dividing the difference of the gage readings, shown in Table 5, by the length of pipe between the piezometers. The losses of head per foot of straight pipe were plotted as ordinates against mean velocities as abscissas on logarithmic cross-section paper, as shown in Fig. 15. In this figure the solid lines, all of which have the equation $h=0.00245\ v^{1.836}$ are intended to show the average loss of head per foot of pipe. A line having this equation seems to fit the points of all three sections of pipe, indicating that the conditions of the pipes were practically the same. In section C–D the observations taken in the spring of 1908 seem to indicate a greater loss of head than was observed later. The points belonging to the earlier observations are averaged by the dotted line. Owing to the shortness of the length of pipe between its entrance and the first piezometer during the first runs, some entrance losses were probably included in the piezometer readings causing a greater average loss of head. The conditions of the later experiments prevented such an error.

The friction loss in the pipes changed considerably during the period covering the various series of experiments. This

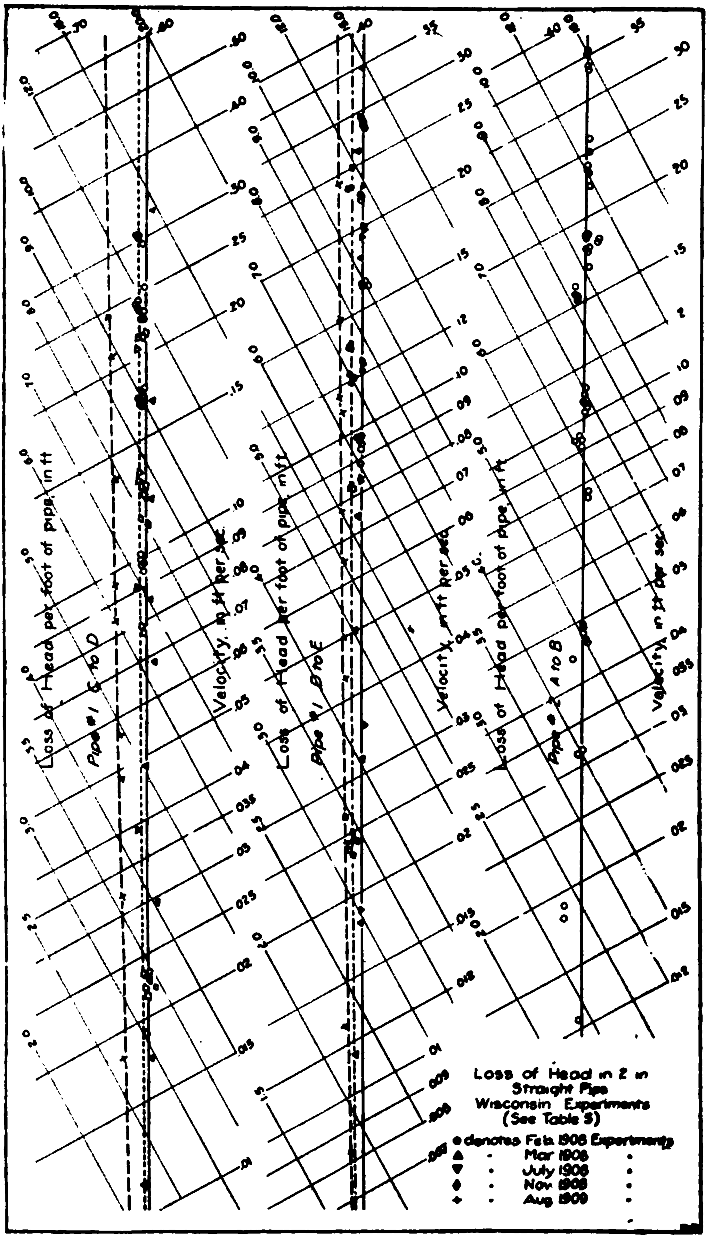


Figure 15.—Logarithmic Plotting of Straight Pipe Losses.

fact is best shown by the results of measurements of friction loss between piezometers D and E. The measurements taken in the spring and summer of 1908 agree so closely that a single line (the solid line in Fig. 15) averages the plotted points satisfactorily, but by November 1908 the friction had increased greatly. The points representing the observations taken at that time are averaged by the dot and dash line in Fig. 15. Another friction test was made on pipe No. 1 in August, 1909, which showed that the friction had still further increased to a considerable extent. The results of this latter test are averaged by the dash line. The pipes were dry during some of the time between experiments so no information can be gotten from this data as to the rate of change of friction loss.

In Figs. 16(a) and (b) the total losses between piezometers B and E, as observed at various velocities for the various elbows, have been plotted on logarithmic cross-section paper and straight lines have been drawn averaging the points. Below these lines were drawn lines representing the loss due to the straight pipe included between piezometers B and E. These lines were located by multiplying the loss per foot of straight pipe taken from Fig. 15, by the proper length of pipe. The total loss occasioned by the curve, corresponding to Case 2, discussed on page 18, was found by taking the vertical intercepts between the line showing the loss of head for a given elbow and that for the corresponding length of straight pipe. This length of straight pipe was different for the different elbows; being 18 feet for curves 1, 2, 3, 4, 5, 6, and 7, 18.9 feet for curve 12, and 19 feet for curves 8, 9, 10 and 11. For elbows 1 to 9 inclusive the results of the straight pipe experiments made in the spring and summer of 1908 were used in computing the loss in the straight part of the pipe, but for elbows 10, 11 and 12, which were experimented on in December, 1908, January and May, 1909, respectively, the results of the November straight pipe experiments were used. To avoid confusion most of the lines representing the straight pipe losses have been omitted in Figs. 16(a) and (b), only two being shown to illustrate the method of computation. This way of computing, over-estimates the curve loss somewhat for the long radius

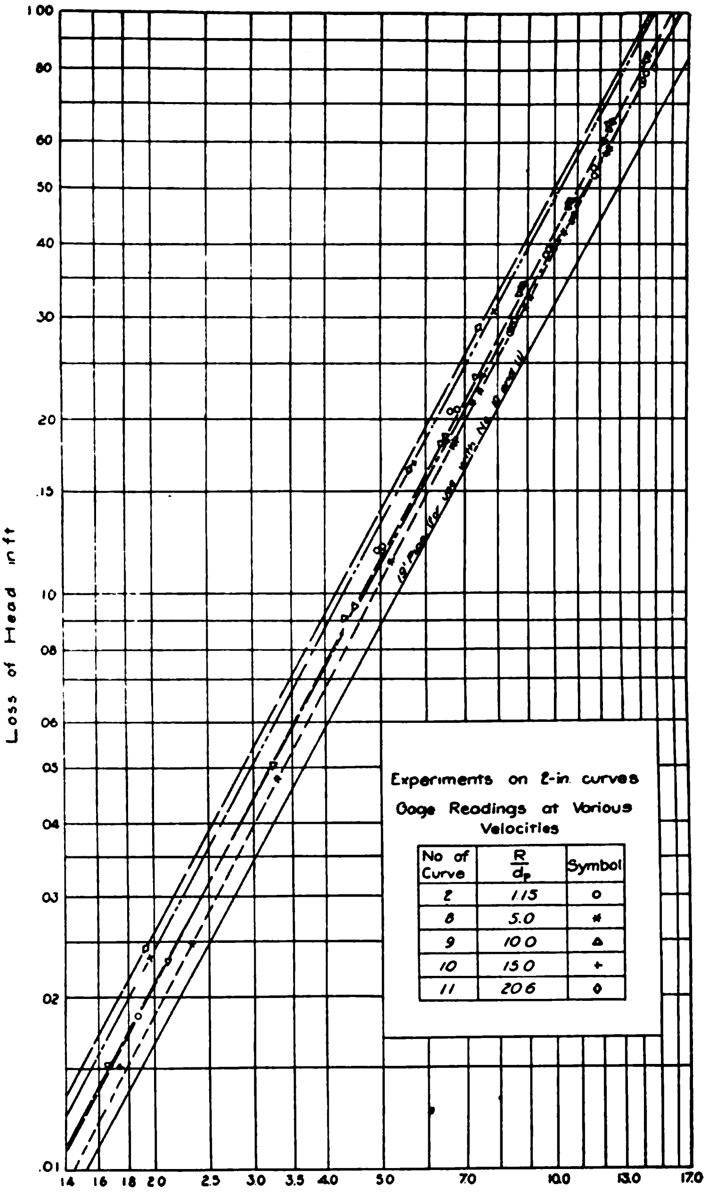
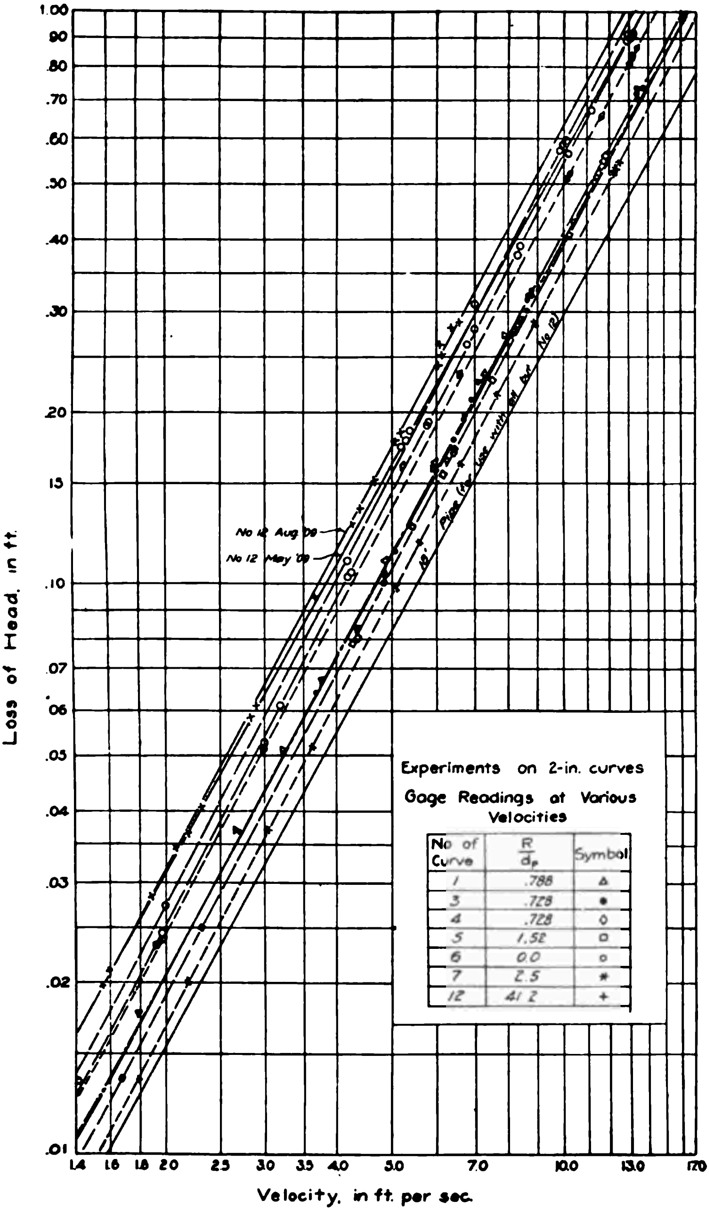


Figure 16(a).—Logarithmic Plotting of Curve Losses.



No of Curve	$\frac{R}{d_p}$	Symbol
1	.788	△
3	.728	●
4	.728	◇
5	1.52	▢
6	0.0	○
7	2.5	✱
12	41 2	+

Figure 16(b).—Logarithmic Plotting of Curve Losses.

curves on account of neglecting the increase in straight pipe friction which occurred between November and the dates when the curves were tested. The curves were tested very soon after erecting, so that they were clean during the test.

The values of the curve loss at a velocity of 10 feet per second, found as described above, have been plotted as circles in Fig. 17 and a curve has been drawn averaging the points.

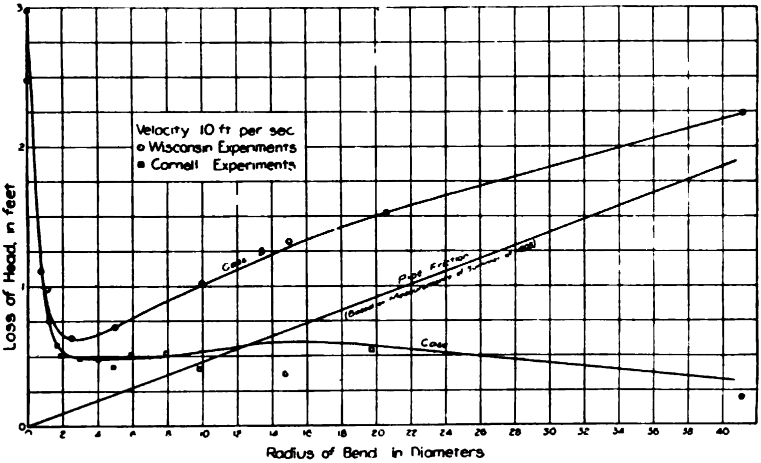


Figure 17.—Relation of Loss of Head and Radius of Bend.

The pipe from which curve No. 12 was made was tested for pipe friction before it was bent. It was placed between pipes 1 and 2 and the gage connected between piezometers B and E just as was done after it was bent. The difference between the losses at 10 ft. per second velocity when curved and when straight was 0.2 ft. This is the value of the net loss due to the curve, in accordance with the definition of Case 1. The point has been plotted on Fig. 17 as a solid black circle. None of the other elbows were tested for pipe friction before bending, so the method used for determining the net friction for elbow 12 cannot be applied to them.

The net friction, according to the definition of Case 1, was determined for the other curves by deducting from the plotted values of the total friction the computed friction loss due to a length of pipe equal to the length of the curve. A curve, marked Case 1, has been drawn on Fig. 17 to represent these net losses. The squares along this curve are not from the Wisconsin Experiments. These points will be discussed later. The vertical intercepts between the curves marked Case 1 and Case 2 are equal to the ordinates of the line marked Pipe Friction which shows the loss of head due to a velocity of ten feet per second in lengths of pipe equal to the length of the elbows.

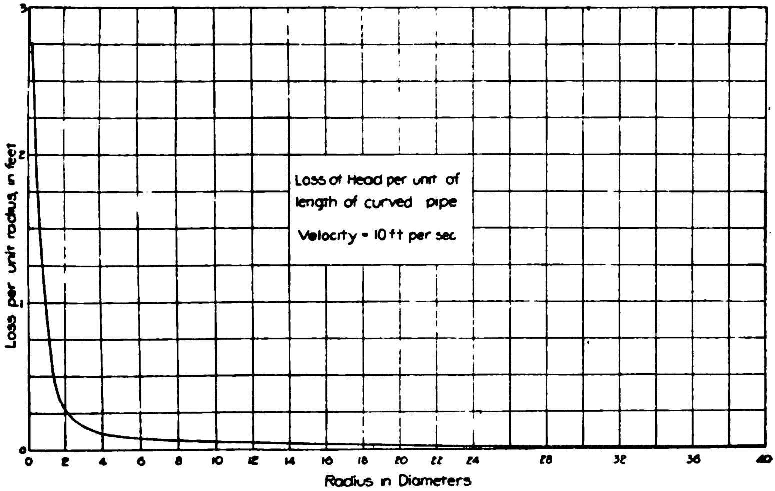


Figure 18.—Loss of Head per Unit Length of Curved Pipe

This line is based on the straight pipe friction losses determined in the spring and summer of 1908 when the pipes were comparatively clean, and approximately in the condition that the elbows were in when they were tested.

The net loss for the various elbows, taken from the average curve on Fig. 17, has been divided by the length of the respective elbows expressed in pipe diameters, as described under Case 1 (b) on page 18, and the results have been plotted in Fig. 18. It may be seen in this figure that the loss per unit length of curve continuously decreases.

THE EFFECT OF ENLARGEMENTS

In the foregoing discussion only the curves having the same diameter as the pipe have been considered. It was found that elbow number 1 gave practically the same loss of head as elbow number 3. This may be seen by comparing the lines for elbows number 1 and 3 on Fig. 16 (b). The increase in loss to be expected in elbow number 1, due to the slight enlargement and contraction of section, seems to have been very nearly balanced by the reduction in loss due to a slightly longer radius of curvature.

An idea of the increase in loss due to the enlargement from 2$\frac{1}{16}$ in. to 2$\frac{3}{8}$ in. and again contracting to the smaller size may be had by comparing elbows numbers 3 and 4 which are both of cast iron, of about the same degree of roughness, as may be seen in Figs. 4 and 5, and both have a radius at the center line of 0.728 diameter. From the curves in Fig. 16 (b) it was found that the total loss due to elbow 4 was about 1.8 times the loss due to elbow 3. Elbow 5 caused about the same loss as elbow number 2.

CORNELL UNIVERSITY EXPERIMENTS

Experiments on elbows and bends were made at Cornell University by Ernest W. Schoder, during 1907. A report of this work by the experimenter, may be found in the Transactions of the American Society of Civil Engineers, Vol. 62, p. 67 (1909).

DESCRIPTION OF APPARATUS

The straight pipes experimented on were 6-inch wrought iron pipes with flange connections. The lengths of the pipes, from face to face of flanges, are given in Fig. 19. The following are the inside diameters—the means of four measurements, two at each end:

Pipe No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
6.106 in.	6.086 in.	6.102 in.	6.078 in.	6.072 in.	6.083 in.

The curves used were made to order, except that Nos. 10 and 12, respectively, were standard "long sweep" and "short turn" 6 in., cast iron, flanged, 90° elbows. Curves numbers 1 to 6, inclusive, were bent from 6 in. wrought-iron pipe. Curves numbers 7 to 12, inclusive, were of cast iron. All were 90° curves; and all were left uncoated. The wrought-iron pipe curves had about 6 in. of straight pipe at each end. The ends had been threaded to receive standard flanges. The cast-iron curves were flanged, faced, and drilled complete, ready for setting up. The dimensions of the curves are given in Table 7.

The piezometers consisted each of two diametrically opposite holes in the pipe wall into which 1/8 in. T-handle cocks were screwed. A three-way connection served to join the short hoses from these cocks and the single long hose to the gage. A water differential gage was used to measure the loss of head during the straight pipe experiments and a mercury differential gage was used during the elbow experiments. These gages were provided with blow-off cocks for the removal of air from the gages and connections. The same up-stream piezometer was used for the curve experiments as for the first straight-pipe experiments. The down-stream piezometer was a new one, of the same type, placed 1.05 ft. up stream from the down-stream end of Pipe No. 5 which adjoined the nozzle. A similar intermediate piezometer was placed 2.00 ft. up stream from the down-stream end of Pipe No. 3.

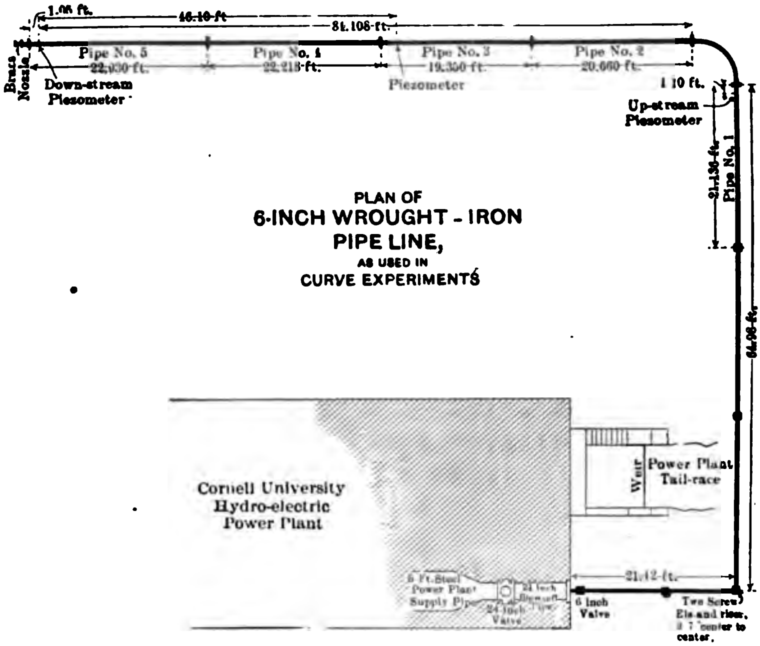


Figure 19.—General Plan of the Apparatus Used in the Cornell University Experiments.

A general plan of the apparatus is shown in Fig. 19.

The experimental pipe line was arranged with the idea of keeping all conditions the same except for the introduction of the several curves. Consequently, the flange joints in the portion downstream from the curves were not disturbed throughout the experiments. The entire length of 85 ft. of 6-inch wrought-iron pipe, together with the nozzle, was shifted bodily when a new curve was placed in the line. The portion upstream from the curves was left unchanged during the experiments on the curves.

During the experiments made for finding the straight pipe friction losses the water discharged through the pipe was measured in a calibrated concrete measuring tank, of 500 cubic feet capacity, but during the experiments on the elbows and bends the water was measured by means of a brass nozzle attached to the discharge end of the 6-inch pipe. It had been calibrated previously with pressure heads at the base of the nozzle ranging from 1.649 to 50.208 feet. The discharge co-efficient determined from the average of 40 experiments was found to be 0.988.

TABLE 7

CORNELL EXPERIMENTS

1	2	3	4	5	6	7	8
No. of curve.	Internal diameter in feet: d.	R. in feet.	$\frac{R}{d}$. d.=.508 ft. =mean diameter of straight pipe.	Length of curve along center line.	Length of elbow from face to face of flanges.	Length of straight pipe between piezometers.	Material.
1......	.508	10.00	19.68	15.71	16.77	86.268	Wrought iron.
2......	.515	7.50	14.76	11.78	12.84	86.268	Wrought iron.
3......	.513	5.00	9.84	7.85	9.01	86.368	Wrought iron.
4......	.509	4.00	7.87	6.28	7.34	86.268	Wrought iron.
5......	.509	3.00	5.90	4.71	5.89	86.388	Wrought iron.
6......	.508	2.50	4.92	3.93	5.08	86.358	Wrought iron.
7......	.492	2.00	3.94	3.14	3.64	85.708	Cast iron.
8......	.496	1.50	2.95	2.36	2.86	85.708	Cast iron.
9......	.492	1.08	2.13	1.70	2.54	86.048	Cast iron.
10......	.492	0.95	1.87	1.49	1.75	85.468	Cast iron.
11......	.496	0.88	1.73	1.38	3.62	87.448	Cast iron.
12......	.494	0.67	1.32	1.05	1.05	85.208	Cast iron.

METHOD OF EXPERIMENTATION

Before the curve experiments were made a straight line composed of six pipe lengths having a total length of 122 feet was set up and the loss of head measured in a length of

99.33 feet, a length of 20.04 ft. being allowed up stream from the first piezometer for the disturbances due to entry from a 12-inch header to die down. The results of these measurements are given in Table 8. After the completion of the experiments on the curves a second series of straight pipe experiments was made on the 46.10 ft. of straight pipe immediately above the down-stream piezometer. Pitot tube traverses had shown conditions of normal flow at the up-stream end of this experimental section, but the results of the second series of straight pipe experiments did not agree with the results of the first series. The use of the first series in computing the curve losses gives results more in harmony with the Wisconsin Experiments than does the use of the second series and the latter will therefore not be considered in this bulletin.

TABLE 8

Experiment on Friction loss in straight pipes.
Data by Schoder, E. W.
Date of Experiment, Sept. 4, 1907.
General data, Temperature of water at start=68° F; at finish=70° F.

No. of experiment.	Observed loss of head in 99.33 feet of pipe, in feet of water.	Loss of head per foot of pipe, in feet.	Mean velocity, in feet per second.
1	1.901	0.0191	6.03
2	1.687	0.0170	5.67
3	1.449	.0146	5.22
4	1.241	.0125	4.80
5	1.000	0.0101	4.29
6	0.831	0.00836	3.88
7	0.704	0.00708	3.55
8	0.473	0.00476	2.87
9	0.307	0.00309	2.27
10	0.160	0.00160	1.61

The losses of head in all the curve experiments were measured between the piezometers just upstream from the curves and the piezometer just upstream from the nozzle, distant 168 diameters downstream from the curves.

METHOD OF COMPUTATION

The losses of head per foot length of straight pipe as given in Table 8 have been plotted as ordinates against the velocities as abscissas on logarithmic cross-section paper, as shown in Fig. 20.

The losses of head in the elbow experiments were measured by mercury differential gages, as before mentioned. These measured losses, as published in Mr. Schoder's article, have been reduced to equivalent losses, expressed in feet of water, by dividing by 12.57. The results are given in Column 4 of Table 9 opposite the corresponding velocity. The loss of head per foot of straight pipe at the given velocities was taken from Fig. 20 and tabulated in Column 3. These values have been multiplied by 101.98 and the results tabulated in Column 6 of Table 9. This distance of 101.98 ft. was the center-line length of pipe between the piezometers when Curve No. 1 was installed. The difference between the values in Columns 6 and 4 gives the net losses of head due to Curve No. 1 according to the definition of Case 1 (a). To find the net loss for the other curves the loss that would occur in a length of straight pipe sufficient to make the center-line length equal 101.98 was added to the observed losses to reduce all curves to the conditions of Curve No. 1. The results are tabulated in Column 5 of Table 9. The differences between the quantities in this column and those in Column 6 are the net losses desired. These net losses, tabulated in Column 7, were plotted on logarithmic cross-section paper, and from the curves averaging the points the losses at a velocity of ten feet per second were picked off. These losses are tabulated in Table 10 and have been plotted as squares on Fig. 17. It may be noted in this figure that the Cornell experiments agree very well with the Wisconsin experiments, so far as the net losses are concerned. The normal six-inch pipe friction loss per diameter of length is slightly less than that of the 2-inch pipe and consequently a direct comparison of quantities cannot be made for Case 2.

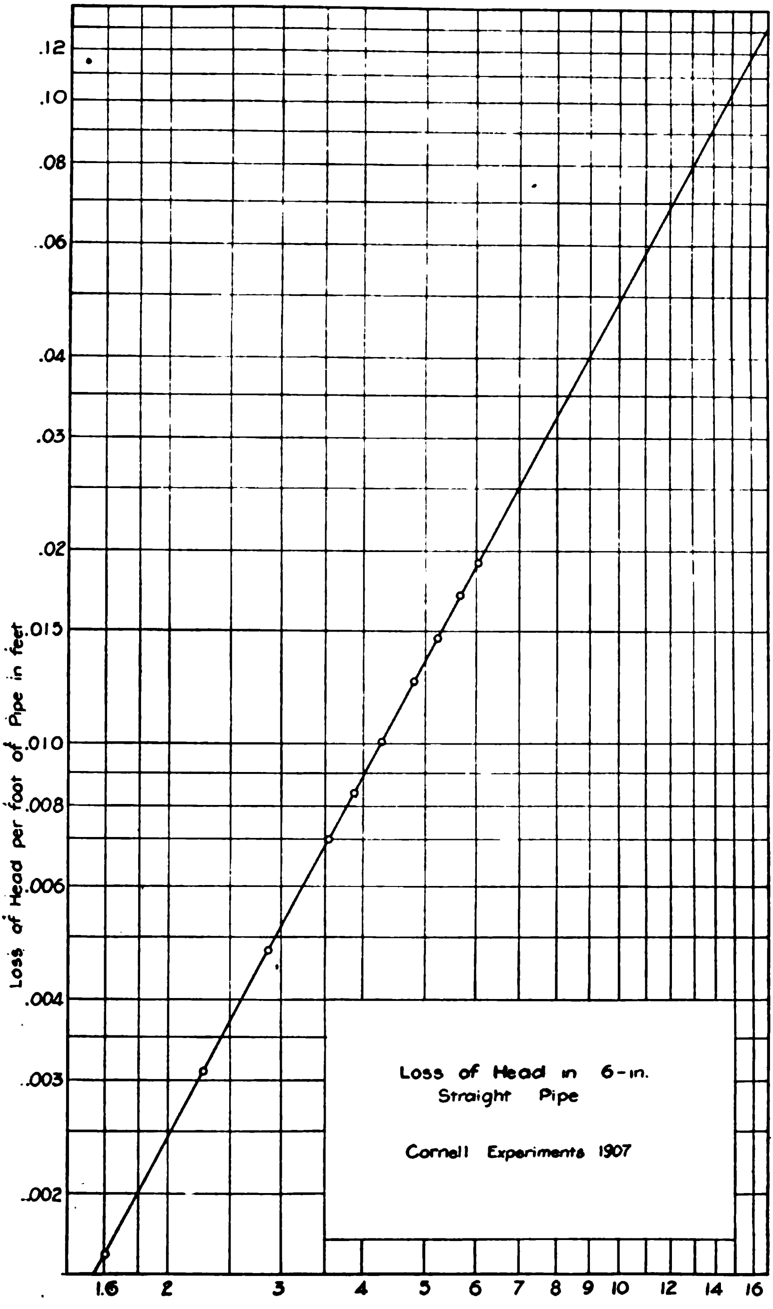


Figure 20.—Logarithmic Plotting of Straight Pipe Losses.

TABLE 9

Experiment on Curve resistance.
Data by E. W. Schoder. Computed by G. J. Kruell.
Date of experiment, 1908.
General data, d. = mean diameter of pipes nos. 1 and 2.

Curve No. 1

Cl. length=101.98 feet. $\frac{R}{d}=19.685$.

1	2	3	4	5	6	7
No. of experiment.	Mean velocity, in feet per second.	Loss of head per foot of straight pipe.	Loss, in feet of water between piezometers.	Loss, in feet of water when total length, Cl.=101.98 feet.	Loss in 101.98 feet of straight pipe.	Difference (5–6).
1	16.56	.1257	14.52	14.52	12.82	1.70
2	16.00	.1180	13.64	13.64	12.03	1.61
3	15.27	.1080	12.44	12.44	11.01	1.43
4	13.41	.0846	9.70	9.70	8.63	1.07
5	12.81	.0780	8.90	8.90	7.95	0.95
6	12.06	.0694	7.86	7.86	7.08	0.78
7	9.29	.0427	4.78	4.78	4.35	0.43
8	7.96	.0320	3.58	3.58	3.26	0.32
9	6.59	.0225	2.49	2.49	2.29	0.200
10	4.98	.0131	1.48	1.48	1.34	0.14
11	3.46	.0068	0.691	0.691	0.693	—.002
12	16.58	.1261	14.56	14.56	12.86	1.70
13	16.58	.1261	14.69	14.69	12.86	1.83
14	16.10	.1200	13.88	13.88	12.24	1.64
15	14.95	.1035	12.02	12.02	10.56	1.47
16	14.13	.0940	10.78	10.78	9.59	1.19
17	13.24	.0828	9.50	9.50	8.44	1.06
18	12.53	.0750	8.55	8.55	7.65	0.90
19	11.64	.0652	7.42	7.42	6.65	0.77
20	9.90	.0487	5.48	5.48	4.97	0.51
21	8.20	.0338	3.87	3.87	3.45	0.42
22	6.38	.0212	2.35	2.35	2.16	0.19
23	3.83	.00815	0.930	0.930	0.831	0.099
24	2.67	.00418	0.427	0.427	0.426	0.001

TABLE 9—Continued

CURVE No. 2

Cl. length=98.05 feet. $\frac{R}{d.}=14.764.$

1	2	3	4	5	6	7
No. of experiment.	Mean velocity, in feet per second.	Loss of head per foot of straight pipe.	Loss, in feet of water between piezometers.	Loss in feet of water when total length, Cl.=101.98 feet.	Loss in 101.98 feet of straight pipe.	Difference (5–6.)
1	16.65	.1270	13.79	14.29	12.95	1.34
2	16.04	.1185	12.81	13.27	12.08	1.19
3	13.35	.0840	9.02	9.35	8.57	0.78
4	14.73	.1015	10.80	11.20	10.35	0.85
5	11.55	.0644	6.90	7.15	6.57	0.58
6	10.62	.0550	5.78	6.00	5.61	0.39
7	9.18	.0418	4.40	4.56	4.26	0.30
8	6.76	.0238	2.46	2.55	2.43	0.12
9	5.02	.0136	1.34	1.39	1.39	0.00
10	3.29	.00616	0.64	0.66	0.63	0.03
11	2.81	.00460	0.45	0.47	0.47	0.00
12	16.65	.1270	13.78	14.28	12.95	1.33
Cl. length=94.22.			CURVE No. 3.			$\frac{R}{d}=9.842.$
1	16.66	.1271	13.39	14.38	12.96	1.42
2	16.11	.1200	12.49	13.42	12.24	1.18
3	14.95	.1035	10.80	11.60	10.55	1.05
4	14.05	.0930	9.68	10.40	9.48	0.92
5	13.28	.0840	8.57	9.22	8.57	0.65
6	12.39	.0730	7.54	8.11	7.44	0.67
7	11.09	.0596	6.13	6.59	6.08	0.51
8	8.85	.0390	4.00	4.30	3.98	0.32
9	7.69	.0300	3.04	3.27	3.06	0.21
10	4.99	.0133	1.38	1.48	1.36	0.12
11	3.45	.00678	0.64	0.69	0.69	0.00
12	16.66	.1271	13.39	14.38	12.96	1.42
13	16.65	.1270	13.35	14.33	12.95	1.38
14	16.16	.1205	12.63	13.57	12.29	1.28
15	15.18	.1070	11.21	12.04	10.91	1.13
16	13.91	.0910	9.43	10.14	9.28	0.86

TABLE 9—Continued

CURVE No. 4.

Cl. length=92.55 feet. $\frac{R}{d}$=7.874.

1	2	3	4	5	6	7
No. of experiment.	Mean velocity, in feet per second.	Loss of head per foot of straight pipe.	Loss, in feet of water between piezometers.	Loss, in feet of water when total length, Cl.=101.98 feet.	Loss in 101.98 feet of straight pipe.	Difference (5—6).
1	16.62	.1267	13.46	14.65	12.92	1.73
2	16.30	,1225	13 00	14.16	12.49	1.67
3	14.59	.0996	10.36	11.30	10.16	1.14
4	13.15	.0820	8.57	9.34	8.36	0.98
5	12.01	.0690	7.19	7.84	7.04	0.80
6	9.29	.0427	4.39	4.79	4.35	0.44
7	6.10	.0195	1.99	2.17	1.99	0.18
8	3.87	.00837	0.78	0.86	0.85	0.01
9	10.46	.0531	5.51	6.01	5.42	0.59
10	16.63	.1268	13.45	14.65	12.93	1.72
11	16.27	.1220	12.87	14.02	12.44	1.58
12	13.67	.0886	9.20	10.04	9.04	1.00
Cl. length=91.10			CURVE No. 5.			$\frac{R}{d}$=5.906.
1	16.63	.1268	13.24	14.62	12.93	1.69
2	16.09	.1200	12.38	13.68	12.24	1.44
3	14.05	.0930	9.58	10.59	9.48	1.11
4	12.85	.0787	8.07	8.92	8.02	0.90
5	10.80	.0570	5.78	6.40	5.81	0.59
6	7.63	.0296	3.02	3.34	3.02	0.32
7	4.86	.0127	1.26	1.40	1.30	0.01
8	3.42	.00664	0.60	0.67	0.68	—0.01
9	16.63	.1268	13.22	14.60	12.93	1,67
10	16.61	.1266	13.14	14.52	12.91	1.61

TABLE 9—Continued

CURVE NO. 6.

Cl. length=90.29. $\frac{R}{d}=4.921$

1	2	3	4	5	6	7
No. of experiment.	Mean velocity, in feet per second.	Loss of head per foot of straight pipe.	Loss, in feet of water between piezometers.	Loss, in feet of water when total length, Cl=101.98 feet.	Loss in 101.98 feet of straight pipe.	Difference (5—6).
1	16.65	.1270	13.02	14.50	12.95	1.55
2	16.40	.1235	12.62	14.06	12.59	1.47
3	15.41	.1100	11.22	12.50	11.22	1.28
4	13.04	.0805	8.10	9.04	8.21	0.83
5	8.67	.0375	3.75	4.19	3.82	0.37
6	10.36	.0520	5.15	5.76	5.30	0.46
7	7.78	.0307	2.99	3.35	3.13	0.22
8	5.53	.0162	1.52	1.71	1.65	0.06
9	3.41	.0066	0.58	0.66	0.67	—0.01
10	16.61	.1266	12.92	14.40	12.91	1.49
11	16.60	.1265	12.90	14.38	12.90	1.48
12	16.61	.1266	13.00	14.48	12.91	1.57
13	16.45	.1240	12.76	14.21	12.64	1.57
14	16.01	.1180	12.10	13.48	12.03	1.45

Cl. length=88.85. CURVE NO. 7. $\frac{R}{d}=3.937.$

1	2	3	4	5	6	7
1	16.61	.1266	12.87	14.53	12.91	1.62
2	14.80	.1020	10.30	11.64	10.40	1.24
3	12.90	.0790	7.93	8.97	8.06	0.91
4	11.94	,0680	6.84	7.73	6.93	0.80
5	8.58	.0368	3.61	4.09	3.75	0.34
6	7.31	.0274	2.66	3.02	2.79	0.23
7	5.62	.0167	1.55	1.77	1.70	0.07
8	4.38	.0105	0.96	1.10	1.07	0.03
9	3.20	.00586	0.49	0.57	0.60	—0.03
10	16.62	.1267	12.88	14.54	12.92	1.62

TABLE 9—Continued

CURVE No. 8.

Cl. length=88.07. $\frac{R}{d}$=2953.

1	2	3	4	5	6	7
No. of experiment.	Mean velocity, in feet per second.	Loss of head per foot of straight pipe.	Loss, in feet of water between piezometers.	Loss, in feet of water when total length, Cl.=101.98 feet.	Loss in 101.98 feet of straight pipe.	Difference (5—6).
1	16.60	.1265	12.67	14.43	12.90	1.53
2	15.26	.1080	10.78	12.28	11.01	1.27
3	14.50	.0980	9.83	11.19	9.99	1.20
4	13.64	.0878	8.72	9.94	8.95	0.99
5	9.26	.0425	4.16	4.75	4.33	0.42
6	10.50	.0535	5.27	6,01	5.46	0.55
7	6.56	.0223	2.16	2.47	2.27	0.20
8	4.99	.0133	1.26	1.44	1.36	0.08
9	3.44	.0067	0.63	0.72	0.68	0.04
10	16.62	.1267	12.77	14.53	12.92	1.61

Cl. length=87.75. CURVE No. 9. $\frac{R}{d}$=2.126.

1	2	3	4	5	6	7
1	16.63	.1268	12.82	14.62	12.93	1.69
2	16.40	.1236	12.33	12.66	12.59	0.07
3	15.40	.1100	11.07	12.63	11.22	1.41
4	14.00	.0920	9.18	10.49	9.38	1.11
5	10.56	.0544	5.36	6.13	5.55	0.58
6	7.58	.0292	2.89	3.31	2.98	0.33
7	5.15	.0142	1.34	1.54	1.45	0.09
8	3.13	.00565	0.52	0.60	0.58	0.02
9	16.63	.1268	12.82	14.62	12.93	1.69

TABLE 9—Continued

CURVE NO. 10.

Cl. length=86.96. $\frac{R}{d}=1.870.$

1	2	3	4	5	6	7
No. of experiment.	Mean velocity, in feet per second.	Loss of head per foot of straight pipe.	Loss, in feet of water between piezometers.	Loss, in feet of water when total length, Cl.=101.98 feet.	Loss in 101.98 feet of straight pipe.	Difference (5—6).
1	16.63	.1268	12.77	14.67	12.93	1.74
2	16.40	.1235	12.44	14.29	12.59	1.70
3	15.65	.1170	11.38	13.14	11.93	1.21
4	14.48	.0978	9.78	11.25	9.97	1.28
5	12.60	.0758	7.48	8.62	7.73	0.89
6	9.26	.0425	4.07	4.71	4.33	0.38
7	8.70	.0377	3.62	4.19	3.84	0.35
8	7.41	.0280	2.89	3.31	2.86	0.45
9	5.60	.0165	1.58	1.83	1.68	0.15
10	3.97	.00873	0.80	0.93	0.89	0.04
11	2.46	.00358	0.30	0.35	0.36	—0.01
12	16.64	.1269	12.77	14.68	12.94	1.74
13	16.66	.1271	12.73	14.64	12.96	1.68

Cl. length=88.83. CURVE NO. 11. $\frac{R}{d}=1.732.$

1	2	3	4	5	6	7
1	16.59	.1263	13.20	14.86	12.88	1.98
2	15.53	.1135	11.64	13.13	11.57	1.56
3	12.40	.0731	7.63	8.59	7.45	1.04
4	8.99	.0400	4.02	4.55	4.08	0.47
5	8.41	.0355	3.46	3.93	3.62	0.31
6	7.75	.0305	3.03	3.43	3.11	0.32
7	7.07	.0256	2.54	2.88	2.61	0.27
8	5.41	.0156	1.51	1.71	1.59	0.12
9	3.78	.0080	0.70	0.80	0.82	—0.02
10	2.87	.0048	0.40	0.46	0.49	—0.03
11	16.56	.1257	13.15	14.80	12.82	1.98
12	16.58	.1261	13.22	14.88	12.86	2.02
13	13.60	.0876	8.99	10.14	8.93	1.21

TABLE 9—Continued

CURVE NO. 12.

Cl. length=86.26. $\frac{R}{d}=1.319$.

1	2	3	4	5	6	7
No. of experiment.	Mean velocity, in feet per second.	Loss of head per foot of straight pipe.	Loss, in feet of water between piezometers.	Loss, in feet, of water when total length, Cl.=101.98 feet.	Loss in 101.98 feet of straight pipe.	Difference (5—6).
1	16.62	.1267	13.15	15.14	12.92	2.22
2	16.31	.1225	12.68	14.60	12.49	2.11
3	14.65	.1005	10.28	11.86	10.25	1.61
4	12.01	.0690	7.01	8.09	7.04	1.05
5	8.92	.0395	4.02	4.64	4.03	0.61
6	6.43	.0215	2.11	2.45	2.19	0.26
7	4.21	.00978	0.90	1.05	1.00	0.05
8	2.74	.00437	0.34	0.41	0.44	—0.03
9	12.61	,0759	7.69	8.88	7.74	1.14
10	16.61	,1266	13.15	15.14	12.91	2.23

TABLE 10.

NET LOSSES DUE TO 6 INCH ELBOWS. V. = 10. FEET PER SECOND.

Number of curve.	Radius of bend, in pipe diameters.	Loss of head, in feet.
12	1.32	0.70
11	1.73	0.57
10	1.87	0.51
9	2.13	0.51
8	2.95	0.49
7	3.94	0.48
6	4.92	0.43
5	5.91	0.51
4	7.87	0.52
3	9.84	0.41
2	14.76	0.38
1	19.68	0.54

BIRMINGHAM EXPERIMENTS

Experiments on elbows and bends were made at the University of Birmingham, England, by C. W. L. Alexander, and were reported by him in a paper presented before the Institution of Civil Engineers.*

The pipes experimented upon were made of varnished wood and were 1¼ inch in diameter. The piezometers were placed only 6 inches (4.8 diameters) from the ends of the curves.

The loss of head in a section of straight wooden pipe was determined, before making the curve experiments, by fitting three piezometers 379.5 millimeters apart, and taking readings between the two extreme piezometers and between adjacent ones.

Having obtained the loss of head per unit length of straight pipe, a table was prepared showing the loss of head at different velocities for lengths of straight pipe equal to the length of the center line between the piezometers on each of the wooden bends, and from these figures a curve was plotted for each bend.

Similarly curves were plotted showing, for each bend, the actual losses of head between the piezometers; and thus that part of the loss due to the bend could be at once read off, being represented by the difference of the abscissas of the two curves. When this had been done, and the net losses due to the bend had been plotted logarithmically it appeared that the loss of head due to the bend is of the same nature as that in the straight pipe and obeys similar laws; a formula of the form $h=k\ v^x$ having the same value of the exponent x for a curved pipe as for a straight one. Such was not the case for the Wisconsin and Cornell experiments, nor in Mr. Brightmore's experiments.

The author does not consider the absolute amounts of the losses of head due to right-angle bends obtained in these experiments to be correct, for the following two reasons:

---

* Min. of Proceedings, Vol. 159, p. 341 (1904).

1. The first piezometer, for perfectly accurate results, should have been preceded by a much greater length of the wooden pipe, because apparently the 2 inches length was insufficient to restore the water to a normal condition of flow before it reached the piezometer. Even although the entrance disturbance in most cases was small, through the connection with the iron pipe having been carefully made, it probably was present in every instance to some extent, while for bends Nos. 2, 3 and 4 it was very marked.

2. The second piezometer should have been farther away from the end of the bend to allow the disturbances set up by the bend to complete their effect. The loss of head must have been greater than that given by the piezometer readings.

The general conclusions reached from the experiments, however, agree with those drawn from the Wisconsin experiments. These conclusions are stated by Mr. Alexander as follows:

"The absolute loss due to the curve decreases rapidly from the elbow, No. 11, to bend No. 6, the radius of which is five times the radius of the bore of the pipe, and that it thereafter commences to increase."

"The loss is dependent on

(1) The roughness of the pipe.
(2) The hydraulic radius of the pipe.
(3) The curvature of the bend.
(4) The length of the bend measured along the center line."

WEISBACH'S EXPERIMENTS

So far as is shown by his published results, the experiments of Julius Weisbach were all made upon pipes less than 2½ cm. in diameter and most of them with very small velocities of flow. In most of the experiments there was no straight pipe down stream from the elbow and in those cases where there was, it was too short to include all the elbow loss. He made some experiments on pipes of larger diameter, but the results have apparently never been printed. The published results of Weisbach's experiments do not show that any curve of radius greater than 1 diameter of pipe were used although some greater ones were doubtless used in his unpublished work. These experiments together with a few made by Dubaut, all on pipes less than 2.2 inches in diameter, served as the basis of Weisbach's formula which has been adopted by most textbook authors.

Weisbach's formula gives losses much larger than were observed in the Wisconsin experiments for small values of $\frac{R}{d}$ and smaller losses for large values of $\frac{R}{d}$.

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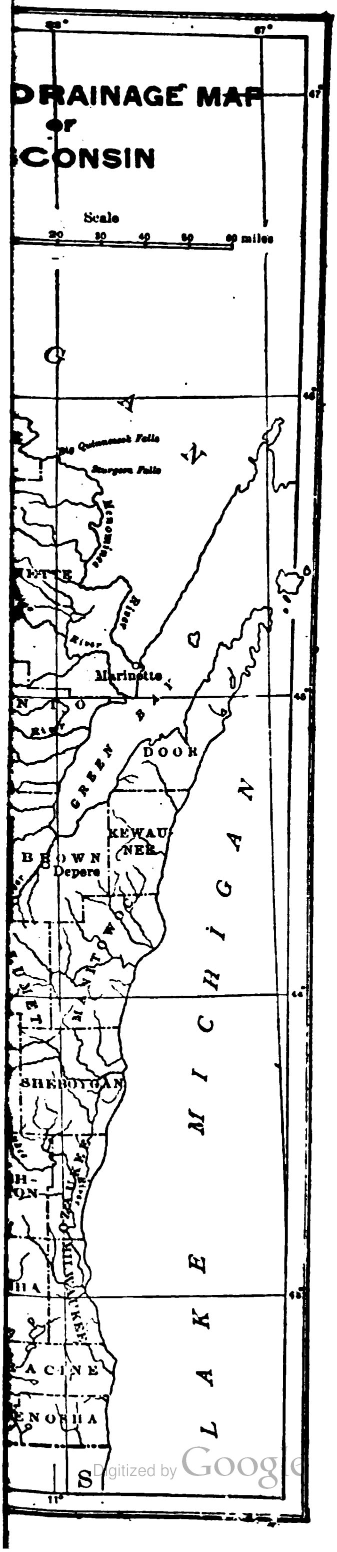
DRAINAGE MAP
OF
CONSIN
Scale
20 30 40 50 60 miles
88°
87°
47°
46°
45°
44°
43°
Big Quinnesec Falls
Sturgeon Falls
Menominee
River
River
Marinette
GREEN
BAY
DOOR
KEWAUNEE
BROWN
Depere
MANITOWOC
SHEBOYGAN
MILWAUKEE
RACINE
KENOSHA
LAKE MICHIGAN

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THE FLOW OF STREAMS AND THE FACTORS THAT MODIFY IT, WITH SPECIAL REFERENCE TO WISCONSIN CONDITIONS

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FLOW OF STREAMS AND THE FACTORS THAT MODIFY IT WITH SPECIAL REFERENCE TO WISCONSIN CONDITIONS

INTRODUCTION

The quantity and regularity of stream flow and its variations are matters of much importance in hydraulic engineering, and equally so to many of the business interests and enterprises of the State of Wisconsin.

Public attention has recently been called to various questions relative to water power and internal navigation, while questions of water supply, drainage, and flood protection are no less important to the public welfare. One of the most important elements in the solution of all of these problems, and of many others almost equally important, is the condition of stream flow or runoff. The application of storage for the prevention of floods and for the promotion of uniformity of flow for navigation. power and water supply purposes, has been much discussed, and many opinions have been expressed as to its possibility, desirability, and value. The value of forests for increasing or equalizing stream flow has been widely discussed, and opinions greatly at variance have been expressed both favorable and unfavorable to such influences. It has also been suggested that water powers of the state be taxed to provide for the reforesting of the head waters of the streams on the theory of direct benefit, and bills providing for such a tax have been offered in the legislature. The arguments that have been presented to sustain the various opinions that have been expressed on many of these questions have seldom been satisfactory or conclusive, except to their authors. Much incomplete and half-considered data have been offered to sustain the contention of the advocates of both sides of many of the debatable questions, and in few cases have the arguments been based on such complete data that the conclusions have not been open to serious question. In many cases the advocates of various theories have apparently sought only for such data as would support their personal views, and have minimized or ignored the data that might suggest a contrary view of the question at issue.

It has appeared to the writer that while most of the questions at issue are exceedingly complicated, and while the data available for the solution of these problems are more or less incomplete and unsatisfactory, yet a careful examination of such data, in considerable detail, on scientific lines might lead to definite and well substantiated conclusions, at least in regard to a limited area. With this idea in view, the writer has made a somewhat detailed study of this subject with special regard to the conditions prevailing in the State of Wisconsin; and while the data examined and considered are too numerous to present in print, he has endeavored to select certain typical locations from which data were available and has endeavored to present the study of the conditions that prevail therein in sufficient detail to warrant the opinions expressed in his conclusions. In most, if not in every case cited, the data on which the opinions rest are presented so completely that the rationale of his conclusions can readily be understood. Where the data have not been presented, the source from which they were obtained and from which they can readily be verified has been stated.

The writer would by no means have it understood that he has not had definite opinions on many of these subjects, but he has approached this study with an open mind and with a strong desire to ascertain the truth, in whatever form it might appear. His personal interest in hydrological science and in practical hydraulic developments makes it most important to him that the truth be determined if possible.

If forests will improve the regularity of streams and prevent floods, if they will conserve and increase the low water flow, if they will prevent denundation and increase the depth of streams, it is most important that these facts be known. If, on the other hand, such effects are purely imaginative or unimportant, it is equally important that that fact be known so that unjust laws and useless expense shall be avoided.

If it is practically possible to construct reservoir systems on the head waters of some or all of the streams of the state, and, by storage, to economically regulate their flow and radically increase the available low water power of the streams and their availability for navigation purposes, such facts and their limitations are of great importance to the citizens of the state. The effects of the draining of swamps and of wet land, the cultivation of agricultural lands, and of various other artificial changes in normal conditions should also be known and appreciated in order that their effects may be obviated if harmful, or emphasized if beneficial, to as great a degree as possible.

This study of Wisconsin hydrological conditions has been made with the importance of these matters fully understood, and with the most earnest desire for the determination of the truth of these theories. It is unfortunate that more complete data have not been available, and that data have not been available on a greater number of streams. It is believed, however, that the conclusions drawn are fully warranted by the data available and that a more complete study would only emphasize the facts.

It should be understood that the conclusions reached are held for Wisconsin conditions only and for other locations strictly comparative therewith. It is hoped that similar studies will be made in other parts of the country and under other conditions so that the broad principle of hydrology may be better understood and appreciated. General principles have been illustrated wherever possible from data and diagrams already available, for neither the time nor the funds has been available to collect the necessary data from Wisconsin for the satisfactory illustration of all of the principles involved in this discussion.

The thanks of the writer are due to Dean F. E. Turneaure for various valuable suggestions; to W. C. Devereaux, Forecaster of the U. S. Weather Bureau, L. M. Mann, U. S. Assistant Engineer, and Lieutenant C. H. Knight, Secretary of the

Mississippi Commission, for furnishing valuable data, also to numerous assistants who have aided in the calculations and diagrams.

THE STUDY OF COMPLEX PHENOMENA

In considering many of the simpler natural phenomena the relations between cause and effect are so direct as to be readily understood and appreciated. The very simplicity of such relation is apt to be misleading when more complex phenomena are under investigation, and, in consequence, undue weight is often given to some single influence that may be only one of many which modify or control the results under consideration.

By complex phenomena are meant those in which the effect is modified by numerous causes, each influencing the ultimate result not only in accordance with its own character and intensity, but also in accordance with the relative character and intensity of other coordinate influences. The effect in such cases may be regarded as the resultant of numerous forces, and the weight, importance, and effect of each must be carefully differentiated in order that its relative importance may be rightly understood and clearly appreciated. Most meteorological and climatic problems are of this class; and many of such problems are so involved in character, the factors that modify their occurrence are so complicated and so irregular, and they are often so modified by unappreciated, and perhaps by unknown causes, as to make their occurence appear to the limited vision of the casual observer as devoid of law and beyond the possible knowledge of mankind.

COMMON ERRORS IN THE CONSIDERATION OF COMPLEX PROBLEMS

The first common error in the consideration of such problems is the assumption of a simplicity of the relations between cause and effect that is not warranted by fact. The second common error is the assumption that when a certain cause is operative under one set of conditions that it is operative to a similar degree under all other conditions. A third common error is sometimes due to the confusion of cause and effect, or in attributing the cause to the effect instead of the true relation of the effect to the real cause.

The relation of the rainfall to the amount and distribution of water flowing from any given drainage area is a complicated problem. The flow from an area is so directly dependent on the rainfall thereon that it seems that some simple and constant relation should be ascertainable between the quantities of each. A brief investigation, however, shows that the modifying conditions are manifold and that the relative importance of each influence varies even more widely than the apparent range in the conditions. These influences are so numerous, and their modifying effect on one another is so direct and important, that no general and constant relation exists between any one influence and the ultimate results. These facts have led to many misinterpretations and erroneous conclusions in the attempt to establish simple relations which cannot from the nature of the case exist.

It is the purpose of this bulletin to point out the numerous influences and their intimate relations; to discuss their relative importance and the reason for the variation of the effects of each; to show as clearly as possible the breadth of the investigation necessary for sound conclusions in regard to such problems and the errors that are likely to be entailed in any estimate based on insufficient study and limited data; and to point out if possible such conclusions as can be safely drawn.

THE PROBLEM OF RUNOFF

The portion of the rainfall that is discharged in the surface flow from a drainage area is termed the ''runoff'' from that area and may occur, in the case of water flowing from areas of considerable magnitude, as a definite stream or, when small areas are considered, merely as the surface drainage not confined to a definite channel.

A brief examination of the subject shows that the factors that modify and control runoff are very numerous and differ in intensity and in effect with every drainage area considered. While the relative runoff from each area will agree more or less closely with that of other areas on which the conditions are similar, there are always variations which occur in these conditions that make them so dissimilar as to create greater or less difference. The causes of such differences are not always manifest as it is often impossible to obtain a sufficiently exact knowledge of the detailed conditions prevailing over an entire drainage area so that more than approximate conclusions can be drawn. Nevertheless, a discussion of this subject, based on scientific considerations, may so clearly set forth the principles involved in any specific case that the dominating influences can be clearly seen and the extent of the influence of any particular factor, if important, may be quite clearly realized and appreciated.

RESUME OF THE FACTORS THAT MODIFY OR CONTROL RUNOFF

The factors that modify or control the quantity and occurrence of the runoff from a drainage area may be briefly summarized as follows:

1. Precipitation—
    - (a) Whether it occurs as rain or as snow.
    - (b) The amount of each, and the total annual precipitation.
    - (c) Its distribution throughout the year.
    - (d) Its intensity or manner of occurrence.
    - (e) The character of storms, including their direction, extent and duration.
2. Temperature—
    - (a) The variations of temperature on the area.
    - (b) The relation of extreme temperatures to the occurrence of precipitation.
    - (c) The accumulation of snow and ice, caused by low temperatures.

- (d) The occurrence of low temperatures causing the freezing of the ground surface at times of heavy spring rains, resulting in excessive runoff.

3. Topography of the Drainage Area—
   - (a) As to whether the surface is level or inclined, and the degree of inclination.
   - (b) As to character of area, whether smooth or rugged.
4. Geology of Drainage Area—
   - (a) Whether pervious or impervious.
   - (b) If pervious, whether such pervious deposits are (a) shallow or deep; (b) level or inclined; whether the outlet or point of discharge of the pervious deposits are (c) in the lower valley of the same river, or (d) in valleys of other rivers, or in the sea.
   - (c) As to the condition of the channel of the stream, whether (a) pervious or impervious; (b) whether or not the bed contains more or less extensive deposits of sand and gravel, permitting of the development of a more or less extensive underflow
5. The Condition of the Surface—
   - (a) Whether bare or covered with vegetation.
   - (b) Whether in natural condition or cultivated.
   - (c) Nature of vegetation, whether grassland, cultivated crops, or forests.
6. The Character of the Natural Storage on the Drainage Area—
   - (a) Nature and extent of surface storage, consisting of lakes, ponds, marshes, swamps.
   - (b) Nature and extent of ground storage, consisting of gravel, sand, and other similar pervious deposits.
7. The Nature of the Drainage Area Considered—
   - (a) As to size, whether large or small.
   - (b) As to shape, whether long and narrow, or short and broad.

(c) The location of the area relative to prevailing winds.

(d) The direction relative to the path of storms.

8. Character of the Stream and Its Tributaries—
   - (a) As to slope or gradient, whether flat or inclined.
   - (b) As to falls and rapids on the stream.
   - (c) As to the section of the stream, whether deep or shallow.
   - (d) As to the arrangement of tributaries, whether joining the main stream at various points along its course or concentrated in a fan-like arrangement at a more or less common point of discharge.
9. The Artificial Control of the Stream—
   - (a) As to dams and storage reservoirs on the drainage area.
   - (b) As to the restrictions of the river sections by dikes and levees.
   - (c) As to the obstruction of the stream by piers, abutments, and other encroachments in or adjacent to the waterway.
10. The Artificial Use of the Stream—
    - (a) For irrigation.
    - (b) For water supply.
    - (c) For the supply of navigation canals.
    - (d) For artificial storage and regulation of the same.
11. Character and Extent of the Winds on the Drainage Area—
    - (a) As to their intensity and direction.
    - (b) As to the modification of the same by mountains and forests.
12. Ice formation—
    - (a) As modifying the winter flows of the stream.
    - (b) As to the formation of ice gorges and their accompanying floods.

PRECIPITATION

That the runoff is the direct result of precipitation, modified in quantity and occurrence by both the quantity and occurence of the precipitation, by its distribution during the year, and by other controlling factors, is so obvious as to require no discussion.

Other factors being the same, the runoff should and does increase with the precipitation and should and does vary in amount and distribution with the varying conditions of precipitation.

The study of precipitation, its distribution througthout the year, and the variations in such distribution in different years and its total annual amount, is most important to a clear understanding of the subject.

The approximate amounts of the average annual precipitation in the United States are shown by Map No. 1 (from Garnett). This map is very general in its nature and shows only approximate average conditions, to which there are many exceptions, which must be considered in detail. Such a map is of little value for the consideration of this subject, but it gives in a general way the limits within which the amount of the annual rainfall is generally approximately equal.

Causes Which Produce or Influence Precipitation

A discussion, in anything like adequate detail, of the causes which produce precipitation and which influence its distribution, its total amount, and its variation, would be too extended for the purpose of this bulletin. In general, the source of precipitation is the water taken up by evaporation from wet surfaces. The immediate vicinity of large bodies of water from which the most extensive evaporation necessarily takes place is therefore one of the most important requisites for extensive precipitation. In general, distance from extensive sources of supply results in a decrease in annual precipitation. These conditions are, however, greatly modified by the direction of the wind. This question can be best discussed by some brief quotation from those who have given this matter much study. The present opinion of meteorologists on the subject of causes of rainfall is set forth by Blanford as follows:

"As a result of a long study of the rainfall of India, and perhaps no country affords greater advantages for the purpose, I have become convinced that dynamic cooling, if not the sole cause of rain, is at all events the only cause of importance, and that all the other causes so frequently appealed to in popular literature on the subject, such as the intermingling of warm and cold air, contact with cold mountain slopes, etc., are either inoperative or relatively insignificant." (*Nature,* XXXIX, 583).*

Curtis in discussing this matter states:

"The ascentional movement of the air requisite to dynamic cooling may be brought about by three different processes; (1) by convective currents; (2) by hills and mountains; (3) by cyclonic circulation; and I classify rainfall as convective, orographic, or cyclonic, according as it is due to the first, second, or third of these causes of ascentional movement. Of course in some cases all of these may be operative at the same time, but for the purposes of analysis they will be separately considered.

"In a region of purely *convective* rainfall, the circulation is primarily vertical, and the moisture evaporated is largely precipitated before being carried away by horizontal currents. Therefore an increased evaporation will be followed by an increased rainfall. Consequently any change in surface condition which increases or diminishes the evaporation will, in such a climate, be followed by a corresponding increase or decrease in precipitation.

"In the case of *orographic* rainfall, currents that are essentially horizontal are forced to become locally ascending. The moisture evaporated in the region lying in the path of the current is partially or entirely precipitated over the region where the ascensional movement is developed. Consequently an increased evaporation will, to a greater or less extent, be restored to the basin by an increased precipitation. The extent to which this will take place, that is, the proportion of the increase of moisture that will be returned to the basin, will depend on the extent and height of the mountains and the relative frequency of the orographic rain-bearing winds. * * * *

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*U. S. Dept. Agriculture, Forestry Division, Bul. 7, pp. 188 et seq.

"The general class called *cyclonic* rainfall includes a great variety of rain types related to cyclonic circulation, some of which are as yet by no means well understood.

"In the ordinary progressive area of low pressure the cyclonic circulation is largely horizontal, but with an upward component. This upward component produces the usual rainfall of our cyclonic storms. In these storms the horizontal component of the circulation is so large that the moisture evaporated over one region is precipitated over another. Consequently in regions where rainfall is of this type, an increased evaporation in any region will not be followed by an increased rainfall in that same locality.

"In local thunder storms we have a type of rain related to a cyclonic circulation in which the vertical component often becomes very large, as compared with the horizontal component. This predominating vertical component is due to convection and the accompanying rainfall is to be considered as partly or largely convective. Convection induces and initiates a cyclonic circulation which may continue after the direct convective action has ceased. * * * *

"The equatorial rain belt is the most prominent region with almost exclusively convective rainfall. The Brazilian forest region, the Aruwhimi district of central Africa, the Malaysian Archipelago, and the valley of Upper Assam in India are in or near this belt. They have light winds, and the moisture evaporated from the surface is precipitated before being carried to any considerable distance by horizontal currents. Under these conditions an increase or decrease in the evaporation will be followed by an increase or decrease in rainfall. * * *

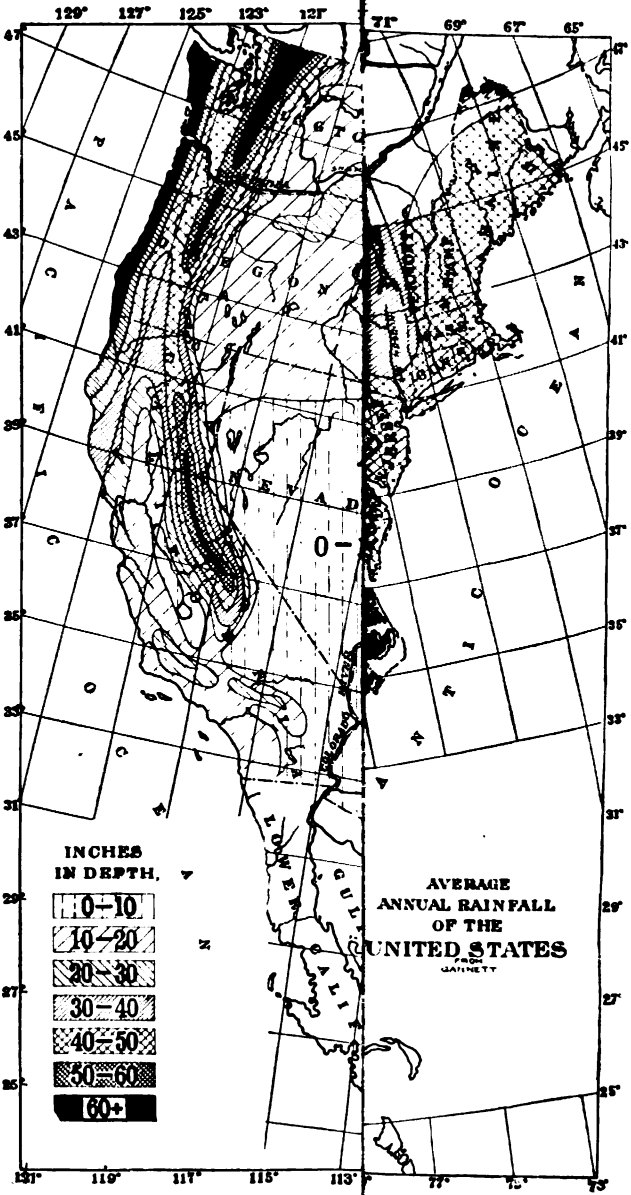
"Blanford estimated that for the Aruwhimi district probably over half of the rainfall is due to the direct restoration of the moisture evaporated. * * *

"Bordering on each side of the equatorial belt are the regions of the trades, which, over the ocean, are almost rainless; but over intercepting land areas, such as Central American and the Antilles, considerable rainfall occurs. This is frequently difficult to analyze, but it is largely convective and in hilly regions partly orographic. The seasonal distribution shows that the rainfall is intimately related to the annual oscillation of the limits of the trade wind, and that the rainy season requires a special explanation. With the exception of the well-known tropical cyclones of the seas, the distribution of pressure over the trade region is unfavorable to the development of a cyclonic circulation, and consequently cyclonic rainfall is seldom presented. * * *

"Passing from the region of trades, we reach latitudes favorable to cyclonic development. Here convective and cyclonic action are frequently combined. In the warmer latitudes and in the summer season, the equilibrium of the atmosphere becomes unstable and convection currents are set up which induce an incipient cyclonic circulation. Then there is a combined convective and cyclonic rainfall. As we go northward the direct convective feature of the rainfall becomes less prominent, and the purely cyclonic rainfall predominates. Over the whole region orographic rainfall is added to the other two classes when hills or mountains are situated in the path of moisture-laden currents. With this general statement the further examination of the effect of surface conditions on the rainfall in middle latitudes must be applied to concrete cases.

"In the United States the Great Plains region of Kansas, Nebraska, and the Dakotas is that for which a variation in rainfall has most often been claimed or anticipated. Many settlers believe that the so-called "rain-belt" is moving rapidly westward with the extension of cultivation and settlement.

"The single condition favorable to such an increase of rainfall consists in the steadier and larger evaporation which no doubt takes place over the cultivated area, but the other conditions necessary to condense and precipitate this moisture over the same region are largely absent. There are no hills or mountains to produce orographic rainfall, and without these barriers the high winds constantly carry away the moisture evaporated



AVERAGE ANNUAL RAINFALL OF THE UNITED STATES FROM GANNETT from the surface and precipitate it far to the eastward. Hence not only is the increased moisture in the air but a small fraction of the total rainfall, but also only a small portion of this is gathered into convective or cyclonic currents and restored again by them to the prairie soil. Evidently a small amount must be thus restored in every summer rain and the total rainfall increased thereby, but quantitatively this must be very small.”

Moore has pointed out the main controlling factors influencing rainfall in Wisconsin and in the Eastern United States as follows:

*“Before one can get a comprehensive idea of the magnitude of the problem involved in the creation of floods of the United States, it will be necessary for him to first study Figure 2, which gives a typical illustration of the cyclonic storms that frequently form on the Rocky Mountain Plateau, either on its northern, central, or southern portions. Under the influence of gravity air flows from regions where the pressure is great toward the regions where it is less. In the case illustrated by Figure 2 the atmosphere, as indicated by the direction in which the arrows point, is flowing from the region marked “high,” which is central over the Carolinas, toward the region where the pressure is low, which is central over Montana, and the vaporous atmosphere that rises from the Gulf of Mexico and the adjacent ocean is carried far into the interior of the continent. Conditions similar to these occur many times each month, and as a result the eastern and central portions of the United States are bathed in a succession of rains which, as shown by Figure 1, gradually thin out and largely disappear on the eastward edge of the Rocky Mountain Plateau, because the currents of air from the Gulf of Mexico do not reach farther inland.”

It is sometimes pointed out that in the United States the precipitation decreases with the elevation of the surface above sea level,† as proof of which the distinct decrease in precipitation from the Atlantic Coast to the mountains, following a somewhat corresponding increase in altitude, is offered. What portion of

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*Report on The Influence of Forests on Climate and on Floods, P. 20. Government Printing Office, Washington, 1910.

† Bulletin C, U. S. Weather Bureau, P. 13.

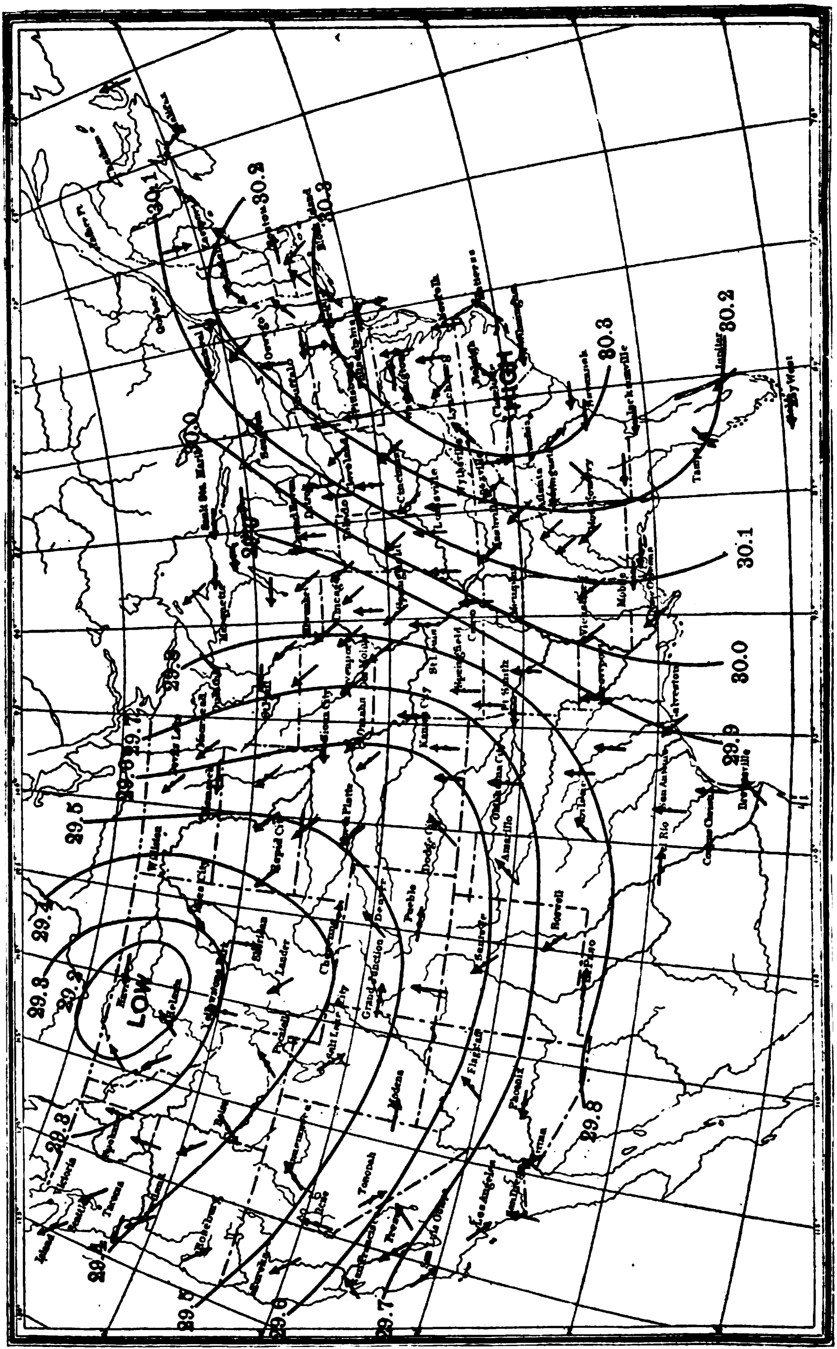

Fig. 2.—Source of Rainfall in Eastern United States. Arrows fly with the wind and show how the moist atmosphere of the Gulf of Mexico and South Atlantic Ocean is drawn inland. (Report on the Influence of Forests on Climate and on Floods.)

this effect is due to altitude and what portion is due to distance from the source of supply is uncertain, but the above discussion by Moore would seem to indicate that distance rather than altitude is the principal influence. Evidence from the Pacific Coast would seem to indicate that the higher elevation assures heavier precipitation when other conditions are equal.

The removal of aqueous vapor by mountain heights, and the decrease in precipitation thereby in areas beyond such elevations, is also illustrated by the area of the country just east of the western mountains. This area, although near an abundant source of supply, is largely deprived of its value by the intervening mountain ranges.

The small rainfall in southern California, in spite of its close relation to the Pacific Ocean, illustrates the necessity of favorable wind currents in order that adequate precipitation shall result.

What influence on the amount and distribution of precipitation is exerted by minor physical conditions is somewhat uncertain. The opinion was formerly quite general that the presence of forests largely affected such distribution. Standing within the shade of a dense forest of mighty trees, such a conclusion might seem warranted, but if the same forest be viewed from a neighboring mountain top, its apparent insignificance in controlling or even influencing such a vast force seems conclusive. Meteorologists have in general concluded that the presence or absence of forests has little or no influence in modifying rainfall.* The occurrence of rain over the dense forests of the Pacific Coast and in the tropics has been even lately advanced as an argument in proof of the effect of forests on rainfall under at least special conditions. It is evident, however, that the forests are due to suitable rainfall conditions, and it is fair to assume, unless further evidence is produced, that the forests are due to the prevailing rainfall conditions rather than that the rainfall is due to the existing forests. It would seem that in this case cause and effect are somewhat confused in the minds of the observers.

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*See *"Meteorology"*—Thomas Russell, P. 86.
"Forest and Rainfall"—H. A. Hazen, Monthly Weather Review, 1897, P. 395.
"The Relation of Forests to Rainfall"—W. F. Hubbard, Monthly Weather Review, 1906, P. 24.

VARIATION IN ANNUAL PRECIPITATION

The variations that occur in the total annual precipitation and in its distribution are still more difficult to trace. The investigator must confine himself largely to a study of the actual variation and the actual distribution, endeavor to determine their limits and the effects due to them.

The average or mean conditions of precipitation are of only general importance. The extreme conditions are those most directly modifying runoff, hydraulic problems and works of all classes. A water supply, for whatever purpose, should be constant in quantity or vary only as the demand for water varies; otherwise constant service will be interrupted or must depend on other provisions; hence the occurrence of minimum conditions will largely modify the nature and extent of works intended to conserve and equalize such supplies. The maximum precipitation resulting in extreme flood flows must, on the other hand, modify works intended for escapement and flood protection.

The question is frequently raised as to whether or not the total annual rainfall is increasing or decreasing, and many believe that the removal of forests has a detrimental effect and that the cultivation of ground, and especially irrigation, has increased the annual rainfall. Meteorologists are generally of the opinion that the causes which influence rainfall are too great and too far-reaching to be modified by any possible works of man, and the records seem to bear out this opinion. Figure 3* shows the progressive mean of the rainfall in various sections of the United States for a considerable period of years, and illustrates the fact that in every section the rainfall is subject to considerable variations. From this diagram it is readily seen that if the rainfall record for only a limited series of years is considered, conclusions might be readily drawn from almost every section that the rainfall is increasing or decreasing, according to the data selected. For example, from the diagram of the rainfall of New England, a continued increase is noted from the year 1838, when the progressive mean was 8.8 inches

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* See Note 1. Page 148.

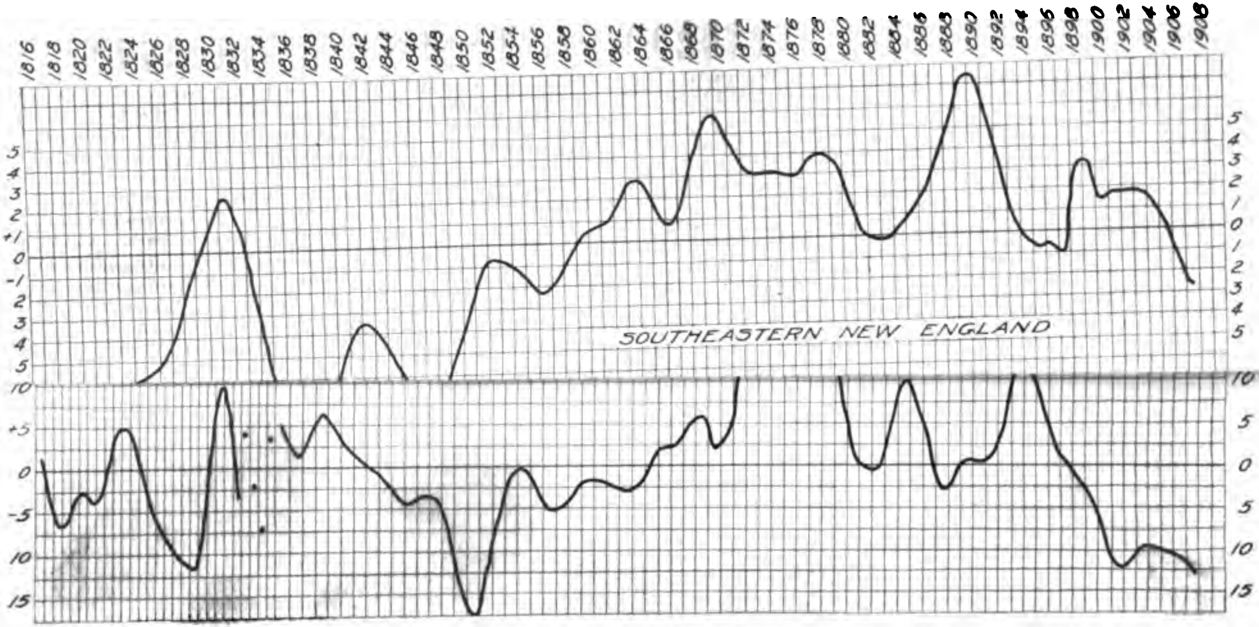

Fig. 3.—Diagram Showing Progressive Mean Annual Precipitation at Various Localities in the United States.

below the mean for the period, to the year 1890 when the progressive mean was 7.5 inches above the mean for the period; while from 1890 to 1907 there is a continued decrease in the progressive mean of over ten inches. In Wisconsin, from the year 1840 to 1881, an increase of 13.5 inches in the progressive mean is shown. while the years 1881 to 1902 show a decrease of 12 inches. The progressive mean for Charlestown, based as it is on the observations at a single station, shows even greater variations. The increase from 1850 to 1877 is 43.5 inches and the decrease from 1877 to 1898 is 39 inches. It is evident therefore that to demonstrate either an actual increase or decrease in the mean annual rainfall, records are required for a much longer period than is available in the United States or elsewhere.

The long periods during which the progressive mean is sometimes below or above the mean for the period should also be noted. For example, in New England the progressive mean was below the mean for the period for 26 years from 1833 to 1859. and above for the 23 years from 1859 to 1882. In Wisconsin, the progressive mean was above the period mean for 20 years from 1867 to 1887, and below the mean for the ten years from 1892 to 1902. Usually, however, the departure in one direction is for more limited periods, as an examination of the various curves will show. The error that must therefore occur in drawing any conclusions from a short series of observations is manifest.

These diagrams of mean rainfall, together with much longer records in foreign countries, lead to the conclusion that while considerable variations must be expected, yet such variations are within certain limits, and that a record of forty years will give a mean from which the mean of any other similar period will hardly depart more than a few percent. While geological research shows conclusively that great changes in climatic conditions have taken place in times past, and undoubtedly will take place in the future. yet these changes have occupied thousands of years; and while the records of centuries may show variations in rainfall conditions, yet so far as the life of man is concerned, the variations in annual rainfall are due essentially to the great swing in the pendulum of conditions, the cause of which we know not but which we can confidently expect when extremes are reached to gradually revert to the opposite conditions. In other words, it is clearly evident that so far as the life of man is concerned, the rainfall conditions remain essentially unchanged except for the variations from average conditions, the extent of which can be fairly well established by an examination of available records. In the changes of ages, the character of which we have not sufficient data to recognize, mankind has no practical interest.

VARIATION IN ANNUAL PRECIPITATION IN WISCONSIN

Having briefly discussed the general variations in annual rainfall, it is essential to the subject to examine the local conditions in Wisconsin in greater detail. Figure 4* shows both the actual departure from the mean for the recorded period of rainfall observations at various stations in or near Wisconsin, and the progressive mean during such period. It will here be noted that the variations in the annual rainfall, even in the limited area considered, are not synchronous. The period of maximum rainfall at Milwaukee was from 1874 to 1879, while at Madison it was from 1878 to 1886. While in some cases a certain similarity exists in the progressive mean, in no two cases are the variations uniformly parallel. The records of precipitation show not only a considerable variation in the annual amount, but also show (see Figures 5 and 6) that the annual distribution throughout the state is never the same. It is seen from these maps that the rainfall does not only vary from year to year at any given locality, but the variations in adjoining localities are often in opposite directions. As the variation in runoff from any drainage area must be due to variation in the rainfall at all points on such area, the data from which the net results must be deduced are, from any extensive drainage area, consequently very complex.

The variation in the total quantity of the annual precipitation on any drainage area is frequently very great, although it is usually less so than at any particular station. At Madison

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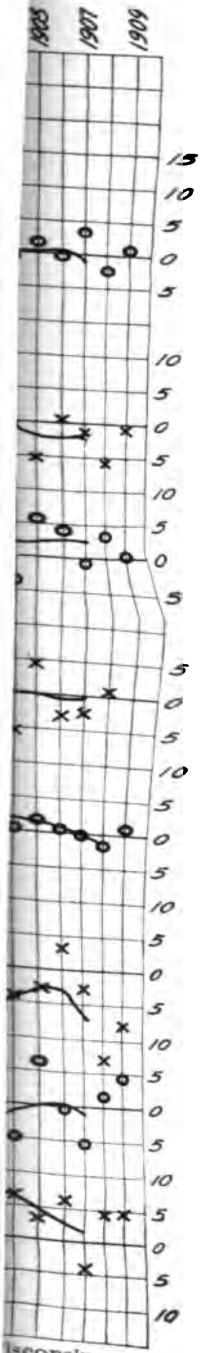
* See Note 2. Page 149.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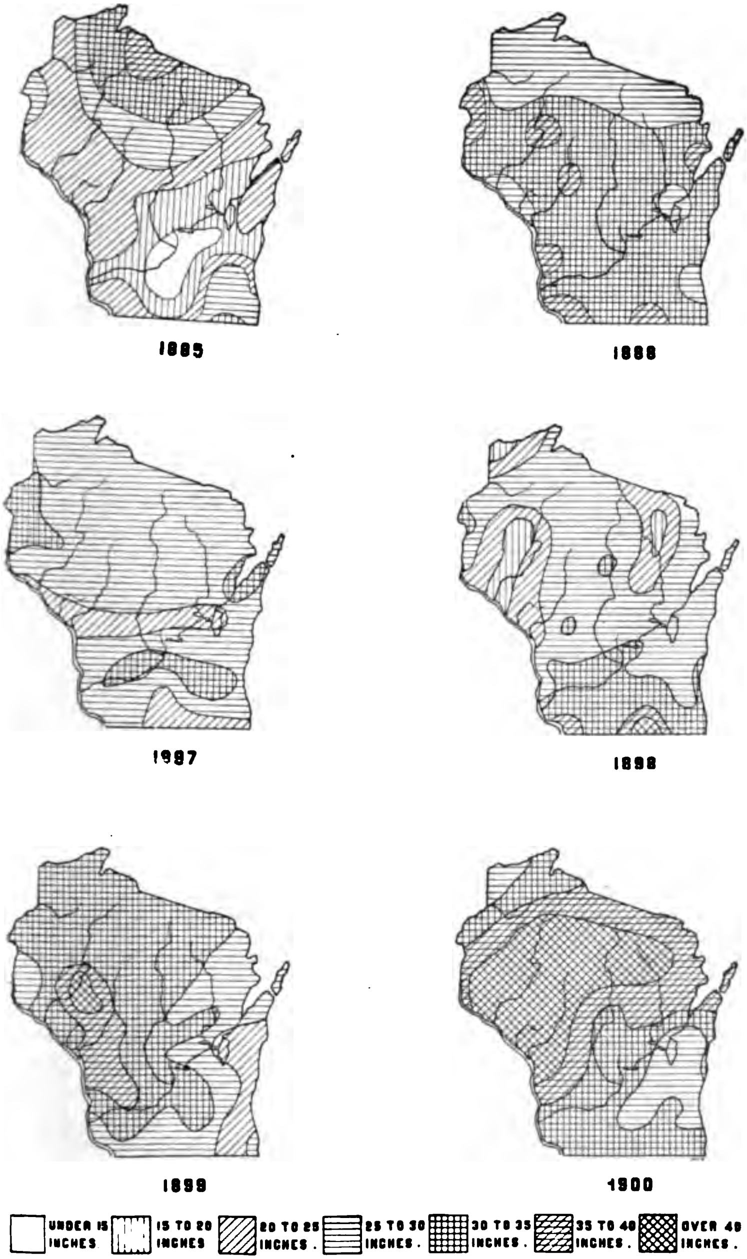


1895

1896

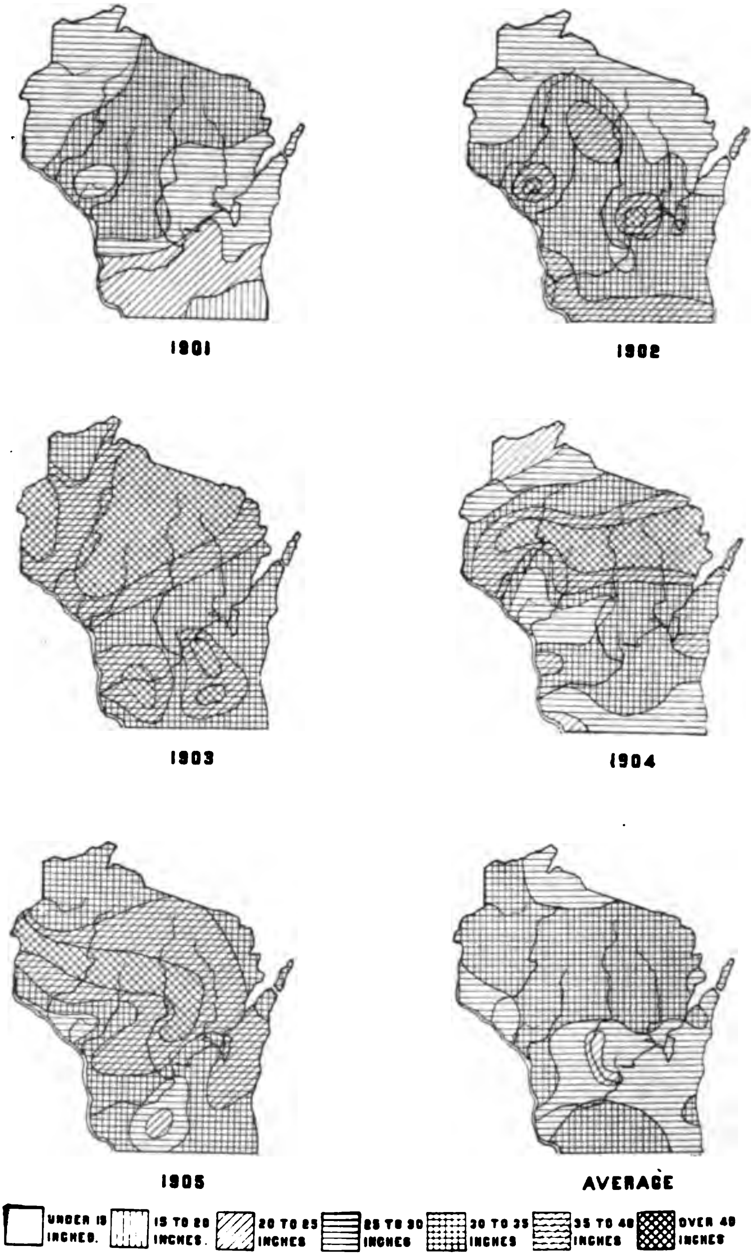
1897

1898

1899

1900

Fig. 5.—Annual Distribution of Rainfall in Wisconsin.



1901

1902

1903

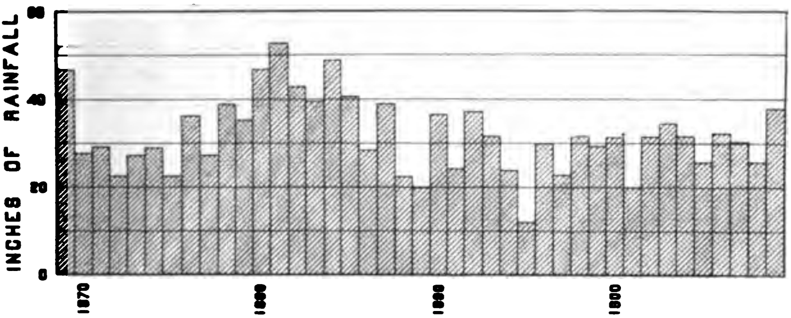
1904

1905

AVERAGE

Fig. 6.—Annual Distribution of Rainfall in Wisconsin.

(see Figure 7), in the last twenty-five years, the extreme variations in the total annual rainfall have been from a minimum of thirteen inches to a maximum of fifty-two inches, the minimum



FLUCTUATION OF ANNUAL RAINFALL AT MADISON, WIS.

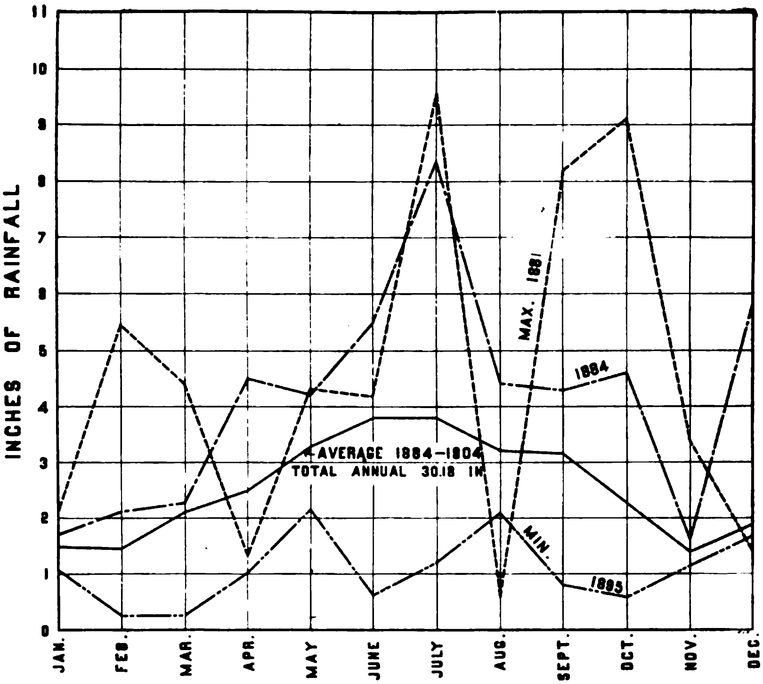
FIG. 7.

being only twenty-five per cent of the maximum (see also Figure 4). Broadly speaking, such a variation is unusual for, as a general rule in the upper Mississippi valley, the minimum rainfall is usually about fifty per cent. of the maximum. Such wide variations in the annual precipitation have a direct and radical effect on the annual runoff.

Variation in Monthly Precipitation

The percentage of variations in any monthly period is always very much greater than for the annual period in the same locality. This is very well illustrated by the record of precipitation at Madison, Wisconsin. (See Table I.) Here the monthly rainfall varies from a minimum of merely a trace (see April, 1877) to a maximum of 9.47 inches (see July, 1881).

The great variation in the monthly precipitation and the great departure of the extreme from the mean conditions are shown by the graphical diagram of the fluctuations of monthly rainfall at Madison (Figure 8).



FLUCTUATION OF MONTHLY RAINFALL AT MADISON, WIS.

FIG. 8.

TABLE I.—PRECIPITATION AT MADISON, WISCONSIN

Year.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov	Dec.	Total
1869	2.69	2.35	0.79	3.08	4.90	6.14	3.63	5.92	2.68	2.66	2.05	2.64	37.53
1870	3.35	1.35	3.85	0.18	1.09	1.92	5.25	3.85	4.00	2.09	0.53	0.67	28.13
1871	2.32	1.43	2.98	2.00	3.31	4.93	2.11	3.35	0.47	3.07	2.35	1.15	29.45
1872	1.20	0.40	2.18	1.82	2.83	2.44	1.28	2.24	5.11	0.60	0.76	1.60	22.44
1873	1.40	0.60	2.07	1.26	3.53	6.60	0.82	2.76	2.54	1.98	2.15	1.80	27.49
1874	3.64	0.95	0.95	1.26	2.14	2.85	5.00	1.40	5.46	1.44	3.51	0.45	29.05
1875	1.10	2.80	0.90	1.87	2.61	3.37	0.97	2.56	2.08	1.96	0.40	2.18	22.80
1876	2.31	1.60	2.27	2.65	5.18	4.57	4.25	3.42	3.41	1.59	2.31	2.59	36.15
1877	1.00	0.30	3.40	T	1.02	4.77	3.84	3.76	0.64	4.12	2.81	2.01	27.67
1878	0.40	1.19	2.45	2.87	4.64	4.20	7.56	4.28	6.54	3.78	0.76	0.79	39.54
1879	0.79	2.54	1.34	3.33	3.91	2.80	5.91	0.99	2.79	2.50	6.02	2.29	35.21
1880	2.75	1.75	2.11	5.48	4.45	9.31	6.00	5.90	4.44	1.68	0.68	1.17	46.72
1881	2.05	5.42	4.34	1.50	4.25	4.15	9.47	0.56	8.17	9.12	2.56	1.32	52.93
1882	1.33	1.74	4.73	4.21	2.89	7.76	2.70	6.83	1.91	4.14	2.62	2.03	42.89
1883	1.01	1.64	0.32	......	6.98	7.57	8.89	2.74	2.39	3.79	2.56	1.95	39.84
1884	1.68	2.12	2.31	4.51	4.21	5.47	8.44	4.39	4.25	4.60	1.53	6.68	49.19
1885	2.44	0.82	0.62	3.45	1.68	5.11	7.30	6.41	4.05	2.37	2.74	3.59	40.58
1886	3.33	2.36	4.67	2.48	2.02	1.08	0.79	5.05	2.29	2.21	1.21	1.30	28.78
1887	3.09	4.34	2.14	0.96	2.12	1.48	5.49	3.75	6.67	3.18	1.16	4.53	38.89
1888	1.74	1.01	2.61	1.85	3.76	2.95	2.26	1.27	1.04	1.68	1.32	1.57	23.06
1889	1.59	1.84	1.48	1.71	3.28	2.00	2.12	0.72	1.93	T	1.17	2.33	20.17
1890	1.81	2.01	2.38	2.22	5.03	7.72	1.81	4.23	2.62	4.59	1.94	0.62	36.98
1891	1.19	1.38	3.62	1.45	1.48	3.69	2.66	1.41	0.38	1.49	3.31	2.24	24.24
1892	2.42	1.94	1.38	3.94	6.98	7.61	2.32	3.43	3.29	0.36	1.24	2.19	37.10
1893	1.06	1.05	2.29	4.53	2.28	6.69	4.64	1.42	2.67	1.85	1.30	1.68	31.46
1894	0.96	0.46	1.73	3.57	3.36	3.94	1.75	0.54	4.21	1.77	1.65	0.67	24.59
1895	1.12	0.26	0.27	1.06	2.58	0.59	1.21	2.08	0.91	0.58	1.03	1.80	13.49
1896	0.88	0.32	0.79	4.91	6.31	2.69	3.62	2.43	4.29	3.03	1.40	0.68	31.35
1897	2.22	0.82	2.38	2.51	0.51	4.03	1.79	2.73	1.73	0.86	1.35	1.67	22.58
1898	3.59	2.42	2.67	2.46	4.71	4.40	2.83	2.56	2.43	3.08	0.55	0.21	31.91
1899	0.16	0.42	1.94	2.69	4.92	2.64	3.22	3.57	3.35	1.58	0.99	1.37	26.81
1900	0.69	1.28	1.32	1.31	1.86	3.20	6.91	2.72	2.89	4.43	1.72	0.47	28.78
1901	0.62	0.59	2.77	0.45	2.41	2.40	1.54	1.33	4.16	2.49	0.46	1.02	20.24
1902	0.17	1.46	0.60	1.17	5.16	4.27	8.18	0.78	4.18	1.23	1.41	1.84	31.25
1903	0.11	1.11	3.50	2.88	4.38	1.39	6.70	6.95	3.54	2.18	1.07	0.60	34.40
1904	0.30	1.87	2.44	1.39	5.03	2.85	3.27	3.20	5.93	1.60	0.03	3.34	31.25
1905	0.77	1.28	1.74	1.43	6.40	2.88	2.27	2.48	0.58	2.25	2.23	1.17	25.49
1906	2.80	0.97	2.13	0.10	3.35	4.55	1.80	7.56	2.04	2.09	2.36	1.23	32.38
1907	1.89	0.28	1.80	3.00	2.69	2.80	5.84	3.59	4.69	1.14	1.22	1.35	30.29
1908	0.97	1.79	1.62	4.41	5.38	1.80	2.85	2.53	0.78	0.64	2.14	0.76	25.67
1909	2.33	1.70	1.12	7.19	2.49	0.29	2.78	4.39	2.20	0.91	2.28	3.15	30.83
Mean for 41 Yrs.	1.64	1.51	2.12	2.44	3.61	3.91	4.01	3.22	3.16	2.32	1.75	1.75	31.45

DETAILED STUDY OF PRECIPITATION

When precipitation is studied in detail it is seen that the variations in the annual and in the monthly precipitation remain within certain limits; but these totals are made up of still greater variations in individual storms. Every rainstorm is of more or less limited extent and also varies widely in intensity and in the amount of the total local precipitation.

The daily maps of the Weather Bureau show the variations in both these factors from day to day, and are worthy of the careful consideration and investigation of this subject.

The net result of the rainfall conditions in Wisconsin during certain weekly periods of 1907 is shown in Figure 9, and the varying conditions for these periods on different portions of some of the drainage areas of the state are clearly shown.

Rainfall Records

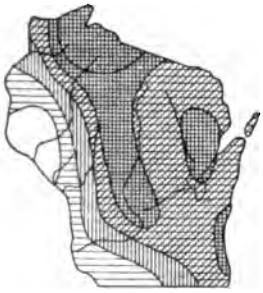
The records of rainfall made by the signal service of the United States Army, and later by the U. S. Weather Bureau, cover a considerable period. The number of points at which such observations are made is increasing from year to year; but when important hydrological questions are under consideration, the data available are still found much too limited as an entirely satisfactory basis for exact conclusions. The degree of accuracy of such observations is far from satisfactory, for although the placing of a rain gage and the observation of the rainfall would seem a simple process, yet the comparative results from gages located at points closely adjoining indicate that elevation and wind currents, due to various conditions, obstruction, etc., greatly modify the results.

The artificial divisions of daily, weekly, and monthly observations are sometimes misleading when they are examined with reference to corresponding runoff records. A rainstorm follows no such divisions of time, and the effect of a heavy rainfall near the end of a month will often result in an increased runoff during the following month for which monthly records of observations, usually available, offer no adequate explanation.

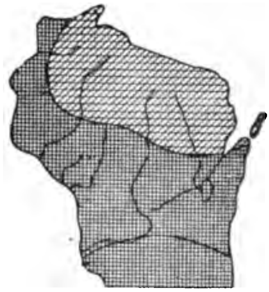
Divisions of the Water Year

For the purpose of studying precipitation and its effect on the flow of streams, precipitation and runoff records are frequently subdivided into what is termed the "water year."

The water year for Wisconsin conditions may be divided into periods beginning, approximately, with December, and ending, approximately, with the following November. The period from December to and including May, is termed the "storage" period; June, July, and August, the "growing" period;



MAY 13 TO MAY 20



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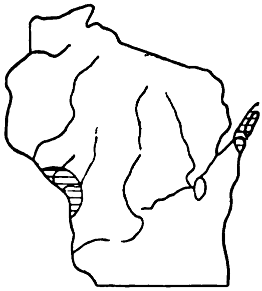

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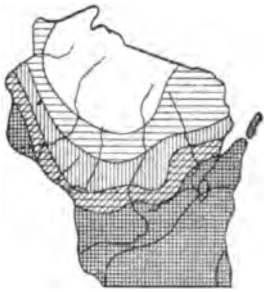

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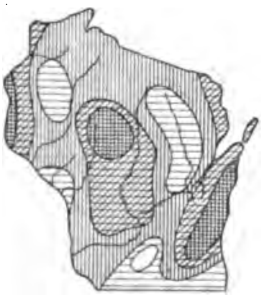

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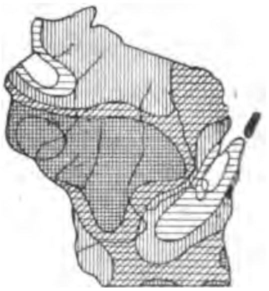

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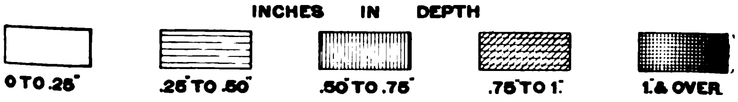

Fig. 9.—Weekly Distribution of Rainfall in Wisconsin.

and September, October, and November, the "replenishing" period.

These divisions of the water year are more fully described in the discussion of the relations of precipitation to runoff. Studies of the rainfall and its subdivisions as they have occurred on the drainage areas of the Wisconsin, Menominee, and Peshtigo Rivers are shown in Figures 10, 11, and 12.

A study of the same character of data showing how they have varied on different areas of the valley of the Wisconsin River is shown in Figure 13.

DISPOSAL OF RAINFALL

The disposal of the rainfall is a subject pertinent to this inquiry, for while some of the rainfall flows away in the stream, the amount so flowing is not always a large percentage of the total amount which has fallen, and the amount which appears in the runoff is evidently the residue after the other demands on the rainfall have been temporarily satisfied. A light rain falling on an impervious rock surface may simply wet the surface and soon disappear by evaporation. A heavier rainfall on the same surface will be accompanied by a considerable runoff, and although the same evaporation may obtain, yet its comparative effect is small. If the rock be pervious, a light rain may simply wet the surface and soon disappear by evaporation; a heavier rain will be absorbed and disappear, except the amount still lost by evaporation, while a very hard rain may be accompanied by more or less runoff, according to the relative conditions. When the surface is covered by vegetation, a certain amount of the rain will be used by the growing plants, and an additional element must be considered. The proportion of the rainfall which is lost in these various ways is greatly modified by surface conditions and by conditions of temperature and wind.

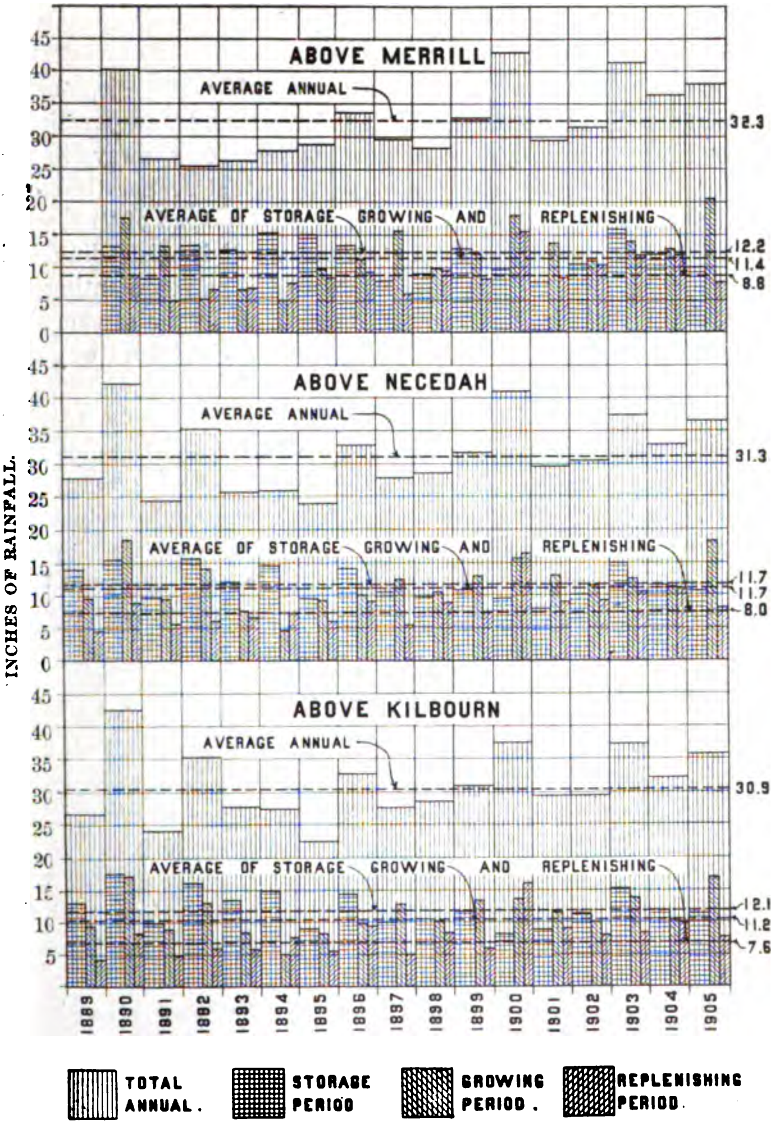

Fig. 13.—The Comparative Annual Rainfall on Various Drainage Areas of the Wisconsin River.

EVAPORATION

Some hydraulic engineers, in considering the disposal of precipitation, divide the same into "runoff" and "evaporation." By "runoff" is meant that part which flows away in the streams. The parts lost by seepage, by actual evaporation, and that used by vegetation, are classed under the head of "evaporation." This use of the term "evaporation" is misleading and seems unnecessary. The term "retention," as representing the portion of the precipitation not running off in the stream flow seems more logical and will be used, so that the term "evaporation" when used herein will have its true significance.

Vermeule endeavors to prove (see "Water Supply of New Jersey") that the amount of water retained on a drainage area is in direct proportion to the average annual temperature on the area. A careful investigation will, however, convince the observer that no direct relation exists between the retention and the average annual temperature. This is well illustrated by the relations shown by Rafter (see Figures 14, 15 and 16) between retention (designated as evaporation on the diagram) and temperature, on the Sudbury and Cochituate areas. The relations between retention and temperature are very indistinct, and the writer believes it is clearly shown that other modifying influences are very much more powerful than the average annual temperature. That temperature increases evaporation is undoubtedly the case; and that temperature materially increases the amount of water taken up by vegetation, and thus results in loss to runoff. is also true. It would seem, therefore, that the summer temperature, rather than the mean annual temperature, would have to be considered.

These diagrams show that the annual runoff as a general rule varies with the annual precipitation. On account of the various other conditions mentioned, there are numerous exceptions to this rule and considerable variations in the average relations between annual precipitation and annual runoff may obtain. In a general way, the retention may also be said to increase with precipitation because under such conditions there is more water

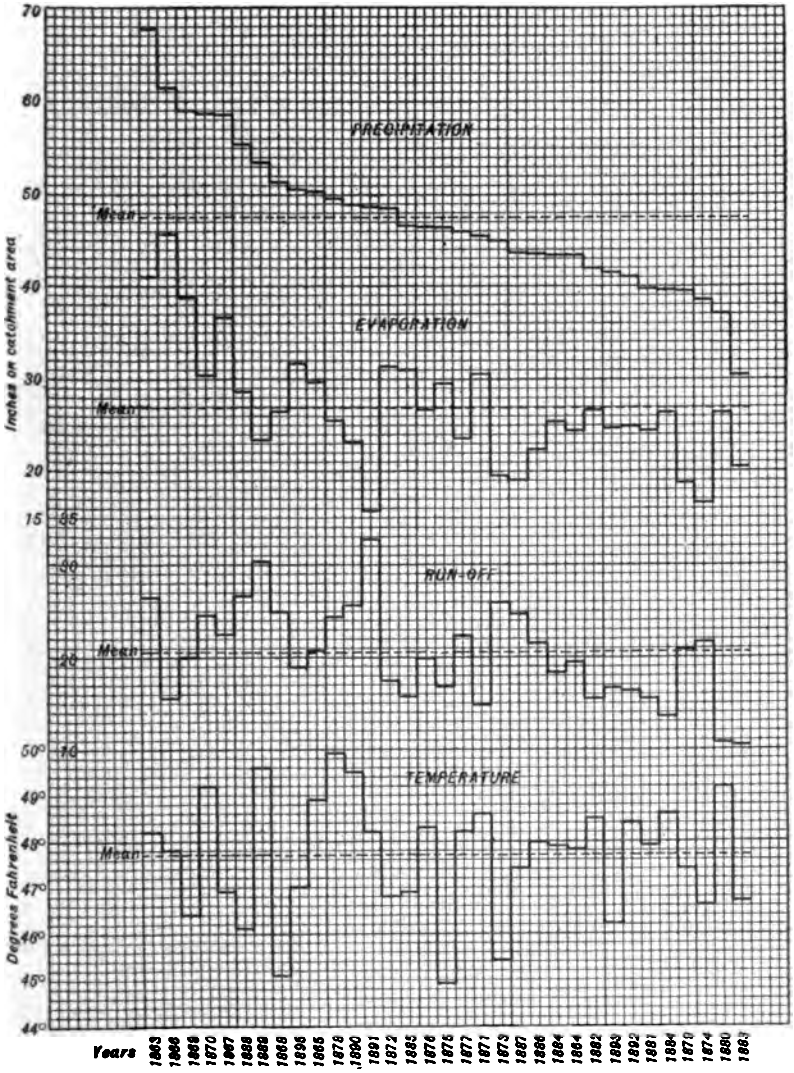

Fig. 14.—Relation Between Precipitation, Evaporation, Runoff and Temperature on Lake Cochituate Basin. Years arranged in order of dryness.

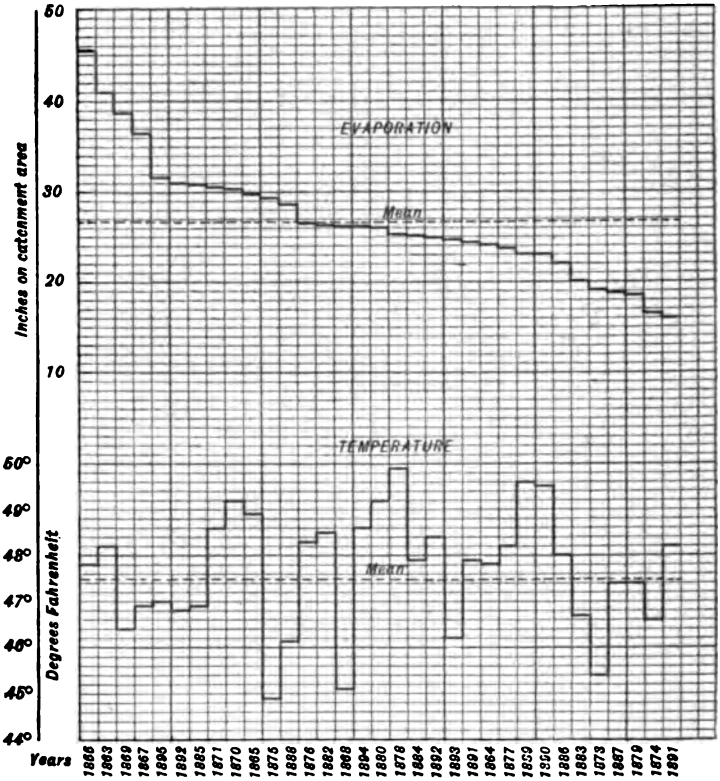

Fig. 15.—Relation Between Evaporation and Temperature on Lake Cochituate Basin. Years being arranged according to amount of evaporation.

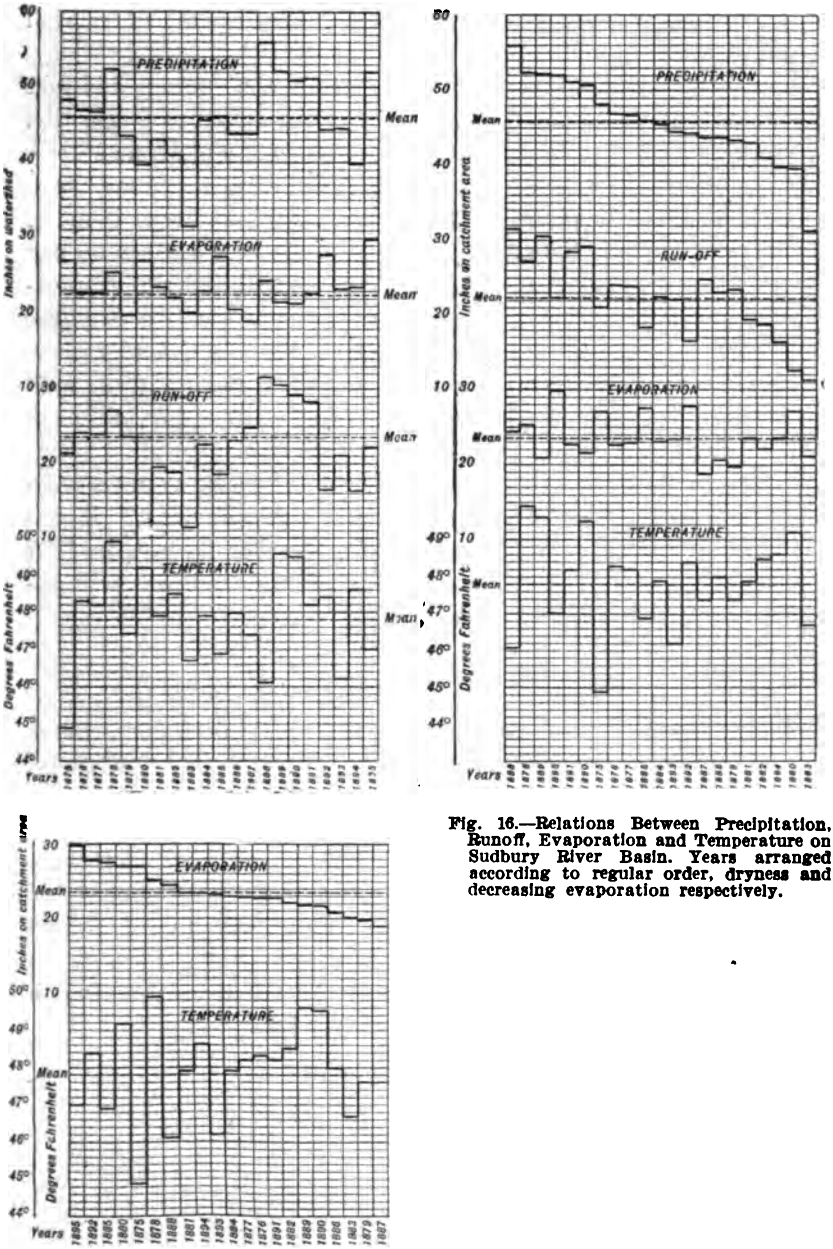

Fig. 16.—Relations Between Precipitation, Runoff, Evaporation and Temperature on Sudbury River Basin. Years arranged according to regular order, dryness and decreasing evaporation respectively.

to be lost in various ways in which the disposal of rainfall may occur.

A large evaporation takes place from the swamp and lake areas, fully equal for the most part to the amount of rain that actually falls on the exposed water surfaces; and if such areas occupied approximately the full drainage area, no runoff whatever would take place. While evaporation is a large item, on exposed surfaces the actual percentage of water surface on any drainage area is usually comparatively small, and the evaporation is therefore not a large factor when the entire drainage area is considered. Swamps and lakes, therefore, tend to conserve and control the flood waters and undoubtedly in most cases, add materially to the regularity of stream flow, although they frequently reduce its total annual amount.

Figure 17 is an approximate map of the mean average evaporation in the United States from free water surfaces. It was made from observations of the U. S. Weather Bureau in 1887 and 1888.* Further investigations of this subject will undoubtedly materially alter the data.

THE WIND

The effect of wind on stream flow is of minor importance. Much of this effect depends on the relative exposed surfaces of water and the protection afforded by topographical and forest conditions. The winds have a decided effect in increasing the amount of evaporation from exposed water surfaces as the saturated air in contact with such surfaces is rapidly removed and fresh unsaturated air is brought in contact with them. Where these water surfaces are small and lie among the hills, or are surrounded by forests, the effect of the wind on the same is greatly reduced and evaporation is diminished.

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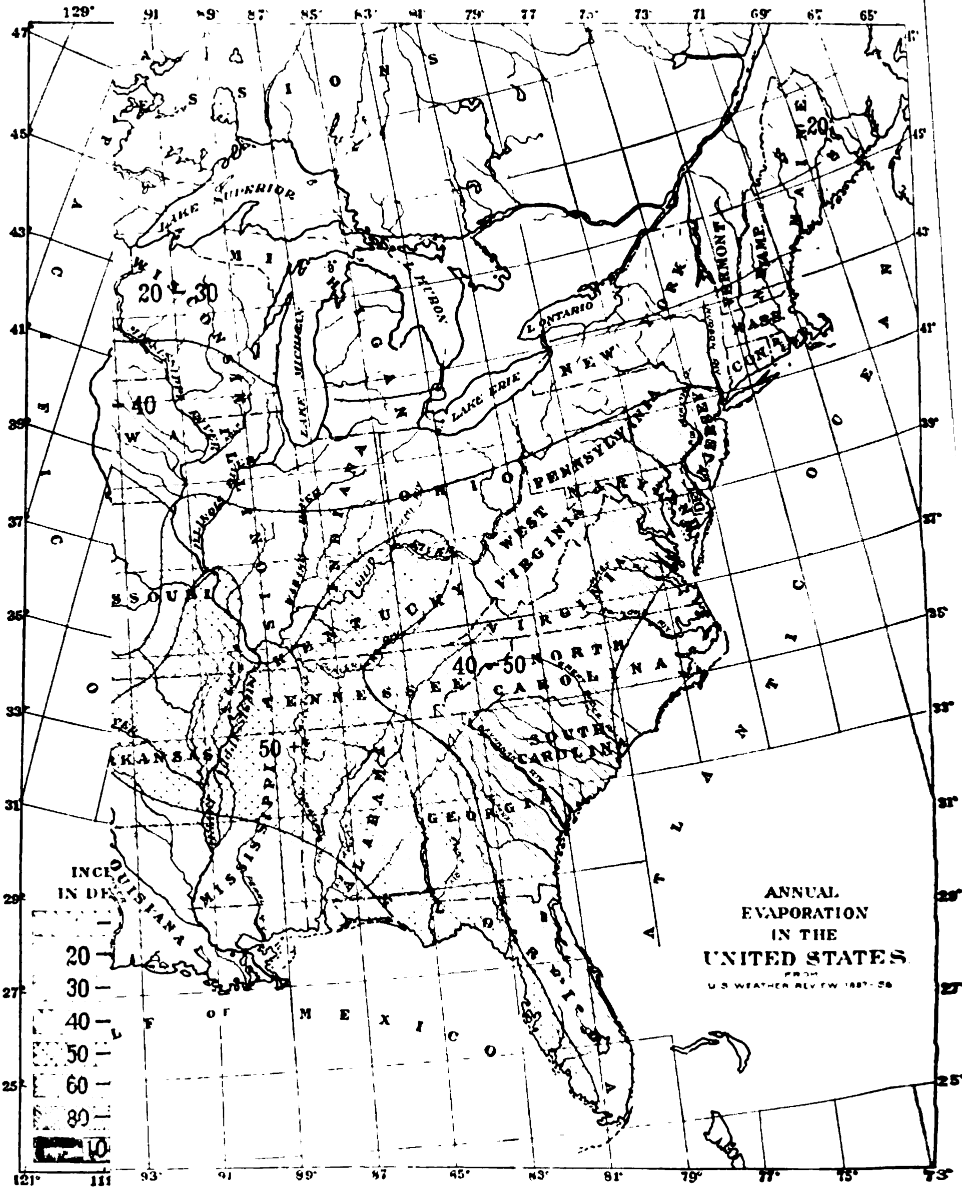
*See Monthly Weather Review, Sept., 1888.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PENNSYLVANIA
NEW YORK
VERMONT
NEW HAMP.
MAINE
MASS.
CONN.
NEW JERSEY
MARYLAND
WEST VIRGINIA
VIRGINIA
KENTUCKY
TENNESSEE
NORTH CAROLINA
SOUTH CAROLINA
GEORGIA
ALABAMA
MISSISSIPPI
LOUISIANA
ARKANSAS
MISSOURI
FLORIDA
PACIFIC OCEAN
ATLANTIC OCEAN
GULF OF MEXICO
DOMINION

TEMPERATURE

Temperature has a most marked effect on the relation of precipitation to runoff. The low temperatures of winter are the cause of the retention of much of the precipitation in the form of snow and ice throughout December, January, and February, in all northern latitudes, until these deposits are melted by the higher temperatures of March and April. This materially increases the runoff, and, even with light rains, produces the spring floods. The extent of the spring flood is modified to a very great extent by the conditions which have prevailed throughout the winter and which may prevail during the spring season. The precipitation of December, January and February commonly assures complete saturation of the ground. All the pervious deposits are usually filled during this period and the warm rains of spring, under such conditions, run rapidly from the saturated surface which even though naturally pervious has become impervious by saturation. Frozen ground, even though pervious and unsaturated, may be rendered impervious by surface freezing and thus permit a rapid flow of the rainfall into the stream, resulting in sudden floods which would not occur were the temperature conditions different.

In the northern streams the winter flow is frequently the minimum flow, for not only is the precipitation held in the form of snow and ice and hence may not materially affect the moderate stream flow, but frequently the shallow ground waters and springs are also congealed and the flow may thereby be greatly reduced and will be fed by the deeper seated waters which, while reaching the stream, do not feel the effect of the low temperature.

The Effects of Ice on Stream Flow

The effects of ice on the flow of a stream are, in a general way, of minor importance; though, under certain conditions, these effects may become serious locally. The formation of ice over a stream offers an extra resistance to the flow of the water and the flow resulting from a given gradient is therefore materially reduced. The water surface, therefore, during the ice period is materially higher than in summer with the same quantity of flow. The effect of ice is materially influenced by its thickness and whether the stream surface is partially or entirely covered by the same. Such conditions introduce considerable difficulties in the determination of stream flows that actually take place in a stream during the winter months. While numerous observations have been made on the effects of this ice covering in stream flow, these effects vary so greatly with the nature of the cross section, thickness of ice, etc., that the accurate gaging of streams demands many local measurements and observations for the correct interpretation of the relation between gage heights and stream flow.

The sudden freezing of streams may also cause an abnormal but temporary decrease in flow, even where the sources are not seriously affected. The friction of flow of a river under ice is considerably greater than with open water; hence when ice is suddenly formed on a stream, not only is a considerable quantity of water congealed and temporarily withdrawn from the runoff, but the river must rise and fill a greater cross section in order to overcome the extra friction caused by the ice formation. In one case on the Wisconsin River the flow was decreased almost 75% for two days after the sudden freezing of the river

This same phenomenon was observed by the United States engineers on the St. Croix River in 1882 (see Report Chief Engineer U. S. Army, 1883, P. 1470) as follows:

"In the early winter it was found that each cold wave which increased the thickness of the ice about one-tenth of a foot at a time, was accompanied by a great falling off of the discharge, to be followed by partial recovery during the next few days, the same phenomenon recurring with great regularity at each cold wave, the recovery of discharge being in each case only partial. The gradual tendency was downward until the apparent minimum was reached when there was no appreciable change for several weeks."

The Movement of Ice

When the ice moves out of the stream in the spring it frequently gives rise to serious local conditions. If the ice has begun to melt and has become soft and rotten before a sufficient rise in the river occurs to break it away from the banks and set it adrift, it may pass from a river without injury to the property along the stream. On the other hand, an early spring rise while the ice is still solid may detach it from its banks and cause it to move down the river while it is still solid.

Damage from ice is usually occasioned by the formation of ice jams. These are caused by the lodgement of ice on shallows, against islands or other obstructions in the stream, and a gradual accumulation of large quantities of ice may act as a dam, obstructing the stream flow and sometimes causing overflows and serious damage. On the other hand, the impounding of water behind a jam will produce great pressures against the jam which will ultimately break away, perhaps releasing great volumes of water and producing serious flood conditions in the river below the jam, frequently causing much damage in its course.

TOPOGRAPHICAL EFFECTS

The influence of topography on the runoff of streams is a matter of common observation. Gradient is an important function of flow. This is demonstrated by the slow flow of water on nearly level areas and in its rapid discharge from steeply inclined surfaces. The discharge is also greatly facilitated by uniformity of surface and greatly retarded by irregular and rough surfaces. The quantity of seepage is frequently a function of the length of time during which the drainage waters are in contact with permeable surfaces: consequently seepage is greatly facilitated by low and moderate grades, and greatly decreased, even where the surface is highly pervious, by high gradient over which the water rapidly passes.

TOPOGRAPHY OF WISCONSIN

While Wisconsin can not be called a prairie state, almost one-half of its area is quite level, and most of the balance is rolling country with only low hills and no mountainous or rugged area.

"Hills over 300 feet in height are rare. A few mounds or isolated steep hills, with extremely narrow bases, rise out of the sandy plains of Jackson and Clark counties, and a few larger, more massive hills, one of 1,940 feet above the sea, occur in the valleys of the large rivers, besides the low broad hills which form the crest of the Penokee and copper ranges. These hilly tracts do not cover over 5% of the total area, while about 45% is level upland, and about 50% is rolling country of which a considerable portion is steeply rolling 'kettle' or 'pot-hole' land." (*Forestry Conditions of Northern Wisconsin,* Bulletin Wis. Geol. & Nat. Hist. Survey No. 1, 1898, by Filibert Roth. P. 2.)

"A wide and comparatively flat highland crosses the northern part of the state. This divide varies in elevation from 1,900 feet in the eastern part to 1,000 feet in the western part, and extends to within thirty miles of Lake Superior. From it the rivers descend radially in all directions except eastward. Owing to the fact that Lakes Superior and Michigan bound the State on the north and east, while the Mississippi River forms the southwestern and a large part of the western border, all the rivers must need find a low trough into which to discharge, and that at a short distance from their sources. * * *

"About 9% of the total area considered belongs to the abrupt Lake Superior watershed, and the remainder to the broad southeast, south, and southwest slopes. The divides between the rivers which drain this southern slope are almost imperceptible in some cases, being entirely lost in the labyrinth of lakes and swamps." (*Water Powers of Wisconsin.* Bulletin Wis. Geo. & Nat. Hist. Survey, No. 20, L. S. Smith, P. 9.)

The elevation of the surface of Lake Superior is approximately 602 feet above sea level while that of Lake Michigan is about 581 feet above sea level. The surface of the Mississippi River varies from 683 feet above sea level at low water stage at St. Paul, to 585 feet at low water stage at Dubuque, Iowa, near the southern boundary of Wisconsin.

GEOLOGICAL INFLUENCES

The character and arrangement of the geological deposits have much to do with the quantity of runoff. If the geological deposits of the drainage area are highly impervious, the surface will receive and transmit the water into the mass only through the cracks and fissures in the rock. Gradient under these conditions may only slightly modify the quantity of water entering the strata, which may be insignificant. Good drainage under such conditions may, however, materially increase the total amount of runoff reaching the stream, for the retention of water on impervious surfaces normally means that a large proportion of the precipitation will be lost in evaporation and comparatively little will reach the streams. On pervious strata, the gradient may, under some circumstances, greatly modify the amount of precipitation seeping into the strata during any storm. If the water stands on the surface of a comparatively level and pervious deposit, or flows slowly across it, considerable opportunity is given for seepage into the rock structure, whereas, if the rock surface is highly inclined, a rapid flow of water over the same surface will materially reduce the quantity of seepage. The amount of precipitation that will be taken up by any geological deposit will, of course, depend on its degree of perviousness. Coarse sand and gravels will rapidly imbibe the rainfall into their structure. Fine and loose beds of sand also rapidly receive and transmit the rainfall unless the precipitation be exceedingly heavy, under which condition some of it may flow away on the surface. Many of the highly pervious indurated formations receive this water more slowly and require a considerable length of contact in order to receive the maximum amount.

The depth of the deposit naturally has a material effect on their capacity for imbibing water. The water taken into the pervious deposits on a watershed constitutes what is known as ground waters. They may, however, be divided into two classes:

1. The ground water proper, which flows underneath and at a greater or less distance below the surface, according to the conditions; and

2. The deep seated waters which enter the pervious indurated formations and may follow such formations to distant outlets, perhaps on other watersheds, or into the sea itself.

The ground water proper usually finds an outcrop in the lower valley of the drainage area on which it is received and frequently reappears to augment the lower stream flow. When these deposits are extensive, they materially affect the regularity of flow. They often form the principal source of the stream flow during the summer months when the greater proportion of the rainfall during the season is lost in evaporation or taken up by plant life, and very little of the precipitation sinks into the ground or flows away on the surface to reach the stream itself. These ground waters are the source of springs when some impervious underlying deposit brings the water in its course to the surface; but the bulk of the ground water flow is received into the stream at or below the stream surface as a slow seepage and its presence and value in augmenting flow are not so readily recognized. Where, however, the pervious outcrop dips rapidly from these impervious deposits, it may conduct a considerable quantity of water away from the stream, and it may, or may not, discharge at some lower point in the stream's course.

Geological Conditions in Wisconsin

The indurated geological deposits of Wisconsin range from the early Archean rocks, consisting of granite, quartz and allied rocks, which have a surface outcrop in central northern Wisconsin, to the Hamilton rocks of the Devonian epoch. The granite rocks are the lower or bed rocks of the known deposits and are believed to form the bed of the primeval sea on which the later deposits were formed from the disintegration of the deposits which formed the earliest known land. On this granitic floor were laid down in past geological ages the later deposits,

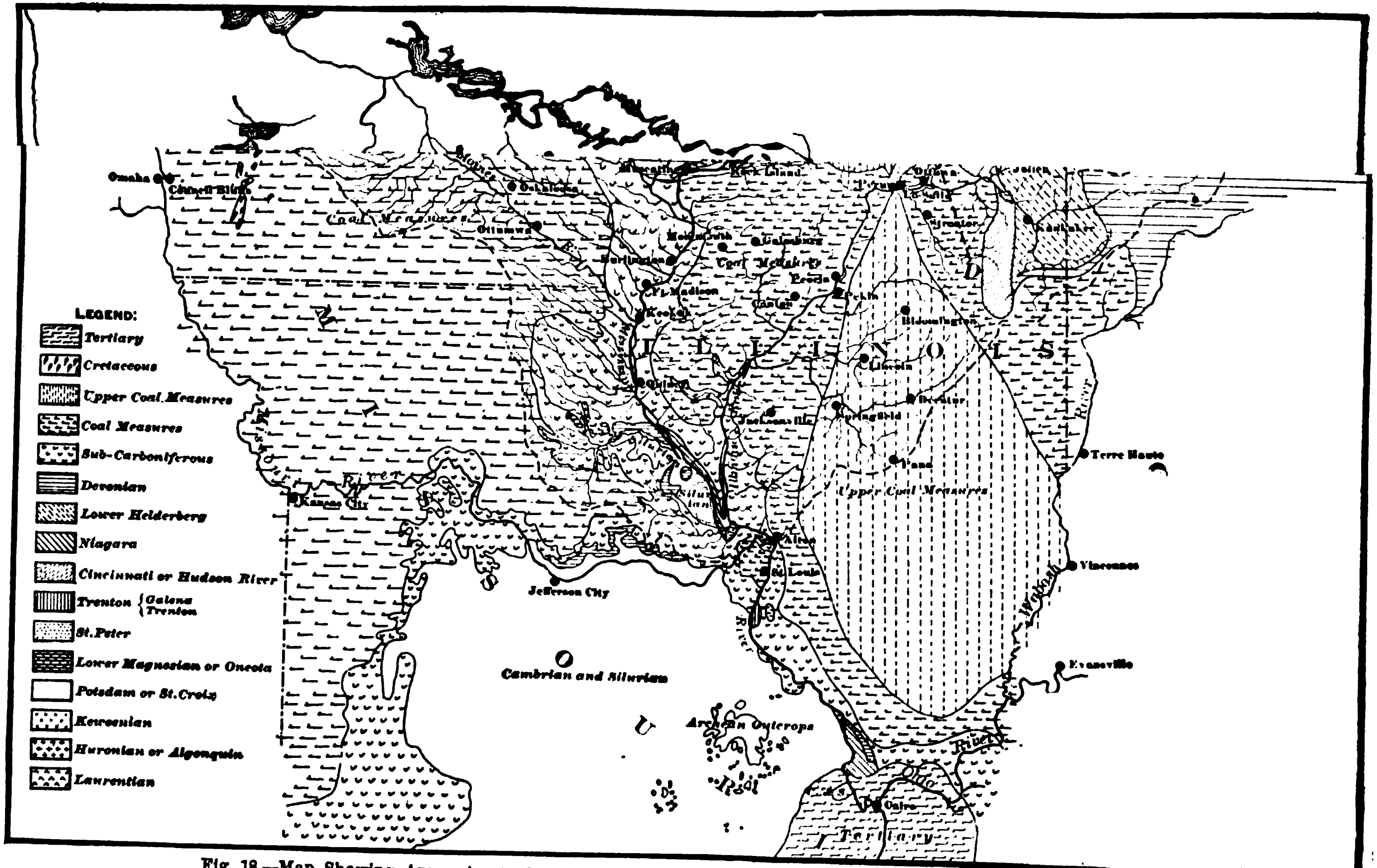


Fig. 18.—Map Showing Approximate Outcrops of the Indurated Formation on the Upper Mississippi Valley.

one upon the other like blankets, varying in thickness with their upper edges exposed at the limits of the gradually extending land surface. The outcrops of these later deposits lie in irregular concentric rings around the granitic nucleus. Since the first land appeared above the sea to the present time, the elements have been at work disintegrating the exposed surfaces, dissolving out the soluble salts, and washing them into the creeks and rivers, and thence carrying them to the sea to form extensions of the land and the later strata. Figure 18 is a map showing the approximate extent of the outcrop of each indurated deposit in the upper Mississippi River and its relative position is shown by the sections AB and DC of Figure 19, the positions of which are indicated on the map, Figure 18. These indurated deposits have been subject to moderate disturbances which have produced cracks and fissures, especially in their surface layers, and these have made them all more or less pervious to water, especially at their surface outcrops. The St. Peter and Potsdam sandstones are as a rule highly pervious in character, although the Potsdam strata contain limestones and shales interbedded with the sandstones. In the ages that elapsed during preglacial times after the surface of Wisconsin had emerged from the sea, the elements were continuously at work disintegrating the surface strata, carrying it to the rivers and thence to the sea and carving deep in the indurated deposits a drainage system much more advanced and complete than that of the present time. The amount of material removed in this manner was enormous, but can be appreciated by a study of Section D. C., Fig. 19. In general, the preglacial topography was much different from the present topography. Deep rugged valleys had been carved in the indurated formation several hundred feet below the present river valleys. The hills were steep, rough and pinnacled, the soils were shallow, and the fauna and flora quite different in many ways from the present.

During the glacial age the surface topography of the major portion of Wisconsin was greatly changed by the passage of great glaciers which brought from the northward more or less foreign material and which in their passage ground off the hill tops and possibly deepened some of the valleys, transporting

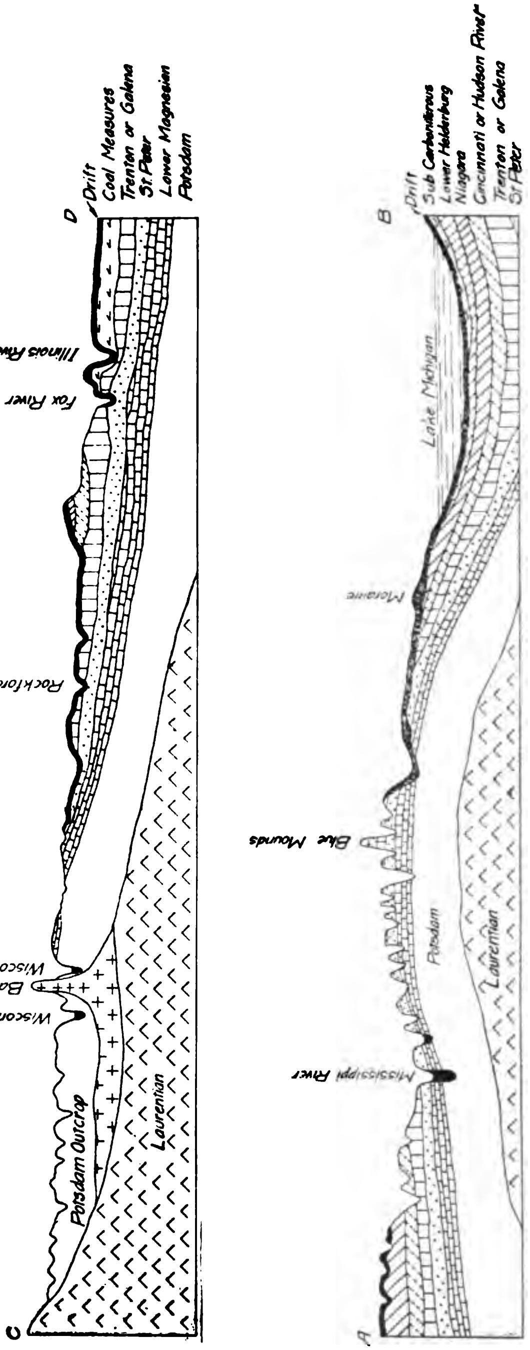


Fig. 19.- Geological Sections of the Upper Mississippi Valley. (See Fig. 18.)

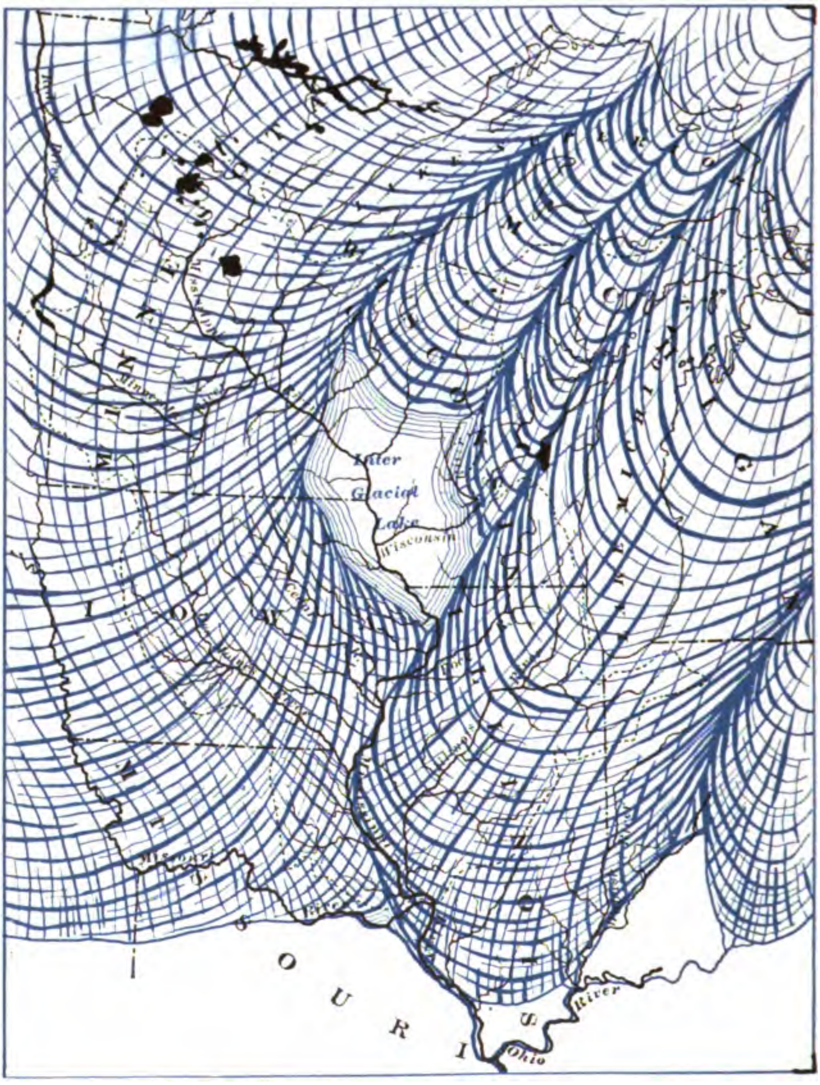


Fig. 20.—Map of First Glacial Epoch in the Upper Mississippi Valley.

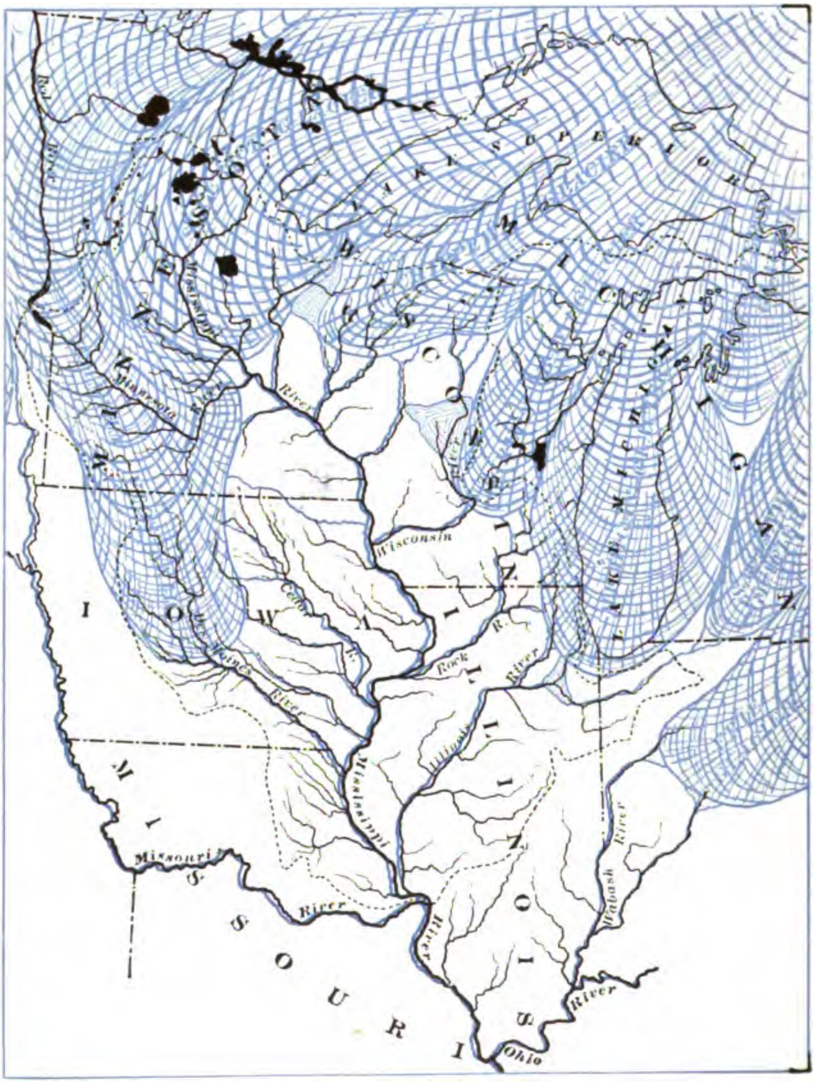


Fig. 21.—Map of the Second Glacial Epoch in the Upper Mississippi Valley.

much of the material to new locations and gradually filling up the original drainage valleys with this material. In this way were covered deep in the indurated strata of the state, drainage channels which are known to have been, in preglacial times, several hundred feet in depth below the hill tops.

In the earliest glacial invasion (see Figure 20), Wisconsin was largely covered with ice except in the southwest portion which was covered by an interglacial lake, hence its hills have been left largely as sculptured by denundating agencies. Its valleys, however, have been filled by materials washed from the glaciers and by deep lacustrine deposits. On the recession of the ice, the resulting floods carved out new channels in the glacial deposits, reworking and redepositing them in varying beds of sands, gravels, and clays. The latest glacial invasion (see Figure 21) has perhaps had a much greater influence on the present surface conditions of Wisconsin; at least it has greatly modified the work of the earlier epoch and has left behind marked traces of its work in the present topographical conditions. The limits of this invasion are clearly marked by the kettle moraine, a line of low rounded hills, formed by the ice, that stretch across the state and include within their limits most of the numerous lakes that occur in the state. From the melting ice of this invasion, the rivers were again more or less altered, the valleys more or less filled, and various terrace systems formed along some of the rivers. The ice melting from its farthest advanced position left an irregular and imperfectly drained condition of the intermoraine country which has given rise to the thousands of lakes and swamps of Wisconsin. The present drainage system of the State is the result of drainage during the glacial recession, more or less modified in the time that has since elapsed. The last invasion was, however, geologically so recent that the present drainage system of the State is new and imperfect and has left most of the lakes and swamps but partially drained.

These various glacial deposits cover in part most of the indurated formation shown in Figure 18, which shows an ideal condition with the drift sheet removed.

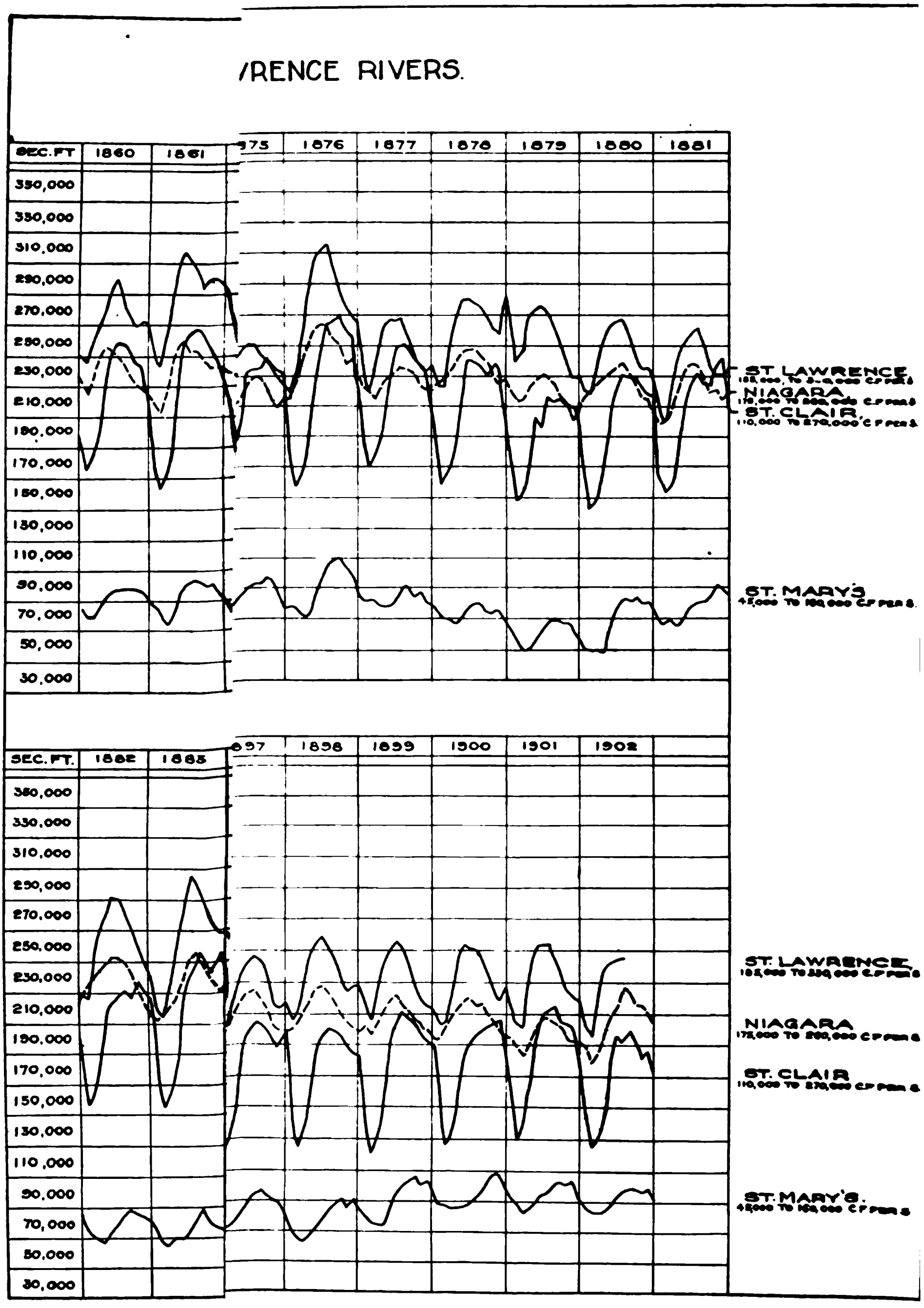
In many places the drift sheet is fairly impervious as evidenced by the lakes and swamps; in other places it is so pervious that depressions considerably below the surrounding levels and with no visible surface drainage are yet perfectly drained through the soil. In many cases the underlying rocks are fairly accessible to the rainfall, although more or less deeply covered by these deposits, on account of this condition.

More or less of the water that falls on the outcrop of the Potsdam sandstone or the more or less pervious drift that overlies it (see Figure 19) sinks beneath the surface, flows slowly in the direction of the dip of the strata and furnishes many of the water supplies of southern Wisconsin and of those portions of Illinois, Iowa, and Minnesota that lie adjacent to the Wisconsin line. With the exception of the waters that are obtained from this source for our water supplies, the ultimate course of the Potsdam waters is unknown. Whether they pass from the underlying Potsdam to other and higher deposits, and through them ultimately reach the sea, or whether they are stopped in their course and simply keep the deposits in a condition of saturation, it is impossible to say.

The Potsdam deposits and the St. Peter sandstone frequently furnish the springs that occur on the line of fracture and upheaval of the overlying deposits, and a quantity of water reaches the surface and ultimately the streams by this means. The quantity reaching the streams from this source is not sufficient to materially augment the flow of important streams.

The St. Peter sandstone is also a highly pervious deposit which will rapidly imbibe water and may deliver it to somewhat distant points as well as to augment the stream flow along the line of its outcropping. Both the St. Peter and the Potsdam strata, when occurring in the bed or banks of a stream, may serve to supply or to deplete the flow according to the conditions of the strata as influenced by their outcroppings, dip, and extent.

Some of the deposits of the Galene limestone are cavernous in character and may receive and transmit more or less of the rainfall. Other limestone rocks are water bearing to a greater or less extent by virtue of more or less extensive cracks and fissures.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50,000
30,000
ST. LAWRENCE.
NIAGARA.
ST. CLAIR.
110,000 TO 270,000 C F PER S.
ST. MARY'S.
45,000 TO 100,000 C.F PER S.
1882
1883
1897
1898
1899
1900
1901
1902
ST. LAWRENCE.
185,000 TO 330,000 C.F PER S
NIAGARA
175,000 TO 260,000 C F PER S
ST. CLAIR
110,000 TO 270,000 C.F PER S
ST. MARY'S.
45,000 TO 100,000 C F PER S

NATURAL STORAGE

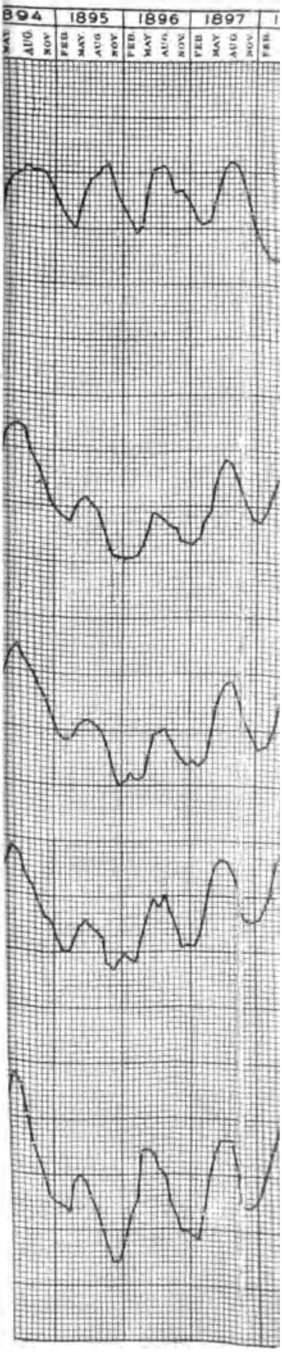
The factors that control runoff are largely interdependent. The natural storage area on any drainage area depends largely on the topography and geology. The presence of ponds and lakes, swamps and marshes, depends on incomplete drainage conditions and the presence of depression with impermeable beds and restricted outlets. Such conditions tend to regulate the flood flows from drainage areas, and to retard the flow of storm waters, delivering them more slowly than would occur from other areas. On the other hand, the evaporation from such swamp and marsh areas, and similarly exposed bodies of water, is considerable. If such bodies of water occupy approximately the entire drainage area, the evaporation from the water surface throughout the year would often be as great as the rainfall itself, and no runoff would result.

The effect of swamps and lakes is to reduce the flood peaks and prolong the high water period. Unless very extensive or unless augmented by extensive pervious deposits, their influence on the low flow of extreme dry periods is not particularly advantageous without artificial regulation.

The Great Lakes of North America and the rivers flowing from them are the greatest example of the effects of natural surface storage on the regulation and flow of streams. The variation in monthly flow of these rivers for the years 1860 to 1902 is shown in Figure 22. Of these rivers the minimum flow is about 62 per cent of the maximum in the St. Mary's River; 60 per cent in the St. Clair River; 60 per cent in the Niagara River, and 67 per cent in the St. Lawrence River. The variation in flow is much less variable and the regulation is therefore much better than any ordinary stream. The ratio of lake surface to drainage area on the Great Lakes is about 25 per cent for the entire system, and the ratio of average annual discharge to mean annual rainfall is about 37 per cent. This is less than the average ratio of discharge to rainfall in the northern streams of Wisconsin, and the reduction is doubtless due to the great evaporation from free water surface. The surface elevations of the Great Lakes are subject to seasonal fluctuations similar but not relatively so great as in the case of rivers. The mean annual gage heights vary closely with the progressive mean of the annual rainfall on their drainage area, and are subject to periodic changes extending through a series of years. (See Figure 23).

Streams on which pervious deposits are small or are entirely wanting are those generally subject to the greatest variation in flow. Under such conditions, the rain falling on the surface moves rapidly toward the thread of the valley and passes away as flood flows. The flows at other times are little or nothing and such streams rapidly dry up and become dry runs during the summer months, or sometimes whenever rain is not actually falling.

On the other hand, extensive pervious deposits on a drainage area generally produce a high degree of regularity in the flow of a stream. On drainage areas where such pervious deposits are extensively developed, the rainfall, especially if the surface be uncovered by vegetation, is rapidly absorbed, becomes a part of the ground water, flows slowly toward the stream, and, dependent upon the character of the deposit, reaches the stream only after a lapse of days, or perhaps months. As the ground water becomes filled, its gradient becomes steeper and the rapidity of its flow is increased. Heavy rainfalls on a pervious drainage area therefore will considerably increase the velocity of the ground water flow and hence augment the flow of the streams to a greater extent during or following such periods. As time passes after the occurrence of rains, the gradient will slowly decrease as the water drains from the pervious deposits into the stream, the velocity and quantity of flow becomes less, and the stream flow gradually decreases. The decrease, however, is not rapid and a sufficient quantity is usually held in storage until further rainfalls augment the stored ground water and again increase the stream flow. Such a stream may frequently flow for several months even without any rainfall reaching the ground water, and the flow is commonly regulated to a greater degree by this means than by any other natural condition on the watershed except large percentage of lake area.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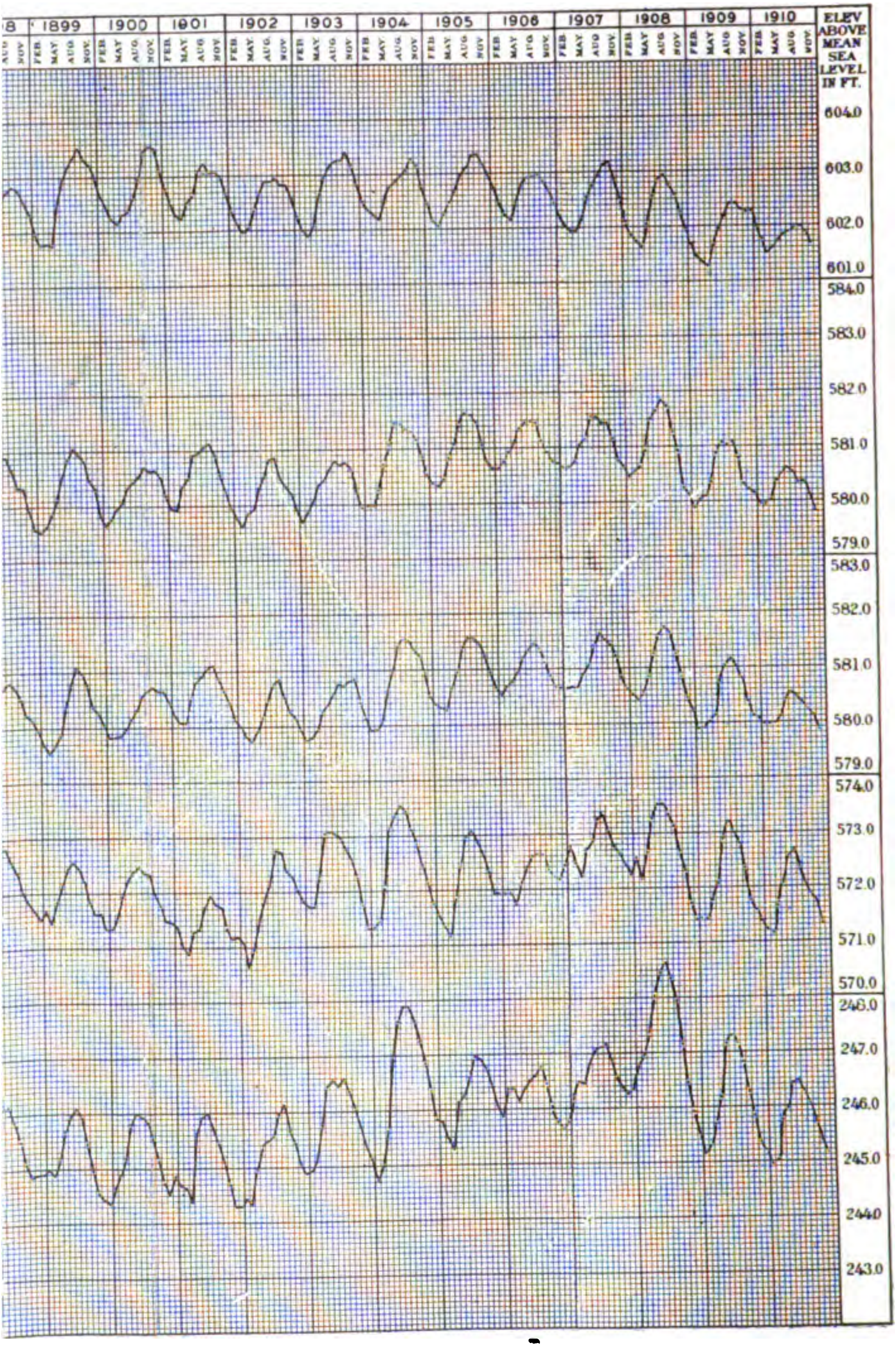
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The flow from such areas is a comparatively large percentage of the rainfall.

Ground storage in gravels and sand, or other pervious deposits, removes, to a considerable extent, all of the water reacning them from the immediate influence of evaporation, and store them under the very best conditions for the future supply of the stream.

The drainage systems of Wisconsin are (see Frontispiece, Fig. 24), geologically speaking, of recent origin. In few places have the rivers reached their base level and the drainage system of the state is new and imperfect; otherwise, the lakes and swamps of the state would not exist. The drainage systems, outside of the ultimate gulleys at the stream head waters, are occupied by perennial streams, fed by the surface and underground reservoirs that equalize and maintain the flow more or less satisfactorily throughout the year.

DRIFT DEPOSITS OF WISCONSIN

The pervious deposits of the drift and mantle rocks present the same characteristics as the deeper indurated deposits in that they materially affect the stream flow and remove much of the water from a surface to a sub-surface flow. The water is diverted from the natural surface flow of the stream itself into underground channels which may discharge at some distant and unknown outcrop.

Such conditions, it is believed, largely affect the flow of the Rock River. The Rock River is known to possess a flow of only about half of that of the Wisconsin River (see Table 2) and of other rivers of northern Wisconsin where the underflow conditions are not largely developed. On the Rock River, on the other hand, underflow conditions are perhaps developed to a maximum (see Figure 25). The river for a considerable portion of its length flows over or parallel to its ancient pre-glacial drainage channel. This ancient drainage channel is known to lie more than three hundred feet below the present stream bed in many places. It is known to be largely filled with sand, gravel, and other pervious deposits, and the quantity of water contained in it is very great. The modern river flows in its upper course to a considerable extent over the general location of the ancient valley as far as Rockford, Illinois, where it flows westerly across a former divide. The ancient channel follows a more southerly direction and leads we know not where. The comparatively small flow of the Rock River is believed to be largely due to these conditions.

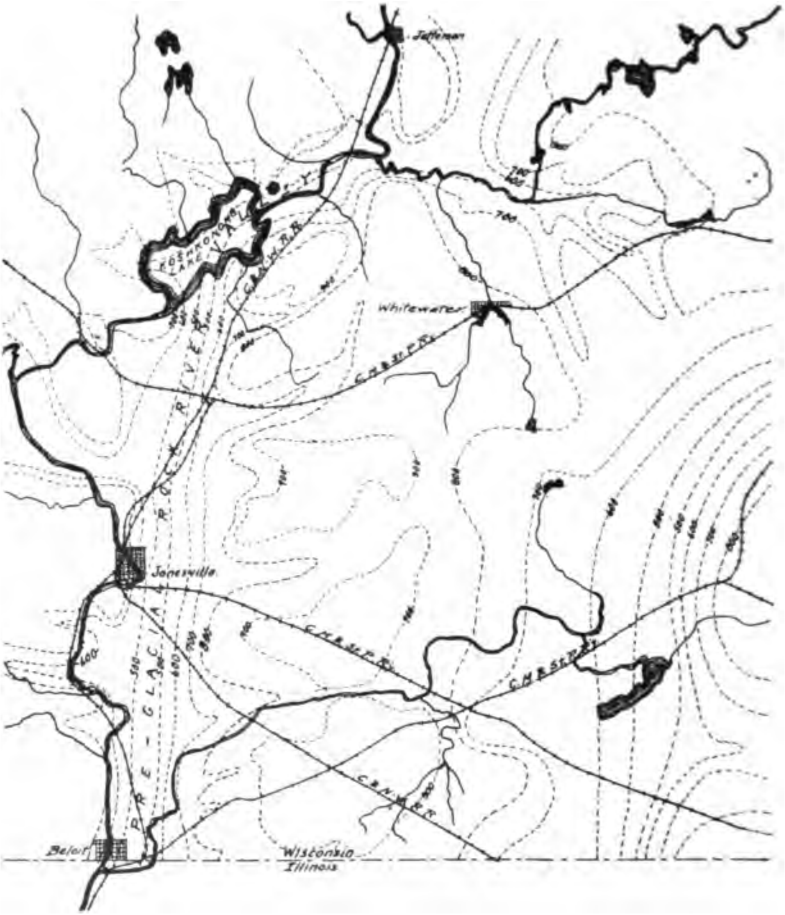


Fig. 25.—Map Showing Pre-Glacial Valley of the Rock River in Southeastern Wisconsin. (From Professional Paper 34 U. S. G. S. by W. C. Alden.)

It is only necessary to point to the Platte River of Nebraska, and to numerous other streams of the same nature, in which, during low water, the surface flow largely or entirely disappears from the stream itself, while the ground flow continues throughout the year in the valley below the bed of the surface stream in which water can always be found from the underground flow by digging deeply enough in the pervious deposits of the stream bed. The importance of geological conditions is very great and can hardly be overestimated, for on no other factor except precipitation does the quantity and character of the flow so largely depend.

SOILS OF WISCONSIN

The general character of the soils of Wisconsin is shown by Figure 26, which is the soil map compiled by Professor A. R. Whitson, chiefly from the geological and soil maps of Professor T. C. Chamberlin, published in 1882, and the soil map of north central Wisconsin, by Dr. Samuel Weideman.

TABLE 2.—COMPARATIVE MEAN MONTHLY RUNOFF OF THE WISCONSIN RIVER AT NECEDAH, WISCONSIN, AND THE ROCK RIVER AT ROCKTON, ILLINOIS, IN CUBIC FEET PER SECOND PER SQUARE MILE.

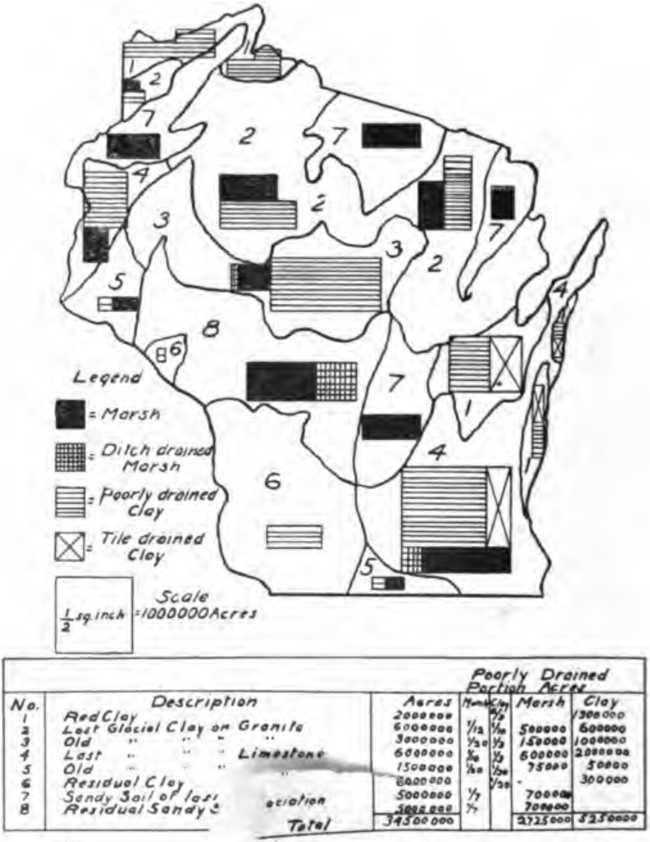
	Jan.	Feb.	Mar	Apr.	May	Jun.	July	Aug	Sept	Oct.	Nov	Dec.
1903.												
Wisconsin River	.45	.44	2 04	1.43	2.50	1.19	1.56	1. 5	2.73	1.83	.86	1.34
Rock River	.....	.....	.....	.. .	.....	.....	,91	.68	.91	.78	.44	.45
1904.												
Wisconsin River	.....	.....	.....	2.21	2.63	1.96	1.02	.66	.90	2.34	.98	.....
Rock River	.45	.77	2.80	1.76	.88	.39	.26	.24	.38	.50	.30	.41
1905.												
Wisconsin River	.....	.....	1.56	2.72	1.91	4.02	1.50	1.05	1.78	.99	.81	1.53
Rock River	.60	.53	2.10	1.63	1.10	1.06	.64	.41	.43	.39	.40	.44
1906.												
Wisconsin River	.....	.....	.....	3.90	1.81	1.86	1.13	.90	.89	.83	1.17	1.41
Rock River	1.56	1.59	1.92	1.49	.58	.37	.38	.....	....	.16	.21	.28
1907.												
Wisconsin River	1.20	1.35	1.52	2.80	1.94	.90	1.09	.51	.76	.57	.42	.48
Rock River	1.02	.....	.49	.85	.37	.55	.66	.37	.45	.43	.20	.19
1908.												
Wisconsin River	.48	.60	.63	1.21	2.20	1.48	1.28	.40	.31	.40	.34	1.01
Rock River	.21	1.06	1 35	.76	1.09	.71	.32	.12	.09	.09	.10	.14

SURFACE CONDITIONS IN WISCONSIN

Swamps and Marshes

There are in Wisconsin about 2,400,000 acres of undrained swamp land. Much of this land can and will be made of agricultural value by proper drainage. Other lands not classified as swamps are materially benefited for agricultural purposes by surface ditches and tile under drainage.

Something of an idea of the relative amount of marsh land in various parts of Wisconsin, and the present drainage conditions, are shown in Figure 27—a map and table prepared by



No.	Description	Acres	Poorly Drained Portion Acres: Marsh	Clay	Marsh	Clay
1	Red Clay	2000000		1/3		1300000
2	Last Glacial Clay on Granite	6000000	1/12	1/10	500000	600000
3	Old " " " "	3000000	1/30	1/3	150000	1000000
4	Last " " " Limestone	6000000	1/10	1/3	600000	2000000
5	Old " " " "	1500000	1/20	1/30	75000	50000
6	Residual Clay	6000000		1/20	-	300000
7	Sandy Soil of last [illegible] eciation	5000000	1/7		700000	
8	Residual Sandy S[illegible]	5000000	1/7		700000	
	Total	34500000			2725000	5250000

Compiled from e[illegible]rimeters made in Bulletin 146 Wis. Exp. Sta.

Fig. 27.—Map Showing Approximate Drainage Conditions in Wisconsin.

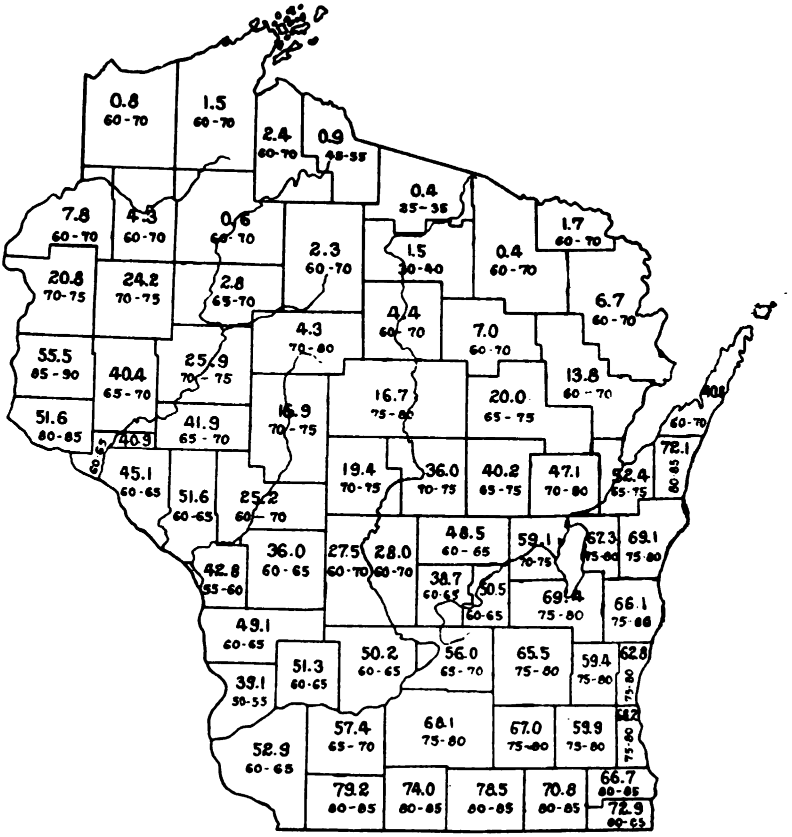
Professor E. R. Jones of the University of Wisconsin, showing in general the soils of Wisconsin and the drainage conditions in 1907. Drainage work affects surface storage only. Open drainage canals and ditches bring the surface flow more directly and quickly into the streams and thus undoubtedly tend to increase the crest of the flood height. Cultivation and sub-drainage, however, have an opposite effect.

THE CULTIVATED AREAS OF WISCONSIN

Figure 28 is an outline map of Wisconsin, prepared by Dr. Samuel Weideman, State Geologist, to accompany the Report of the Conservation Commission of the State of Wisconsin. This map shows the percentage of the total land improved in each county of the state and is based on the State Census Report of 1905. The lower figures in each county show the approximate percentage of land which may be ultimately improved. Cultivation of the ground produces two effects which influence runoff from a drainage area. The breaking up of the smooth surface of a field, especially in a semi-impervious deposit, undoubtedly will greatly facilitate the seepage of water into such deposits and will also decrease evaporation from the ground water. Cultivation therefore is favorable to stream regulation.

FOREST EFFECTS

A bare. smooth surface is favorable to runoff and therefore to flood conditions. The presence of vegetation on a drainage area may materially decrease the rapidity with which water flows therefrom, and such vegetation may or may not be favorable to the uniformity of the flow of a stream. A surface covered with vegetation may, to a considerable extent, delay the removal of rain during the smaller storm. The humus, due to forest growth, may form to some extent a minor storage which will prevent the immediate removal of the water of limited storms. When, however, the rain is extensive, the humus, or covering of vegetation, becomes rapidly saturated, and the water then runs from the surface almost as rapidly as though no vegetation existed. The effect, therefore, of vegetation on heavy storms is to retard the earlier flows from the surface which, however, may be delivered to the streams at a later period. Such conditions may be favorable or unfavorable to higher flood conditions, according to the intensity of the storm and the discharging time and capacity of other tributary streams of the same drainage area.



Upper figures (as 51.3) indicate percentage of total land improved in each county.

Lower figures (as 65-70) indicate estimated percentage of land which will finally be improved in each county.

Fig. 28.—Map Showing Percentage of Lands Improved in the Various Counties of Wisconsin (Report of Wisconsin Conservation Commission).

Of the amount of water actually held or retarded by vegetation it is obvious that the larger portion may be taken up by the vegetation and used in vegetable growth, expired by the leaves, and the retention of the water in this way is bound to increase evaporation, and, while in a minor way this vegetation may be considered as affecting stream flow, it prevents a certain amount of precipitation, which would otherwise flow away, from ever reaching the stream. On an area of a pervious nature and with considerable gradient, the vegetation, holding the water on the slope, may assure a considerable extra amount of seepage which, with satisfactory geological conditions, may be returned to the stream through the ground water and thus assist in the regulation of the stream. Where, on the other hand, the deposits lie in a comparatively low gradient, and are highly pervious, the presence of vegetation on the surface will limit the flow of water into the underlying deposits and prevent the rapid seepage which would otherwise occur. Under such conditions the presence of vegetation on the drainage area will assure a large loss to the stream flow caused by the increase in evaporation and vegetable use.

According to French* measurements, from 10 to 40 per cent of the rain falling on a forest never reaches the ground but is caught by the trees and reevaporated. On the other hand, evaporation is greatly reduced within the forest shade, and the loss either from wet ground or water surfaces thus protected is much less than in the open field. The melting of snow is retarded by the shade of the forest, and the same is true of ice in thickly forested swamps; but the delay in the melting of snow or ice is not sufficient to reserve the supply until needed in the extreme low water periods of August or September.

Outside of its direct influence on the immediate disposal of the rainfall, it must be noted that the roots from vegetable life, penetrating deeply into the ground, take from the soil and the underground reservoirs much water which otherwise would ultimately reach the stream. After a considerable period of dry weather, one has only to make a comparatively shallow excavation in the field and forests to ascertain how greatly the deep roots from the forest trees drain the soil of its water to considerable depths, for the use of forest growth, and consume instead of conserve the supply they are said to store. It is sometimes claimed that the penetration of these roots into the ground is favorable to seepage, but the writer has never observed that there is any appearance of channels produced in this manner.

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*Forest Influences. Bul. No. 7, Forestry Div. U. S. Dept. Agric., P. 131.

THE FOREST BED AS A RESERVOIR

Much stress is laid by foresters on the reservoirs provided by the forest beds which are said to conserve the rainfall and to regulate runoff. Such reservoirs are largely ideal and are ordinarily of small relative importance. The investigator will hunt in vain in the forest bed for the stored water which is claimed to regulate and augment the flow of streams.

There is no mystery about the actual reservoirs which supply the streams of Wisconsin. A visit to the lakes and swamps will show a quantity of water therein contained which is manifest and not to be questioned. An excavation into the drainage will uncover the underground sand and gravel and the water is there in great quantities. Its presence or importance can not be questioned. The surface waters from lakes and ponds, and the ground waters from sands and gravel, are well known resources from which extensive public and private water supplies are often obtained; but the history of water supply engineering fails to furnish one instance where a forested area, as such, has been given any consideration in the selection of a water supply.

General Chittenden after a somewhat detailed study of this question arrived at the following general conclusions, which the writer believes to be substantially correct:

CONCLUSIONS OF GENERAL CHITTENDEN ON FOREST INFLUENCES

(1) *"The bed of humus and debris that develops under forest cover retains precipitation during the summer season, or moderately dry periods at any time of the year, more effectively than do the soil and crops of deforested areas similarly situated. It acts as a reservoir moderating the runoff from showers and

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*_Forests and Reservoirs_, H. M. Chittenden, Am. Soc. C. E., Sept. 1908.

mitigating the severity of freshets, and promotes uniformity of flow at such periods.

(2) "The above action fails altogether in periods of prolonged and heavy precipitation, which alone produce great general floods. At such time the forest bed becomes thoroughly saturated, and water falling upon it flows off as readily as from the bare soil. Moreover, the forest storage, not being under control, flows out in swollen streams, and may, and often does, bring the accumulated waters of a series of storms in one part of the water shed upon those of another which may occur several days later; so that, not only does the forest at such times exert no restraining effect upon floods, but, by virtue of its uncontrolled reservoir action, may actually intensify them.

(3) "In periods of extreme summer heat forests operate to diminish the runoff, because they absorb almost completely and give off in evaporation ordinary showers which, in the open country produce a considerable temporary increase in the streams; and, therefore, while small springs and rivulets may dry up more than formerly, this is not true of the larger rivers.

(4) "The effect of forests upon the runoff resulting from snow-melting is to concentrate it into brief periods and thereby increase the severity of freshets. This results (a) from the prevention of the formation of drifts, and (b) from the prevention of snow-melting by sun action in the spring, and the retention of the snow blanket until the arrival of hot weather.

(5) "Soil erosion does not result from forest cutting in itself, but from cultivation. using that term in a broad sense. The natural growth which always follows the destruction of a forest is fully as effective in preventing erosion, and even in retaining runoff, as the natural forest.

(6) "As a general proposition, climate, and particularly precipitation, have not been appreciably modified by the progress of settlement and the consequent clearing of land, and there is no sufficient reason, theoretically, why such a result should ensue.

(7) "The percentage of annual runoff to rainfall has been slightly increased by deforestation and cultivation."

ORIGINAL FOREST CONDITIONS IN WISCONSIN

Roth, writing in 1907, of the original forest conditions in Wisconsin, says:*

"An uninterrupted forest, extending from Michigan through Wisconsin into Minnesota originally covered almost the entire surface of these 27 (northern) counties. Along the southern and southwestern border these forests faded into oak and jack pine openings, and in places gave way to regular prairies. It was generally a mixed forest of white pine and hardwoods on all loam and clay lands; it approached the regular pinery on the tracts of sandy loam and the red clays of Lake Superior, and in all sandy and loamy sand districts, it was invariably pinery proper, generally a mixture of white and red (Norway) pines. This great forest changed in character along a line extending approximately through Range 7 W. from Lake Superior to Township 31 N., from here to the southwest corner of Marathon county, and thence east to Green Bay. To the east and north of this line, the hemlock joined the hardwoods and pine on all gravel, clay, and loamy lands; the birch disputed precedent among hardwoods, so that we may designate the forests as birch forests with admixture; the red oaks were thinly scattered, and the white oaks practically wanting. To the south and west of this line, the hemlock generally did not grow at all; the birch became scattering, white oaks were abundant, and the oaks gave character to the hardwood mixture, making the bodies of pure hardwood distinctly oak forests.

"Along the edge of the forests, to the south and west, the dense cover of a variety of tall hardwoods and conifers gave way, rather suddenly, to monotonous brush woods, composed of scattered bushy oaks, either alone or mixed with jack pine."

* * *

FOREST CONDITIONS IN 1897

"At present these forests are materially changed. More than 1,000,000 acres have been cleared and put in cultivation. During forty years of lumbering, nearly the entire territory has been logged over. The pine has disappeared from most of the mixed forests, and a greater proportion of pineries proper have been cut. There is today hardly a township where no logging has been done. In addition to this, the fires following all logging operations or started in new clearings of the settlers, have done much to change these woods. Nearly half this territory has been burned over at least once. About 3,000,000 acres are without any forest covering whatever, and several million acres more are but partially covered by dead and dying remnants of the former forests. In the better hardwood areas (Taylor. Marathon, and Langlade counties) the least change has occurred; the former existence of the pine is scarcely noticed, and the forest is damaged by fire only where it borders on pine slashings or spots where quite a body of pine occurred and has been removed. * * * Where the pine formed a heavy admixture, the remaining hemlocks and hardwoods are badly damaged. and often entirely killed, over entire tracts (parts of Price, Chippewa, Sawyer, and Washburn counties). In most of the pine areas proper, the repeated fires have largely depleted the land of all the heavy debris in slashings. (Oneida. Marinette, Washburn counties, near Lake Superior at Ashland and Bayfield, and in Douglas county.) Here are large tracts of bare wastes, stump prairie, where the ground is sparsely covered with weeds and grass, sweet fern, and a few scattering, runty bushes of scrub oak, aspen, and white birch. These alternate with thickets of small pine which, in spite of repeated fires, have escaped destruction or have re-established themselves.''

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*See Bul. 1. Wis. Geo. & Nat. Hist. Survey.

The above is the forest condition as described by Roth in 1898, and is further illustrated by Figure 29. Since 1898 a great deal of timber has been cut. but no information is available at this time, from which the present approximate forestry conditions in the state can be mapped. Such conditions are, of course, changing from year to year.

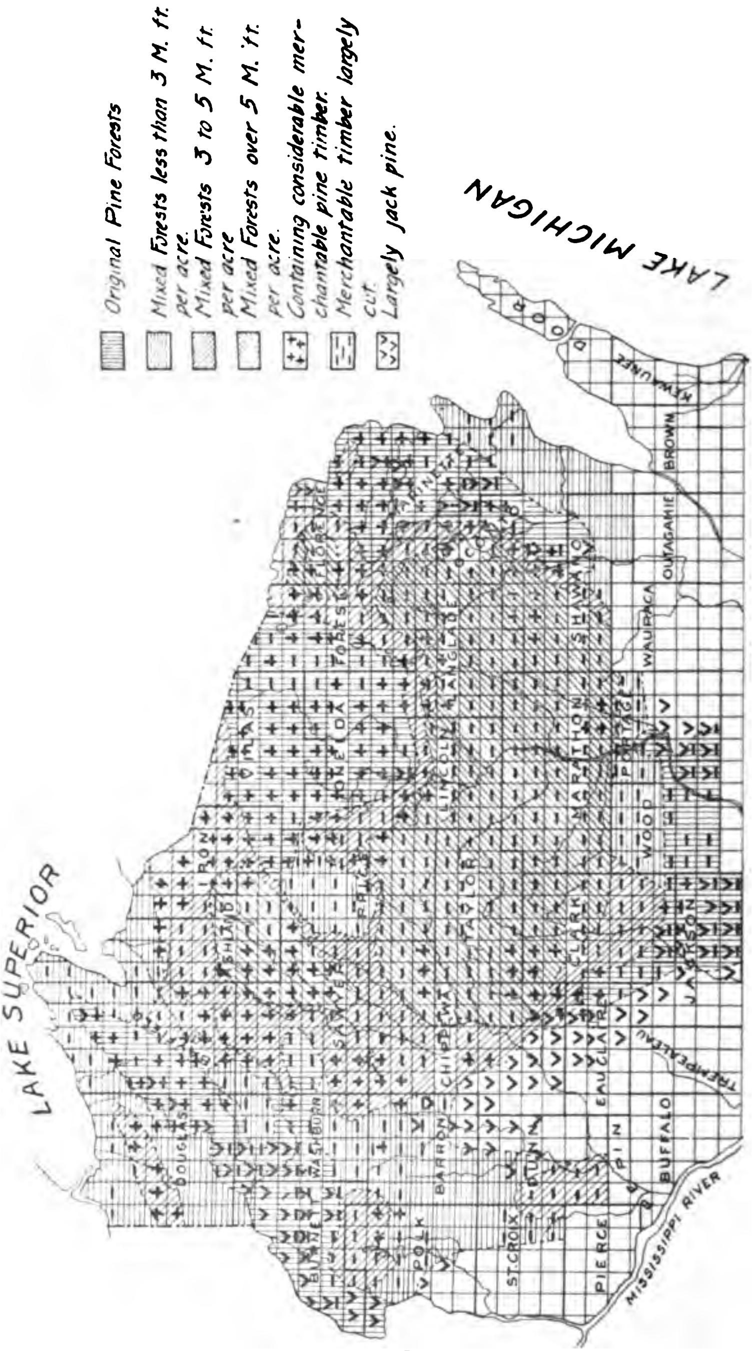


Fig. 29.—Map Showing Forest Conditions in Wisconsin in 1898.

FORESTRY IN WISCONSIN

In 1895 a State Forestry Department was created and a State Forester appointed. The following extract from the report of the State Forester for 1907 and 1908 describes the forestry policy of the state and the claims made for the benefit of reforestation on the improvement of stream flow.

"The question is often asked as to just what is the forestry policy of Wisconsin and for what we are working. In reply it can easily be shown that for certain reasons which will be explained, the main work which Wisconsin should do in forestry is clearly defined.

"In many states the demand for forestry comes almost entirely from the rapid destruction of the forests and the feeling that the state must take a hand in conserving the timberlands. This, of course, in itself is a most important reason for establishing state forest reserves, but in Wisconsin in addition to the question of providing a future supply of timber, there is the even more important necessity of protecting the headwaters of the important rivers, which means also the conservation of the water powers and navigation as well. In order to make clear the connection between forests and streams, the following is taken from the report of this office for 1906:

"Many people do not understand in just what way forests preserve and regulate stream flow, and they imagine that it is brought about largely through the influence of forests in causing an increasing rainfall. In the arid plains, where for months the soil is parched dry, the cooling influence of great bodies of forest have unquestionably a slight effect in causing rainfall. But in Wisconsin, surrounded as we are on the east and north by great lakes, the influence of the forest in causing or increasing rainfall is probably so very slight that it need not here be taken into consideration. However, everywhere and under all conditions, the forest acts as a great sponge in holding moisture and giving it off gradually, thus saving forested countries from the disastrous effects of floods at one season and low water at another. Anyone who will go into the dense forest in early spring will find the snow melting slowly when it has long since disappeared in the open country. The melting snow and the falling rain in the forest work down slowly through the humus and soil and are carried by the roots to some depth, often to underground channels, to appear weeks or months later in the form of springs or in small streams. Thus, streams which rise in forested regions have a fairly even flow throughout the year, while others, rising in regions where the timber has been destroyed, have violent floods in the spring while the snows are melting, or during a rainy season, and shrink to a mere trickle of water during hot, dry months.

"This most important function of the forest in regulating stream flow is so well known in foreign countries, where the forests have been intelligently managed for hundreds of years, that in many sections the owner of forests on an important watershed is prohibited by law from clean cutting it. He is allowed to cut conservatively under the advice and management of the forestry department, but it is wisely held that no person has the right to destroy forests on an important watershed, thus causing floods and jeopardizing the property of so many others.

"In this country the most careful records have been taken for ten years of stream flow in forested and non-forested watersheds in the San Bernardino mountains of California, and the results are most interesting, as will be seen from the following tables:

Annual Rainfall and Runoff on Forested and Non-Forested Catchment Areas in the San Bernadino Mountains, California.

Area of catchment basin. Square mile.	Condition as to cover.	Precipitation.	Run-off per square mile.	Run-off in % of precipitation.
		Inches.	Acre Feet	
0.70............	Forested..................	46	731	28
1.05............	Forested..........	46	756	30
1.47........ ...	Forested...................	46	904	36
.53	Non-forested	33	1,192	69

"It will be noted from the above that the runoff in the non-forested basin was 69%, while in the forested basins it was only from 28 to 36%, a tremendous gain for the streams protected by forests. But still more instructive is the following table:

Rapidity of Decrease in Runoff After the Close of the Rainy Season.

Area of Catchm't Basin	Condition as to cover.	Precipitation.	April Run-off per sq. mile.	May run-off per sq. mile.	June run-off per sq. mile.
Sq. Mi.		Inches.	Acre feet	Acre feet	Acre feet
0.70	Forested	1.6	153—	66—	25—
1.05	Forested	1.6	146—	70—	30—
1.47	Forested	1.6	166—	74—	30—
.53	Non-forested	1.	56—	2—	0

"This table shows the wonderful influence that forests have in regulating and sustaining stream flow. The second month after the rain had ceased to fall, the forested areas were giving off from 66 to 74 acre feet and the non-forested only 2. In June, three months after the rain had ceased to fall, the forested areas were still showing from 25 to 30 acre feet, while the flow from the non-forested area had entirely ceased.

"The water powers of our rivers are one of the most valuable assets of the state but, as can readily be seen, unless the forests at the headwaters of these rivers are protected, the melting snows in the spring will cause freshets for a few weeks and then during the summer months the water will be so low that mills using water power must either shut down entirely or else run only a portion of the time. The development of the water powers of Wisconsin is in its infancy and as the lumber industry dies out and we look more and more to manufacturing, all water powers, large and small, will become increasingly valuable and the necessity of protecting them more apparent." (Report of the State Forester of Wisconsin for 1907 and 1908.)

DISCUSSION OF STATE FORESTER'S STATEMENTS

The principal reason assigned in this report for the necessity of reforestation is the protection of the head waters of streams. If forests are really to be considered of such great advantage to stream flow, it is unfortunate that the relations assumed to exist between forests and stream flow have not been discussed at greater length and some evidence produced which would tend to show some possible relation between them. If forestry is of such great value to stream regulation in Wisconsin, it would seem that proof of these facts might be found in the state itself and without seeking evidence under conditions so obviously different from Wisconsin conditions as they must be on the Pacific Coast.

In view of what has already been presented in these pages, it must be clear to any reader that the simple comparison of the flow of two streams, one forested and one deforested, will be of no value as proof of forest influences unless it is clearly shown that in all other physical conditions the drainage areas are similar. The rainfall on the forested areas described by the state forester is almost 50 per cent greater than on the unforested areas; so in this respect the areas are not comparable, and no other physical data except area and flow are given.

If these tables are of any value whatever in the discussion of this subject, they show that the forests on the San Bernardino mountains have greatly decreased the annual flow of the streams, but have stored and distributed the remaining water to greater advantage throughout the year.

It is worthy of note that the annual runoff of the deforested stream is much greater than of the forested, a "tremendous gain," according to the state forester, which a hydraulic engineer who is interested in the conservation of water may not be able to recognize, for it is evident that if the forester's deductions from these tables are correct, the planting of a forest on a watershed involves a tremendous loss of water to the stream, and that stream regulation can be secured without such loss by a reservoir system. However, it is not believed by the writer that such an assumption is just to the actual results of forestation, and that no such sacrifice of runoff will follow reforestation.

IMPORTANCE OF THE QUESTION OF FOREST RUNOFF RELATIONS

The great value to stream flow which is claimed for forests makes this question of much importance to the citizens of the state. These claims have been stated or assumed so frequently that they have almost become a popular superstition; and as so much seems dependent on them, it seems pertinent to inquire "Are such claims warranted?" "Have they any basis in fact?" If so, we can surely find in the records of Wisconsin streams all the proof necessary to substantiate or disprove this important relation.

PRESENT FOREST CONDITIONS IN WISCONSIN.*

(See also Appendix D.)

While logging, in this state, began perhaps fifty years ago, satisfactory data are not available showing the progress of the lumbering industry. Roth gives the following figures for the cut of lumber in Wisconsin during the twenty-five years ending in 1897, and the data for 1900 to 1908 are taken from the U. S. Census Bulletin:

	Million feet B. M.		Million feet B. M.
1873	1240	1888	3210
1874	1200	1889	3270
1875	1250	1890	3660
1876	1340	1891	3010
1877	1000	1892	4010
1878	980	1893	3490
1879	1470	1894	3100
1880	1920	1895	2800
1881	2190	1896	2080
1882	2580	1897	2430
1883	2750	1900	3389
1884	2950	1905	2543
1885	2710	1906	2331
1886	2680	1907	2003
1887	2890	1908	1613

From these figures it is evident that the heaviest cuttings of timber took place from 1880 to perhaps 1900; and while logging is still in progress, the best timber is cut and the industry is on the decline. It is evident that the effects of deforestation on the flow of streams should therefore be manifest by any records which cover the flow of streams in the northern part of the state from 1890 to date, and that record going back to 1880 or to 1870 would cover a still greater period of deforestation and should show, to a still greater extent, the effects, if any, of the removal of forests on the flow of the streams of the state.

DRAINAGE AREA

The shape and size of the drainage area has much to do with the regularity of stream flow. The greater the area the more apt are the conditions thereon to vary on various portions of the same. Almost, or quite all, of the various modifications which are discussed in this bulletin may exist on various portions of the valley of a great river; and these local conditions will greatly modify and radically affect the character of the runoff on the immediate streams. Most storms for example are more or less limited in extent, and, while they may occur uniformly over a small area, they will differ widely over a broad area (see Figure 9). The natural result is that the smaller drainage areas will usually possess wider variations from minimum to maximum flow, while the greater drainage areas are subject to less relative variation or at least are not subject to such extreme variations.

The arrangement of the tributary streams may also largely affect this condition on the main river. If the branch tributaries leave the main stream at many different points along its course, even a general storm will assure the discharge of the various tributaries reaching the main stream at different times, and will thus modify the flood conditions on the same. Where, however, the tributaries enter the main stream at or near the same point, and the system is fan-shaped in character, a general storm may concentrate the flow from the various tributaries which will affect, in this manner, the flow of the main stream and occasion larger floods than would occur under other conditions.

The result of all these conditions however, is not to prevent flood conditions in the larger streams but to materially reduce them when compared with the flood conditions on the various tributaries, except occasionally in the rare case of the fan-shaped arrangement of tributaries. A good illustration of the effects at different points due to the flow of a tributary stream was shown in the emptying of the Tomahawk Reservoir on the Upper Wisconsin River during the summer of 1908. For purposes of repair, this reservoir, which contains a large quantity of water, was opened and its waters wasted. The effect of opening the gates was said to raise the water at Wausau about three feet. Notice was sent along the river that this flood was on its way. Construction was in progress at Kilbourn, on the lower river, and the probable effect at that point was looked for with interest and with some anxiety. The actual effect at Kilbourn, after a flow of perhaps one hundred miles, was of small importance. By the time the flow reached Kilbourn it had spread out and had become so small in proportion to the flow of other tributaries that it raised the water at Kilbourn not to exceed two inches. It is probable that the discharge of this reservoir, while occupying a comparatively brief period at the controlling gates, actually affected the flow for a week or more at Kilbourn. Similar conditions result from the occurrence of local storms on any of the river tributaries. Their local effects may be large but the effect on the flow of the lower or main streams is modified both by the spreading out of the flow and by the reduced flow of other streams. On most large rivers, however, many of the conditions which modify rainfall, and to some extent runoff, are more or less common to the whole valley. Thus, for example, while the gradual approach of the spring season first influences the flow of the more southern rivers, yet its approach is sufficiently rapid to cause spring rises through the Mississippi Valley. Heavy rainfalls are also likely to be quite common throughout the valley during the months of March, April, and May, and usually cause freshets of greater or less magnitude. The same condition is also true to a less extent of the fall rains which produce September rises which commonly occur.

The condition of the area relative to prevailing winds and storms, and the direction of flow of the river may also modify the conditions of runoff. If the storms commonly approach from the direction of the head waters of a stream and progress in the general direction of the flow of the stream, the conditions become favorable for higher floods on the lower river. The effect of winds in the direction of flow is to augment the rapidity of flow. This is, however, a minor and usually insignificant effect.

CHARACTER OF THE STREAM BED

The slope of the bed of the stream may also have much to do with the flood conditions in its lower portion. A high slope of the tributary streams may bring the storm waters rapidly into the main valley, and, on account of the lower gradient in the main stream, a more or less extensive flood condition may result. The section of the stream itself has very much to do with the character of the flow. Narrow gorges with steep sides result in high rises, where a broad valley will permit the overflow and a broadening out of the stream with comparatively little rise. Falls and rapids, instead of high gradients, will result in a less rapid flow of streams in the immediate reaches, and prevent the more rapid discharge which would occur if the gradient was more uniformly distributed along the entire length of the stream. The direction of the flow of the stream may have much to do with the flood conditions. In the Mississippi Valley, the spring conditions, approaching first from the south, melt the ice from the lower river, and the river rapidly clears from south to north without producing congested flood conditions. The Ohio River, on the other hand, running in a westerly direction, is affected by the approach of spring more uniformly throughout its drainage area, and the result of the spring floods on this river is therefore more intense. On the Red River of the north, which flows from Minnesota north into Hudson Bay, the ice on the headwaters is more apt to break up and start out earlier than the ice on its lower courses. Ice jams and floods are apt to result from these effects which are seldom found on the rivers flowing from the north to the south.

ARTIFICIAL CONTROL OF STREAMS

The normal effects of dams and similar structures on the flow of a stream are much the same as natural falls and rapids. The gradient of the river is reduced, the velocity decreased, and the effect of minor variations is materially modified. A series of water power dams in normal use may, from the contingencies of such use, exert a more or less radical effect on normal flow. The effects of the use or disuse of the water for power purposes on the flow of a river is well illustrated by any hydrographs of the lower Fox River (see Figure 30) which show plainly the effects of the shut down of such plants Saturday night, even during flood conditions.

Reservoirs

Artificial storage reservoirs, by means of which the flood waters can be retained by controlling work and discharged during periods of minimum flow, have a much greater effect on the control of a stream than can possibly obtain by the same area of natural lakes, ponds, swamps, and marshes, the flow from which is not artificially controlled. The effect of extensive reservoir systems may materially modify and control low water flow of a river and when extensive, may be made the principal factors in river control. Such reservoirs where they occupy but a small proportion of the drainage area supplying them, are not seriously influenced in their effects by evaporation. Evaporation may be materially increased by the construction of reservoirs but such evaporation occurs from that portion of the water that would otherwise pass away in flood flows so that the net result from the reservoir is to materially modify and increase the low water flow of the stream.

Many writers have described the great results that might be obtained by the storage of water in the improvement of navigation, the increase in water powers, the prevention of floods, etc. As a rule, the results as described by these writers are ideal only and cannot be realized. The practicable limits of storage are not so extensive as are usually imagined. Figure 31 shows

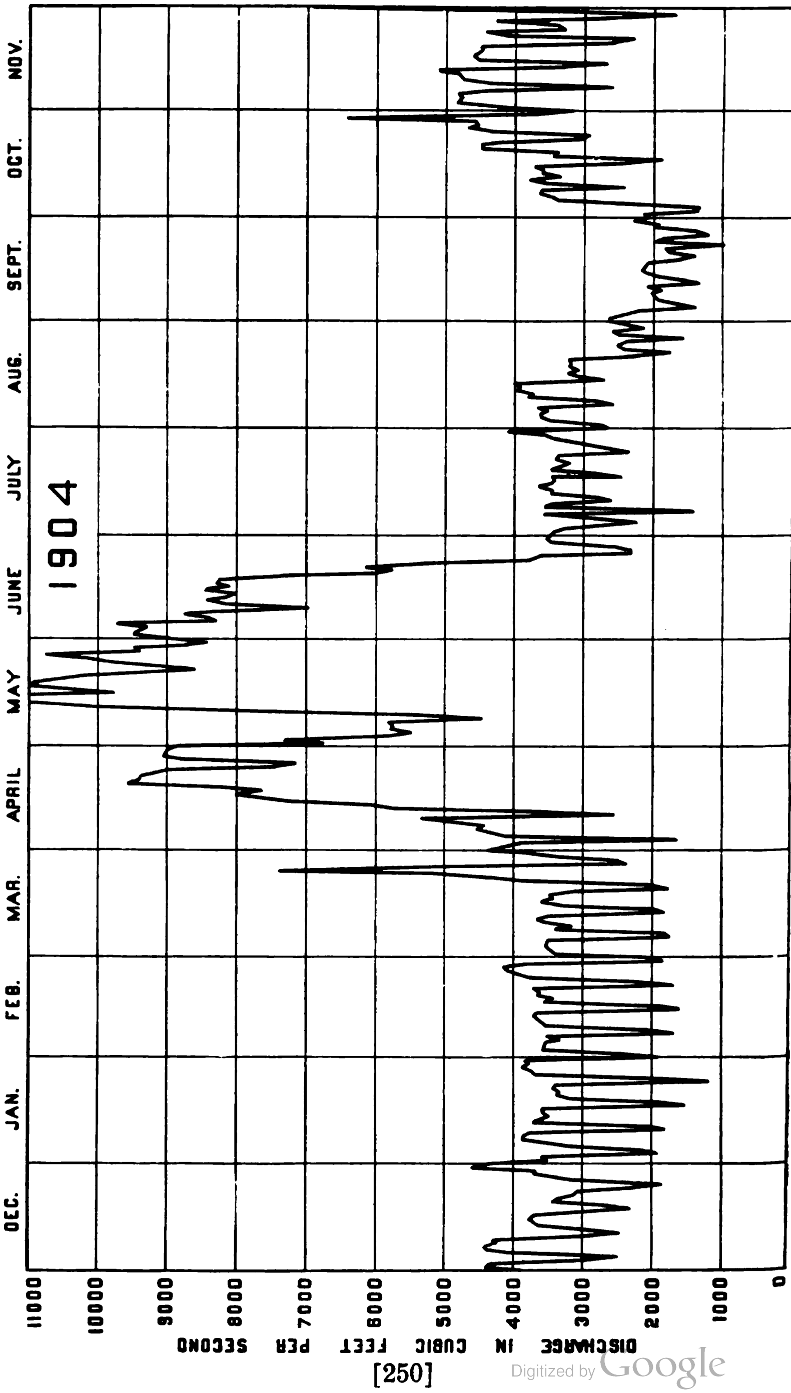


Fig. 30.—Hydrograph of the Lower Fox River Showing Influence of Sunday Closure of Plants on Stream Flow.

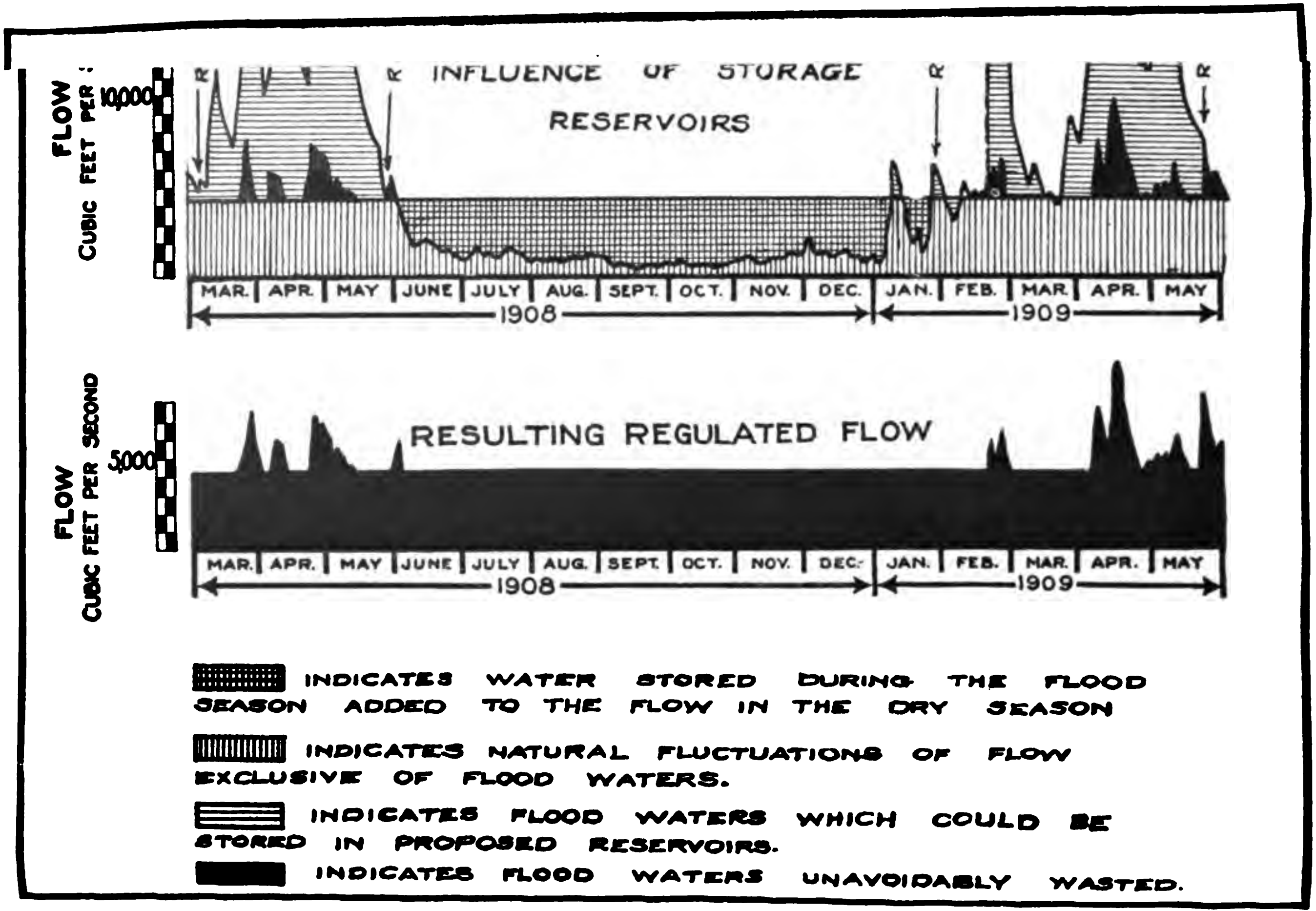
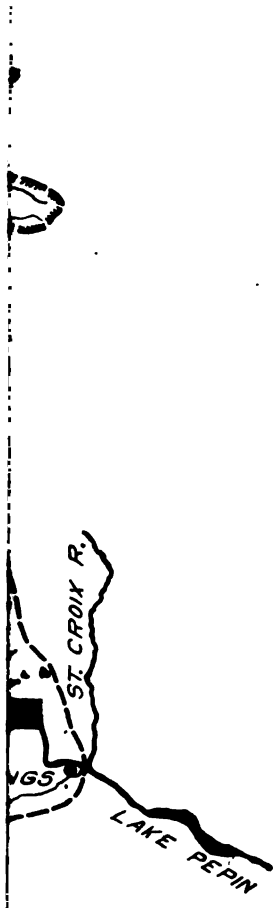

INFLUENCE OF STORAGE RESERVOIRS
FLOW CUBIC FEET PER
10,000
MAR. APR. MAY JUNE JULY AUG. SEPT. OCT. NOV. DEC. JAN. FEB. MAR. APR. MAY
1908
1909
RESULTING REGULATED FLOW
FLOW CUBIC FEET PER SECOND
5,000
MAR. APR. MAY JUNE JULY AUG. SEPT. OCT. NOV. DEC. JAN. FEB. MAR. APR. MAY
1908
1909
▦ INDICATES WATER STORED DURING THE FLOOD SEASON ADDED TO THE FLOW IN THE DRY SEASON
▥ INDICATES NATURAL FLUCTUATIONS OF FLOW EXCLUSIVE OF FLOOD WATERS.
▤ INDICATES FLOOD WATERS WHICH COULD BE STORED IN PROPOSED RESERVOIRS.
■ INDICATES FLOOD WATERS UNAVOIDABLY WASTED.

FIG. 31.



Hastings,

the ideal regulation of the Hudson River, based on a proposed extensive reservoir system and the stream flow for the years 1908–09. From what has previously been shown, it will readily be understood that with a reservoir system designed for such regulation for any given year, the regulated flow secured in other years of greater or less flow will be far different. Such regulation, which in this diagram shows an increase of four or five times the low water flow, can seldom, if ever, be realized on account of the great cost involved.

The reservoir system at the head of the Mississippi River (see areas 1, 2, 3, 4 and 5 in Figure 32) is perhaps the greatest artificial reservoir system in the world. It consists of five reservoirs, having a total capacity of about ninety-three billion cubic feet. It is utilized principally to add to the depth of the water during the navigation season. The effect of this great reservoir system on the flow of the Mississippi River at Hastings can be seen by examining the hydrographs of the river at that place from 1885 to date (see Figures 92 to 99), the reservoir system having been put in use about 1885. In 1905 a board of engineers was appointed to determine the value of this reservoir system. Major Judson, one of the board, in discussing this system before the Western Society of Engineers, says:

"Each interest claimed that it was suffering, as a result of the system. * * * Some of the mills claimed that they got less water occasionally than they had before the system was established; navigation interests contended that they received but little benefit, and did not especially care for the maintenance of the system. Riparians about the reservoirs wished them abandoned, so that they might cultivate land for which they had received flowage damages. Riparians below the reservoirs contended that floods were increased. or at any rate not diminished. None of the different interests ran parallel.

"Suppose reservoirs are operated exclusively to prevent floods. Then it is desirable to keep the sluices open and the reservoirs empty at all times except when floods are threatened. Only at such times would water be accumulated and it would be released as soon as practicable to make room for succeeding floods. If reservoirs be operated for the exclusive benefit of water-power interests, then the gates are kept closed and water is accumulated except during very low stages when water is released to increase the low water discharge. These releases would occur, on northern streams, nearly all winter and during the low water period of the late summer. If reservoirs were operated to promote the navigability of a stream below, then releases would only occur during low water of the open season of navigation.

"Notwithstanding these conflicts of interest, it is indisputable that reservoirs can be manipulated so as to return some measure of good to every interest, except that centering around the reservoirs themselves. The distribution of benefits, however, must always give rise to controversy.

"The good that can be done to the navigation interest, and to flood sufferers as well, is relatively so small that probably in most cases it would be most in accordance with public policy to sacrifice nothing in the operation of the reservoirs to these interests, but to operate the reservoirs exclusively to benefit water-powers. Of course this suggests that in most cases reservoirs should be built by associations of water-power owners."

The only practicable method of regulating stream flow for purposes of water power, irrigation, navigation, and other uses of civilization is by the construction of reservoirs where suitable sites for the same are available. The construction of an extensive reservoir system on a large river is usually a very expensive matter and, as a general rule, it is found inexpedient to carry such construction to a sufficient extent to result in a very marked increase in the low water conditions of flow of a river either for water power or navigation purposes. On small streams reservoir systems that will conserve the waters for water supplies, irrigation, and even for water power, when high heads are available, are often practicable. If the value of these hydraulic resources increases, it may be found that much greater developments of reservoirs on large streams will be desirable in the future than have been finnacially practicable in the past. A careful study of this general subject, therefore, is of great interest and value to all interested in hydraulic resources. It is essential in studying and investigating any subject of this class, where the modifying conditions are complex and differ widely in importance as the conditions differ, to bear in mind that general conclusions are exceedingly dangerous; that conclusions must always be modified by the local conditions; and that the conditions which are known to exist on one drainage area may give results greatly at variance with entirely different conditions which may exist on some other drainage area. Broad conclusions are often misleading and should seldom be made. Specific conclusions, based on known conditions and applying to restricted areas, can, however, be safely drawn.

DIKES AND LEVEES

The restriction of the channel by the construction of dikes and levees to prevent the overflow of low land may result in a material elevation of the flood plain, and must be considered where such constructions are extensive. The obstruction of streams by piers and abutments, and the encroachments of cities and buildings on the waterway, will have the same effect and may result in serious overflow conditions due to the elevation of the flood plains by reason of such obstruction. Many of the serious effects of the flood of Kansas City in 1893 were due, partially at least, to the obstruction of the Kaw River by bridge piers and the encroachment caused by building into the stream, thus taking up the river channel which was needed for the passage of extreme floods.

DIVERSIONS

It is only necessary to point out briefly the radical effect of the diversion of water for irrigation, water supply, and the feeding of navigation canals. On the lower Fox River the use of water for power purposes is occasionally withdrawn for weeks at a time as all the water available is needed for the uses of the canals. Such effects are greater or less in accordance with the proportion of water removed from the stream, and the amount returned thereto by the seepage from irrigated lands, from irrigation ditches, and the discharge of waste weirs and overflow water from the canals.

The drainage of swamps and marshes, and of agricultural lands, also modifies to a considerable extent the natural flow of a stream. Every means provided for the rapid removal of storm water from the land which formerly retained it will have a greater or less effect on the resulting flood heights and consequently on the regimen of the stream.

CONDITIONS FAVORABLE TO MAXIMUM RUNOFF AND TORRENTIAL FLOW

In general, it may be concluded that the conditions most favorable to maximum runoff and irregular flow are as follows:

On a bare rocky drainage area, with scant soil, the rainfall, unhindered by the soil, gravel, vegetation or forests, will flow rapidly into the stream, and except for a small amount of absorption and evaporation, the runoff will equal a large percentage of the rainfall. Under such conditions the stream will be torrential in character. The flow will gradually increase to a maximum just after the rainfall has reached its maximum, and as the rain ceases the flow will slowly decrease until nothing but a dry run is left, shortly after the rain has ceased, and the conditions will so remain until the next rainfall occurs.

CONDITIONS FAVORABLE TO MAXIMUM RUNOFF AND EQUALIZED FLOW

For conditions favorable to maximum runoff and equalized flow, consider this same rocky valley, filled deep with sand and gravel, with the stream meandering through the center of the pervious plain. Here different conditions will obtain. The rain, falling on this area, will sink rapidly into the pervious deposits and move slowly toward the river. Little of the water will be lost in evaporation, because the rainfall will immediately sink below the surface and reach the ground water, and is not subject to evaporation effects. The great subterranean storage will hold back the water but permit it to flow slowly towards the stream which will be fed slowly and with great uniformity; and while the ratio of rainfall to runoff in this stream will be high, it will at the same time be distributed with considerable uniformity throughout the year and the stream will become perennial.

Modifying Effects of Vegetation on a Pervious Drainage Area

If the pervious drainage area just considered, be covered by vegetation, the roots and fibres will obstruct the pervious surface, prevent the free access of water to the subterranean reservoir, utilize almost fully the summer rains for their own development, and produce a surface runoff directly into the stream at times of heavy storm, and the proportion of rainfall that reaches the stream will be materially reduced and the stream flow diminished. On such an area, forests or vegetation of any kind will have a decided effect, and such an effect will be wholly detrimental.

Where Forests and Vegetation May Be of Benefit to Runoff

On the rough, rugged slope of the drainage area first considered, the scant soil will have little effect on the runoff. and the planting of forests or the presence of vegetation may tend to reduce to a limited extent the flood flows and would hence be beneficial in equalizing the runoff. The same beneficial effect would also result from the planting of forests on any impervious drainage area of high gradient.

All things considered, there can be little doubt but that the total amount of water taken up directly by a forested area and lost by evaporation from its foliage is very much greater than from any similar area of other kinds of vegetation. As has already been stated, vegetation of any kind on a flat and pervious surface is a detriment to rapid absorption into the strata; on the other hand, on impervious surfaces and steep slopes, a forest or other forms of vegetation may be of material benefit in aiding, controlling, and regulating runoff during moderate rainfall conditions. Impervious areas. whether flat or steep, may possibly be benefited by forests. Small lakes surrounded by forests undoubtedly lose less water by evaporation than those exposed directly to the unmodified action of the winds.

FLOODS

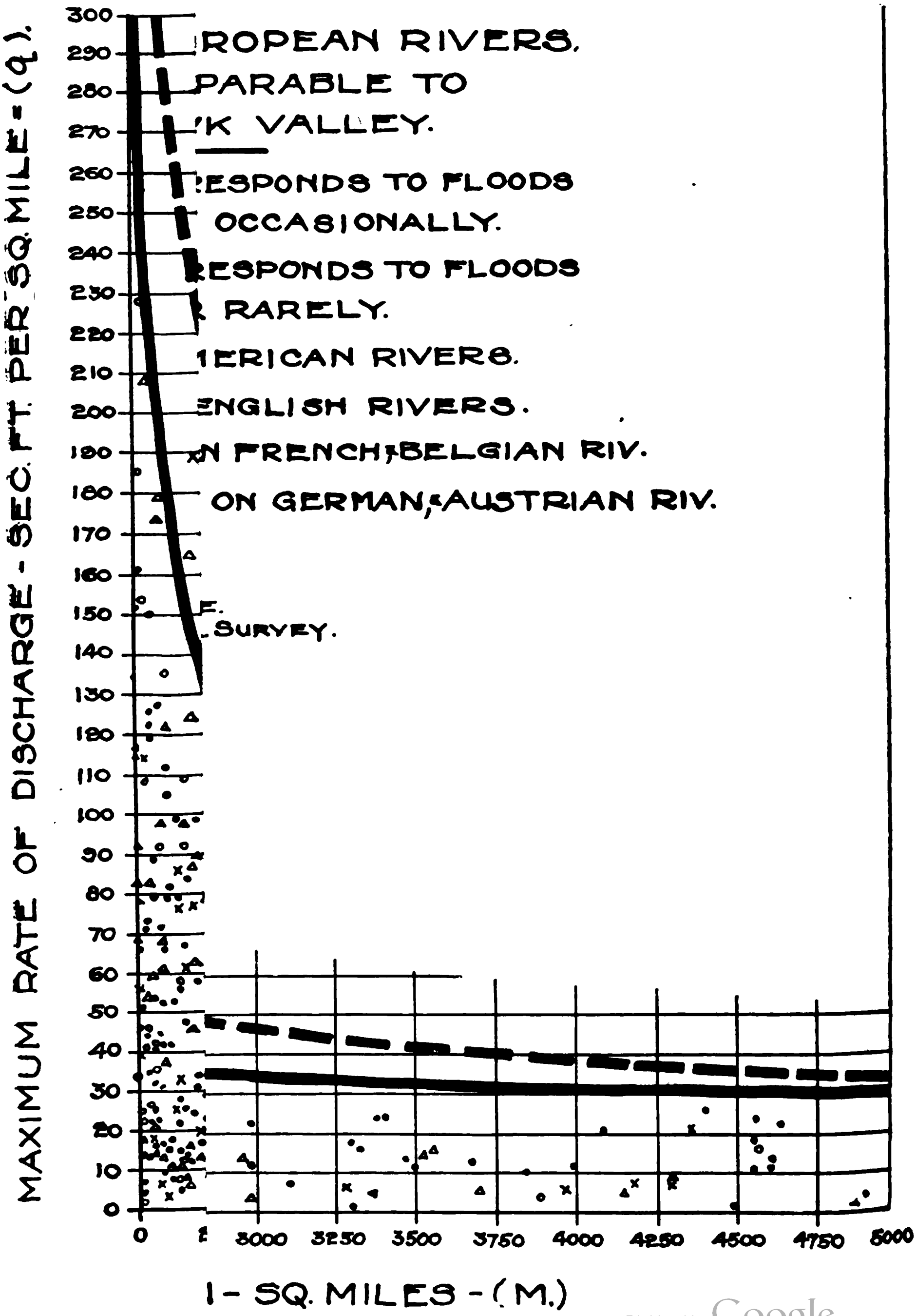
As already indicated, the size of the drainage area has very much to do with the intensity of floods. On account of the averaging effect of extensive drainage areas, the discharge of large streams is much less per unit of area than that of small streams. This fact is well illustrated by the various observations of the rates of maximum flood discharges of certain American and European rivers which cover numerous observations on rivers varying in drainage area from a few square miles to 5,000 square miles. (See Figure 33 prepared by Keuchling.) Taking into consideration all of the rivers of the United States on which observations have been made, it is found that these discharges vary from almost two hundred and sixty cubic feet per second per square mile on small drainage areas, to a discharge of about thirty cubic feet per second per square mile on drainage areas of five thousand miles.

The extreme flood conditions are due to the simultaneous occurrence of various circumstances favorable to maximum runoff. Among these are the conditions which have been previously discussed, including, primarily, long and continued rainfall generally prevalent on the drainage area, impervious condition of the ground due to frozen, saturated soil, heavy snow deposits left from the winter, and similar favorable conditions on all of the tributary streams. General H. M. Chittenden (See *"Forests and Reservoirs,"* Am. Soc. C. E., Sept., 1908) describes the causes of various recent floods as follows:

"The great Kaw River flood of 1903, which wrought such havoc in Kansas City, was caused by a wholly exceptional rainfall over nearly all the watershed of that stream. In the first three weeks of May, 1903, more than the normal amount (4.5) for the entire month fell. This was followed in the next five days by 3.4 in., and upon this was piled 4.7 in. in the succeeding five days, by which time the flood had crested.

"In the flood of 1906 in Western Washington, which did enormus damage and stopped railway traffic for upward of two weeks, the crest of the flood occurred about the 15th of the

CAXIMUM FLOOD DISCHARGE.



month. The month of October had been very wet, and the ground and forest storage was exhausted. In the first half of November, 25% more rain fell than in the normal for the entire month, and of this about one-half came on the 13th, 14th, and 15th.

"In the flood season of 1905 on the watershed of the Upper Mississippi, there fell in the month of April above Pokegama Falls 2.55 in., in May, 4.95 in., in June, 8.03 in., and in July, 6.88 in., a total of 22.41 in. The normal for the entire year is 26.5 in.

"In the record-breaking flood of 1907 in the Sacramento Valley, 88% of the normal for the month of March (based on 21 years' observation) fell in three days (17th-19th), and, on one day, the precipitation ranged from 5 to 8 in. at the different stations.

"In the extraordinary flood of May and June, 1908, in Western Montana, the precipitation for May, at four selected stations, was 6.5 in., and for June, 4.2 in. The greater portion of this fell late in May and early in June. The normal for May is 2.6 in. and for June, 2.3 in."

In the records of a hundred years or more of stream flow one record of maximum, and one record of minimum flow would probably be found that had never been equaled during the period.

In records for limited periods it can never be asserted that conditions resulting in the greatest possible extremes have occurred. It may be reasonably anticipated, however, that a record of thirty years will give within a few per cent all conditions that may be expected to obtain except under the most unusual and exceptional circumstances.

On the Wisconsin River, records are obtainable of the flood heights for thirty-seven years, and at Kilbourn, with a drainage area of some 8,000 square miles, the extreme flood seems to have been approximately ten cubic feet per second per square mile. On the Rock River, at Sterling, Illinois, where the drainage area is approximately equal to that at Kilbourn, the maximum flood flow experienced was about 45,000 second feet, or approximately 5.6 cubic feet per second per square mile. It is quite probable that neither of these is an extreme condition and that more favorable conditions may sometimes give rise to floods of greater magnitude.

In the estimate of flood heights and flood flows, it is important that all facts be very carefully considered, and that sufficiently large estimates of floods be made to guard against the occurrence of loss and damage from this cause. Such estimates should be increased in accordance with the seriousness of the results liable to occur from extraordinary flood conditions. If the loss of life from an underestimate is probable, the conditions become more serious and estimate should be radically increased. If loss of property and resulting damages are liable to occur, such estimates as the conditions seem to warrant should be made.

THE FLOW OF STREAMS

In the previous discussion, the various physical conditions that affect runoff have been considered at some length and the general theory of rainfall and its relations has been briefly discussed and illustrated. Many of the principles that underlie and modify the flow of streams have also been discussed and illustrated.

The proof of any theory is that it shall logically explain all the relations of phenomena to which it applies. The question may properly be raised as to the truth of the principles previously discussed as shown by the actual flow conditions of Wisconsin rivers. In the following page it is proposed to illustrate as completely as practicable the variation in flow or runoff of certain Wisconsin streams, and the relations of that flow to the rainfall and its variations. The actual runoff itself will be considered together with the corresponding rainfall conditions and such other physical conditions as have apparently affected the rainfall-runoff relations. The rainfall-runoff condition of certain Wisconsin streams for the last thirty to forty years will be examined, and any change in the relations of the conditions will be noted. This period represents the period of greatest activity in the settlement of the country, the improvements of lands, the cutting of timber, the drainage of marshes. Surely if deforesta-

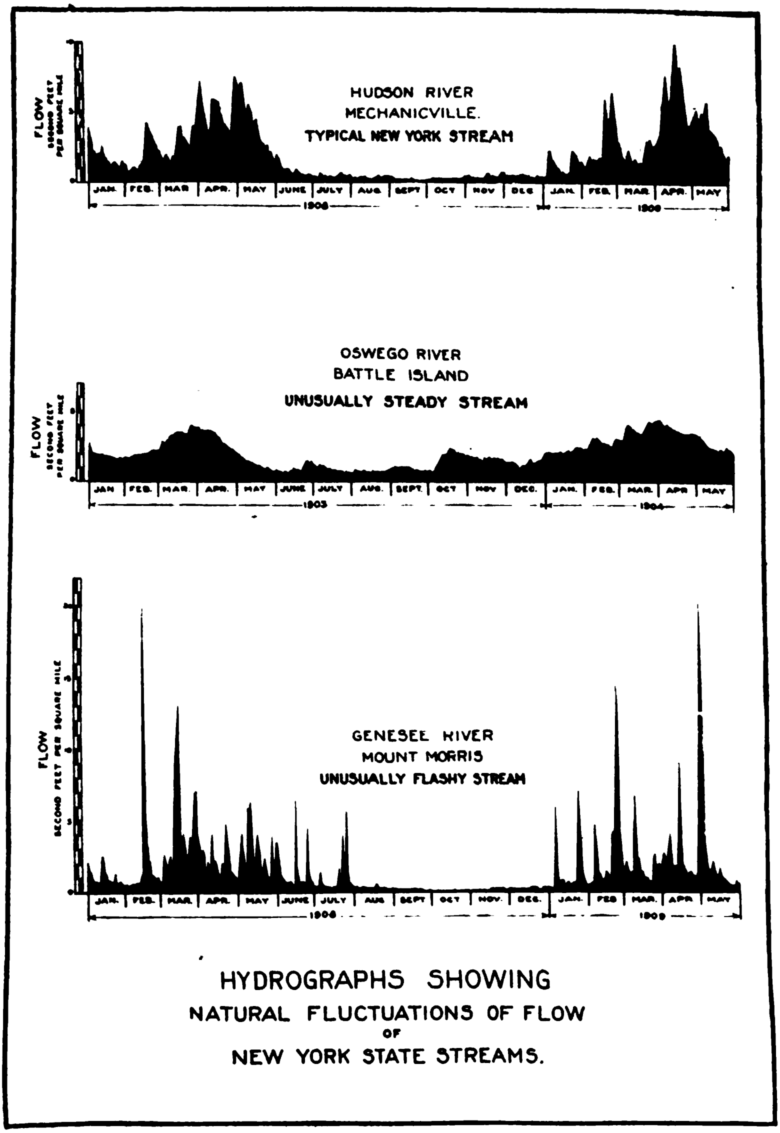

FLOW
SECOND FEET PER SQUARE MILE
HUDSON RIVER
MECHANICVILLE.
TYPICAL NEW YORK STREAM
JAN. FEB. MAR. APR. MAY JUNE JULY AUG. SEPT. OCT. NOV. DEC. JAN. FEB. MAR. APR. MAY
1908
1909
OSWEGO RIVER
BATTLE ISLAND
UNUSUALLY STEADY STREAM
1903
1904
GENESEE RIVER
MOUNT MORRIS
UNUSUALLY FLASHY STREAM
1908
1909
HYDROGRAPHS SHOWING
NATURAL FLUCTUATIONS OF FLOW
OF
NEW YORK STATE STREAMS.

FIG. 34.

tion has the effect claimed for it, such effect must be fully manifest within the period discussed; otherwise we can but conclude that the effects of deforestation and other works of man are, so far as stream flow is concerned, insignificant and unworthy of consideration. Surely the expenditures of valuable energy and large sums of money are only warranted by tangible results.

THE FLOW OF STREAMS FROM DRAINAGE AREAS OF VARIOUS CHARACTERS

The fluctuations in the flow of streams having various characters of drainage areas are well illustrated by Figure 34, which shows hydrographs of various streams in New York State. The hydrograph of the Hudson River closely resembles that of most

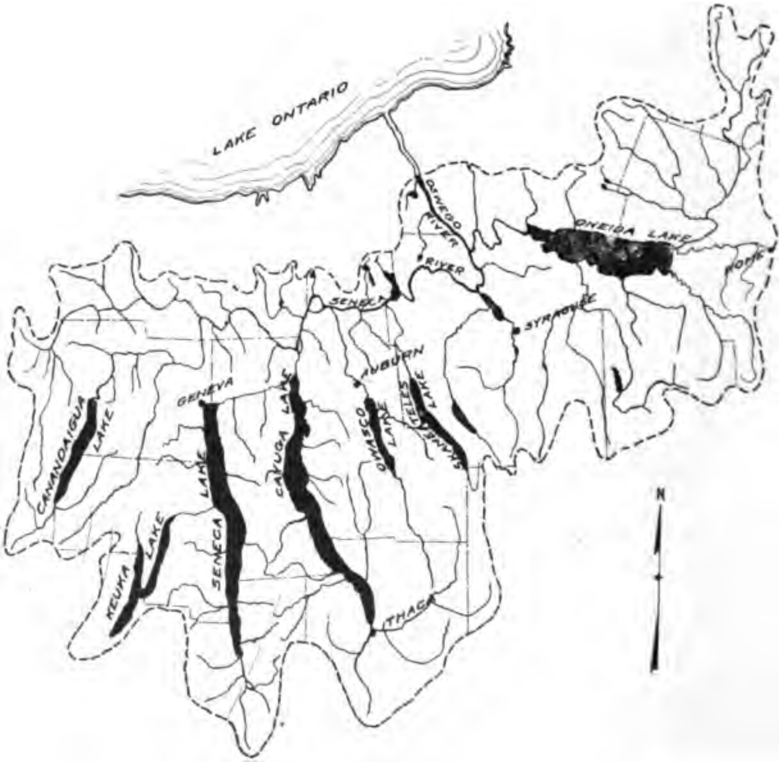


Fig. 35.—Map Showing Drainage Area of the Oswego River, New York, and Natural Storage of Same.

Wisconsin streams. The drainage area of this stream is in the Adirondack Mountains, and its character is similar to that of the streams of Wisconsin. The Oswego River has a large amount of lake storage on its drainage area (See Figure 35), and the flow is unusually steady. The Genesee drainage area (See Figure 36) has little storage and its flow is unusually variable.

General Relations of Precipitation and Runoff

It has been previously stated that other things being equal, the runoff will increase with the rainfall. The relative runoff during the year while of necessity varying with the rainfall is so greatly affected by other factors that on an average the maximum runoff does not occur in the month of maximum rainfall. The average monthly rainfall and the average monthly gage height on three rivers of Wisconsin are shown in Figure 37.

Example Showing the Relation of Precipitation to Runoff

The principle that the maximum runoff does not necessarily occur in the month of maximum rainfall is well illustrated by Figures 38 and 39 which show the hydrographs of the Passaic River at Little Falls, New Jersey, for the seventeen years from 1877 to 1893 inclusive. On these diagrams are also given the monthly rainfall on the Passaic drainage area. From these hydrographs it will be seen that the relations between rainfall and runoff are not direct but are materially modified by other factors. It will be noted that rainfalls of three or more inches per month, during the earlier portions of the year, give rise to flow of considerable magnitude, whereas similar or even greater rainfall during the summer months has little or no effect in augmenting the flows of the stream. These facts can readily be appreciated by noting the comparatively small rainfall during February and March of 1878, 1882 and 1884 and the considerable flows during these months resulting therefrom, augmented, of course, by the snow deposited on the drainage area during the previous winter which snows are melted and carried off by the rains of these months. If these flows are com-

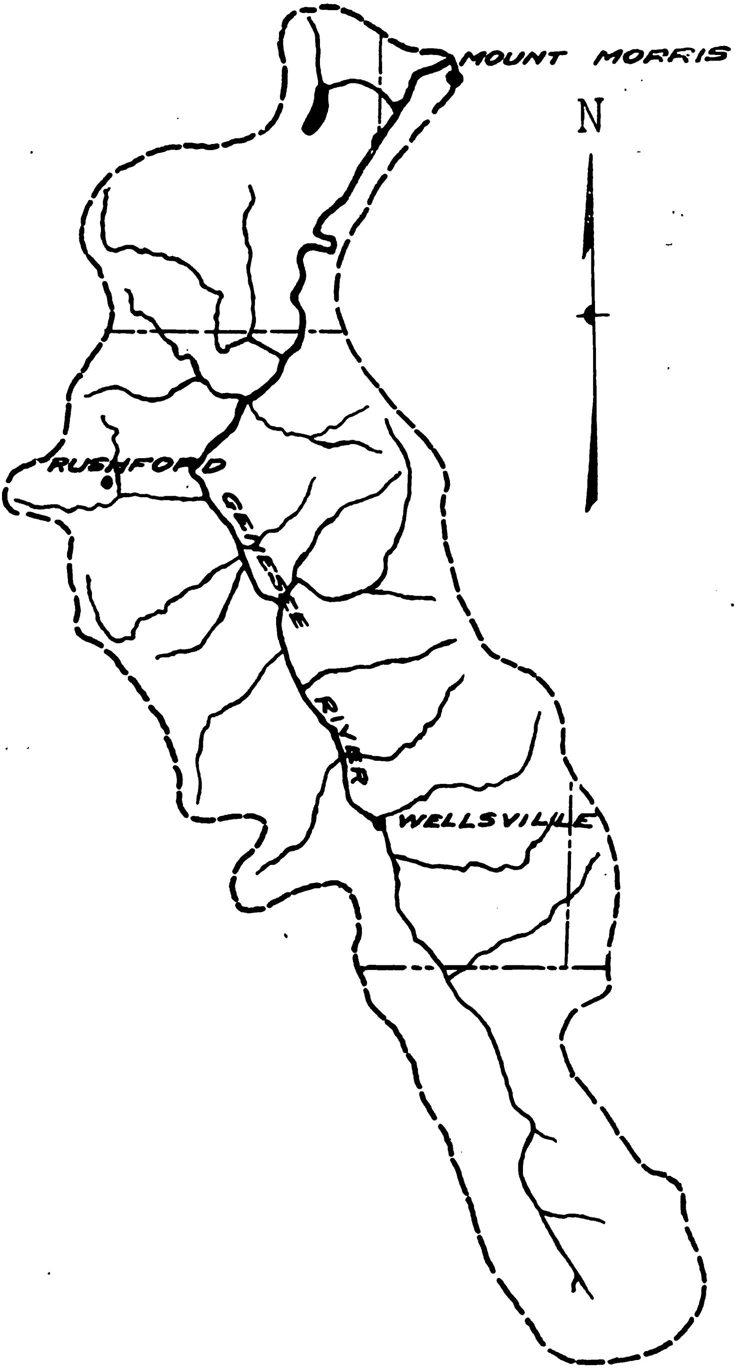


Fig. 36.—Map Showing Drainage Area of Genesee River, New York (limited storage).

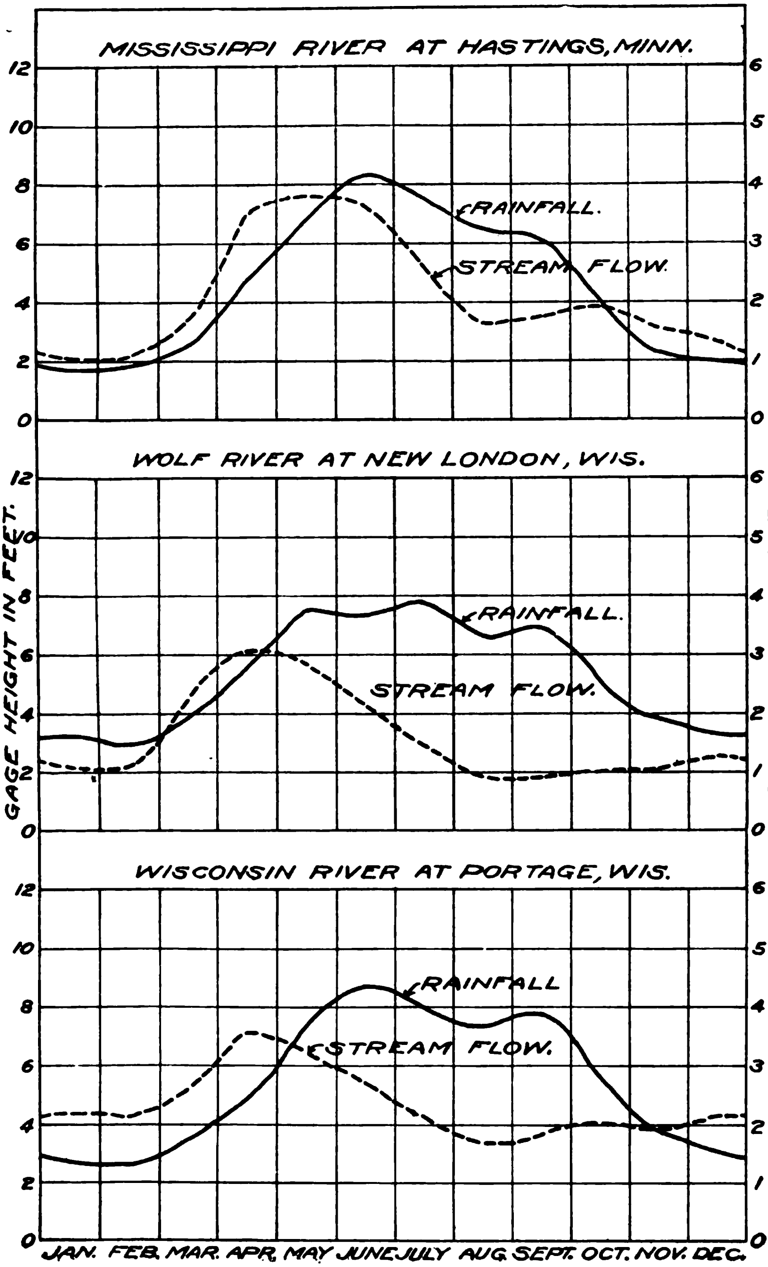


Fig. 37.—Diagrams Showing Relation of Average Monthly Rainfall and Average Stream Flow on Various Rivers of Wisconsin.

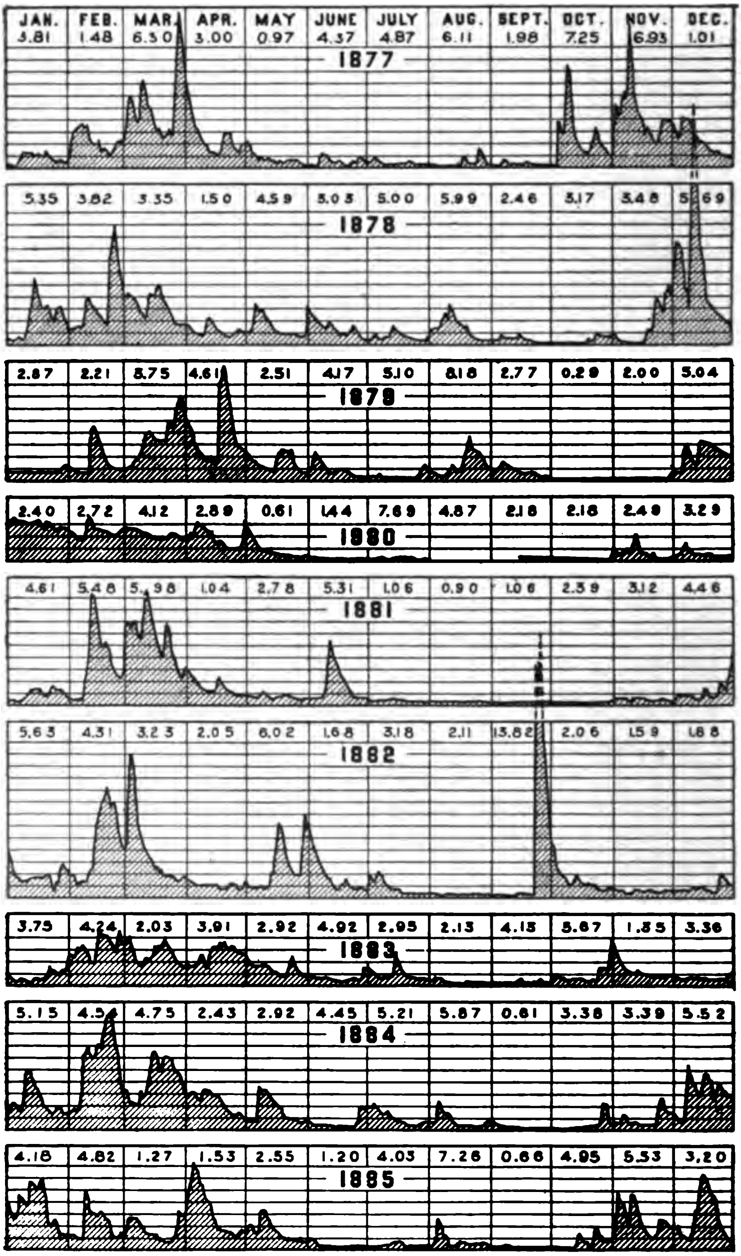


Fig. 38.—Daily Flow of Passaic River, Little Falls, N. J.
Figures near top of each diagram show total monthly rainfall.
Discharge in Cu. Ft. per Second per Square Mile.

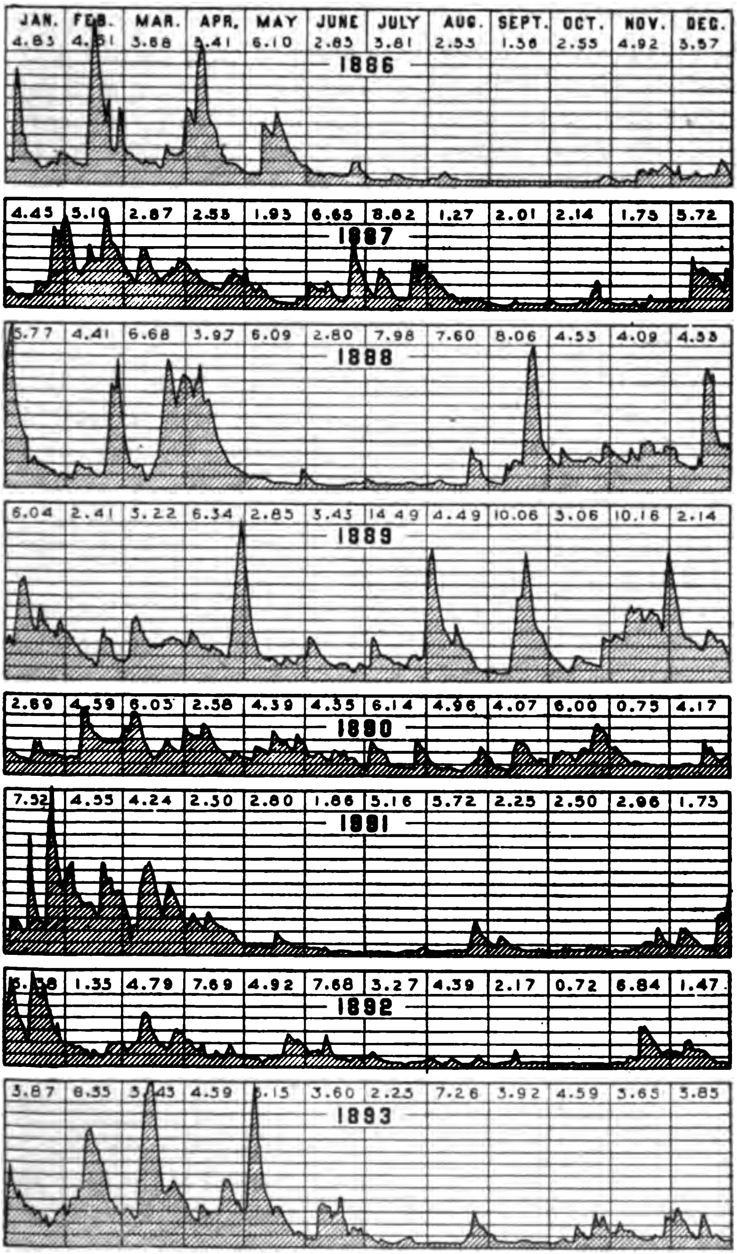


Fig. 39.—Daily Flow of Passaic River, Little Falls. N. J.
Figures near top of each diagram show total monthly rainfall.
Discharge in Cu. Ft. per Second per Square Mile.

pared with the small ones arising from 7.26 inches of rainfall in August of 1885, and 1893, the effect of other factors will be manifest. Occasionally years of high precipitation will be accompanied by floods of considerable magnitude throughout the season. In this connection the high water during 1889 should be noted. The floods early in August of that year resulted from a rainfall during the preceding months of nearly fourteen and one-half inches. If such a rainfall had occurred in March, with a large amount of snow from the previous winter's precipitation already deposited on the drainage area, the extent of the flood would undoubtedly have been very great.

On this drainage area, fall floods are not uncommon but do not always occur. The exceedingly high rainfall in September, 1882, gave rise to apparently the highest flood flow on this stream shown in these seventeen years of record. The cause for such a condition is not entirely obvious from the chart but was apparently due to concentration of the unusually heavy rainfall of the month.

These hydrographs clearly prove that the runoff is modified by other factors almost or quite equal in importance to the amount of precipitation for any period. Among other things, it is obvious from these hydrographs that the runoff is modified not only by the total quantity of precipitation but by its occurrence or distribution throughout the year.

GENERAL DISCUSSION OF PRECIPITATION-RUNOFF RELATIONS

It is desirable with these diagrams in view to summarize briefly the causes of variation in the rainfall-runoff relations. The causes of the variations in the ratio of runoff to rainfall at various seasons of the year are, in general, quite evident.

During the "Storage" period (December to May) the winter snow and the spring rains usually saturate the ground where pervious deposits exist to a great depth, and a large amount of water is held in storage both in lakes and swamps, and in soils and gravels, where such exist on the drainage area. A heavy rainfall in the latter part of this period of the year, after the ground is thoroughly saturated, finds a quick response in large stream flows, for the saturated ground rejects further water, and the water runs rapidly from the surface. The stored soil water, at this period of the year, which lies within the boundaries of a watershed, and above the level of the surface of the stream, is, or may become, available to supply the stream. Such waters will feed a stream until exhausted, regardless of the amount of rainfall during the "Growing" period (June to August), and may with favorable conditions cause a stream flow for several months, even with deficient rainfall. Such conditions, however, vary widely with each watershed. When these conditions are favorable, large rainfall during the storage period will generally assure a more uniform flow during the summer months, when the mimimum flow of the river usually takes place.

The ground water, when pervious deposits exist on the drainage area, usually furnishes more or less of the stream flow, beginning some times early in May. During June, July, and August the rainfall is rarely sufficient to take care of evaporation and plant life without something of a draft on the ground water, and the stream flow is usually entirely dependent on the ground water for such purposes during this period except with unusually heavy rainfall conditions.

RELATIONS OF DAILY PRECIPITATION AND STREAM FLOW

To illustrate graphically the average daily rainfall on a drainage area and its effect on the daily flow of the stream, diagrams have been prepared (see Figures 40 and 41) on which have been plotted the varying flow from day to day of the Wisconsin River at Necedah, Wisconsin (see Map, Figure 42). The drainage area above this station is 5,800 square miles. The flow has been computed from daily gage height observations and relative stream flow observations made under the direction of the U. S. Geological Survey and substantiated by flow observations and measurements made under the direction of the writer at Kilbourn, Wisconsin, and at the Necedah station. It is believed that the resulting hydrographs are fairly accurate within a few per cent. The rainfall observations were compiled from the reports of the U. S. Weather Bureau. Five stations were taken, well distributed over the drainage area, and the

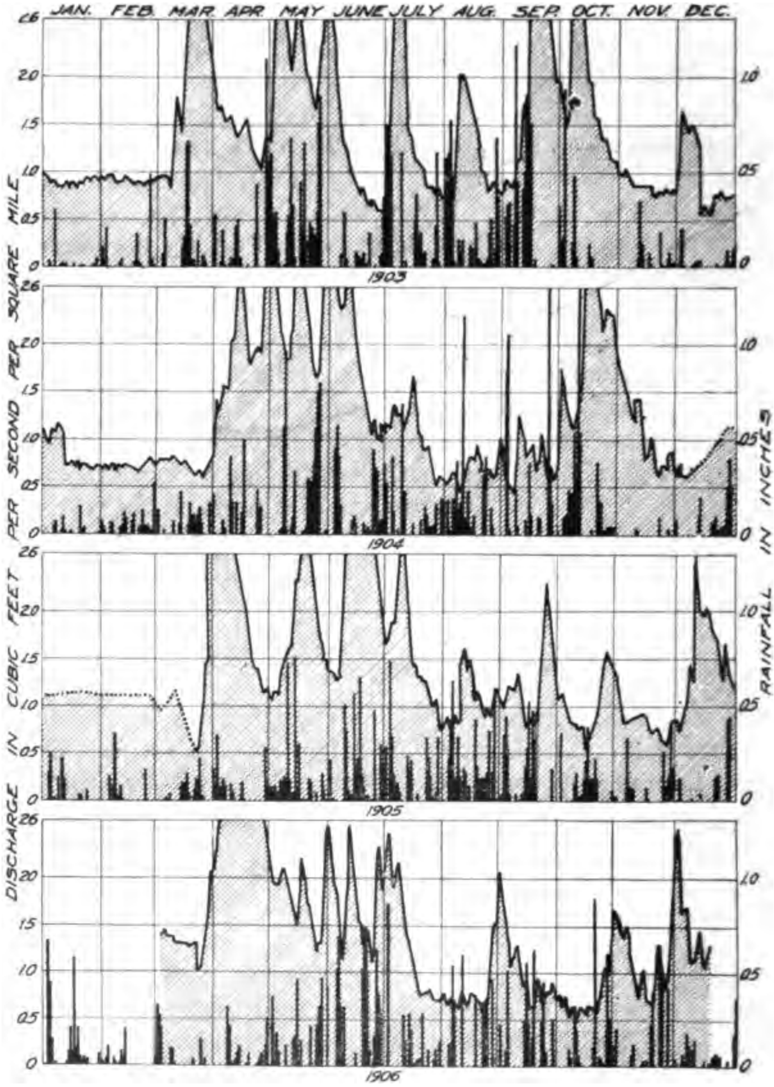


Fig. 40.—Hydrographs Showing Daily Precipitation and Runoff of the Wisconsin River Above Necedah, Wis. (1903—1906).

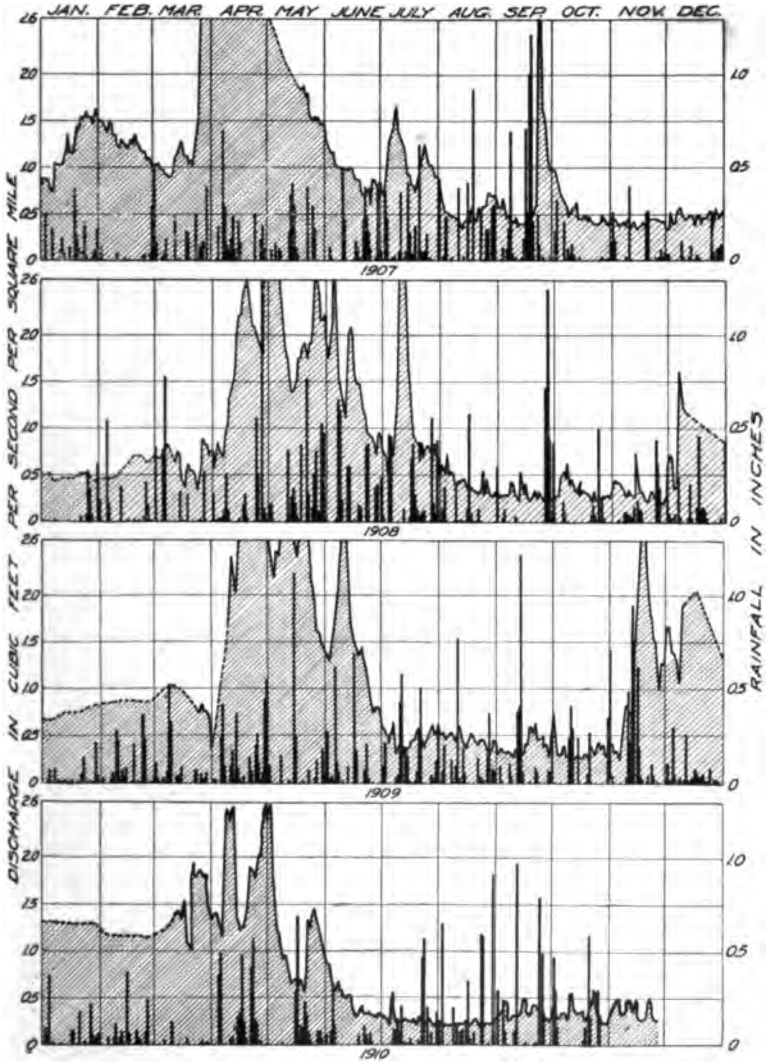


Fig. 41.—Hydrographs Showing Daily Precipitation and Runoff of the Wisconsin River Above Necedah, Wisconsin (1907-1910).

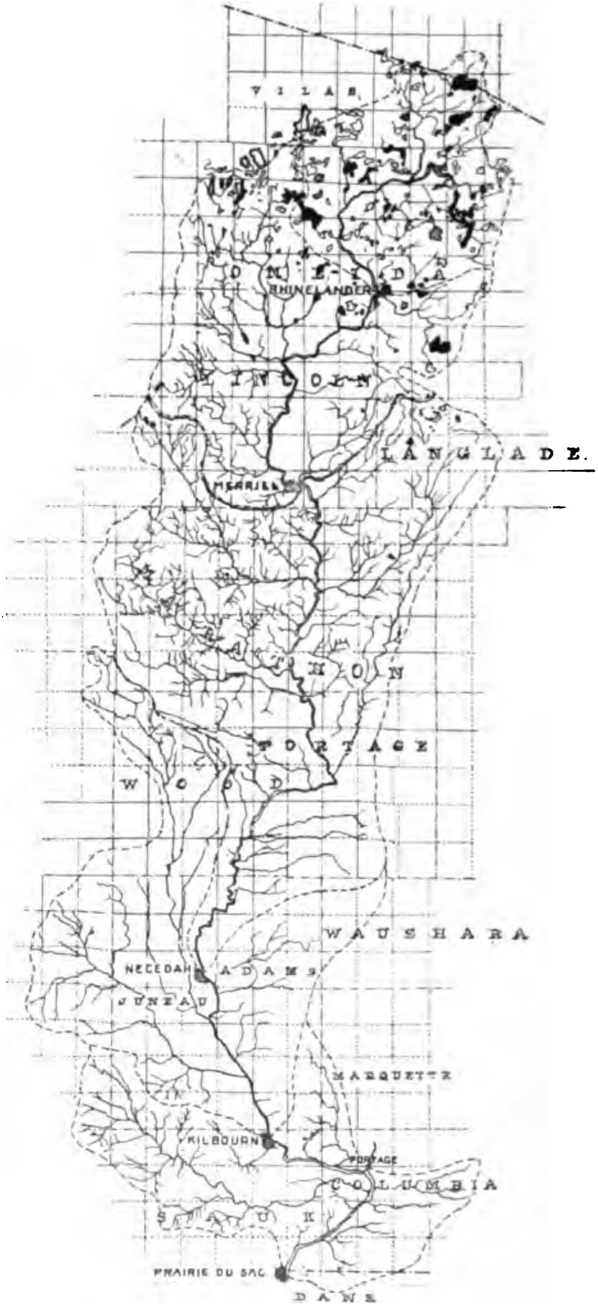


Fig. 42.—Drainage Areas of the Wisconsin River Above Prairie du Sac, Wisconsin.

observations of the daily rainfall at each station were averaged for the daily rainfall of the area (see Table 3). It is believed that the results are fairly representative although the errors involved are probably greater than in the case of the runoff.

The effects of precipitation of various degrees of intensity and at various seasons of the year are well illustrated by these diagrams, and the discussion on the general relations of rainfall to runoff (see page 120) are emphasized and demonstrated thereby. The effect of precipitation on the flow of the stream at the point of observation is less immediate in the case of a large area, as considered in these diagrams, than it is in the case of smaller areas.

The year 1903 was the year of maximum rainfall of those included in these diagrams. The rainfall was well distributed and unusually heavy in the months of May, July, August, and September. The summer flow was therefore unusually favorable, and the ground water was well maintained, and although the rainfall of October, November, and December was below the average, the fall flow was sustained from the ground water (see Figure 43, Diagrams for 1903 and 1904). Similar conditions obtained for the years 1904, 1905 and 1906, although the rainfall of these years was only at or near the mean. In the falls of 1905 and 1906 the rainfall was heavier and the draft on the ground water was not so great. For the years 1907 to 1910, the rainfall was below the average. The low rainfalls of October, November, and December of 1907 resulted in low flows for those months and for January, February, and March of 1908. The rainfall of April, May, June, and July of 1908 was fairly high, and the flow of the streams in consequence well maintained, although a considerable draft on the ground water was necessary to maintain it. The rainfall of the balance of the year was unusually low, and as the ground water had already been partially exhausted, the latter months of the year were among the lowest in flow on record. The rainfall conditions in 1909 were better, although the spring flow was low on account of the depletion of the ground water during the previous fall. The highwater of April, May, and June was moderate, and the summer flow was poorly maintained as the rainfall was apparently only sufficient to supply evaporation and vegetation. The November rains were above normal, resulting in a fairly heavy flow from the middle of November to the end of the year, and a fair flow in January, February, and March of 1910. The peak of the high water of April, 1910, was the lowest ever experienced on the Wisconsin, and the early exhaustion of the ground water due to deficient rains and the low rainfall through the season, resulted in the lowest year's flow experienced on the Wisconsin River since the beginning of flow records in 1873.

TABLE 3.—AVERAGE DAILY RAINFALL ON THE WISCONSIN RIVER ABOVE NECEDAH, MAY, 1903.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Grand Rapids	....	.70	....	.20	.07	.03	...	...	.05	....	.70	1.30	.60	....	...	...	.15	....	.10	.10	...	.50	....	....	.15	....	.70	1.50	...	...	...
Wausau............	....	.60	....	.43	....	....	...	...	....	....	.21	1.00	...	....	...	...	....	.80	.69	....	...	.75	....	....	.33	.05	2.01	....	...	...	...
Koepenick..........	....	.60	.40	.30	.20	....	...	...	....	.20	.30	.10	....	....	...	...	.50	.10	.30	....	...	....	.60	....	.10	.40	.20	....	...	...	...
Prentice	....	1.10	....	.30	....	....	...	...	....	.30	...	1.00	.65	.20	...	...	1.50	....	....	.60	...	....	....	.55	.10	.65	.90	1.40	...	...	...
Florence...........	....	....	....	.22	.17	....	...	...	....	.50	.13	....	.38	....	...	...	.09	....	2.19	....	...	....	.50	....	.15	.39	....	1.51	...	...	...
Average	....	.60	....	.29	.09	.01	...	...	.01	.20	.27	.68	.33	.04	...	...	.45	.18	.66	.14	...	.25	.22	.11	.17	.30	.76	.88	...	...	...

CUMULATIVE DIAGRAMS

The same data shown in Figures 40 and 41 are shown in different form in Figures 43 to 46 inclusive. In these diagrams cumulative progressive curves of rainfall and runoff are shown from which the total amount of either can be determined between any dates by reference to the vertical scale. On these diagrams are also shown "retention" curves. These curves show the difference between the rainfall and the runoff and represent the quantity of the rainfall retained on the drainage area, including the quantities evaporated, used by vegetation, or temporarily held in storage in lakes, swamps, gravel, etc. The retention curve of the diagrams of 1903 and 1904, during October, November, and December, shows plainly the release of one to two inches from storage, and the same phenomena are seen in the early part of the season of 1905, 1906, 1907 and 1910, as well as at other times. This phenomenon is often observed through the summer season when the losses by evaporation and the amounts used by plant life are large and the rainfall comparatively heavy.

In 1905, the discharge to the last of April was greater than the rainfall to that date, and the retention curve shows that the storage as snow and ice retained from December, 1904, has been returned to augment the runoff.

The retention curve in April, 1906, shows a return of fully three inches to the discharge.

MONTHLY RELATIONS

The consideration of rainfall-runoff relations, from the standpoint of daily observations, involves such a multiplicity of data as to be almost or quite impracticable except for a most detailed study. In addition to this, records of daily observations are only available for a brief term of years, while monthly and yearly records can often be secured for much longer periods. It is important, therefore, to determine as clearly as possible what relations may be established between the total precipita-

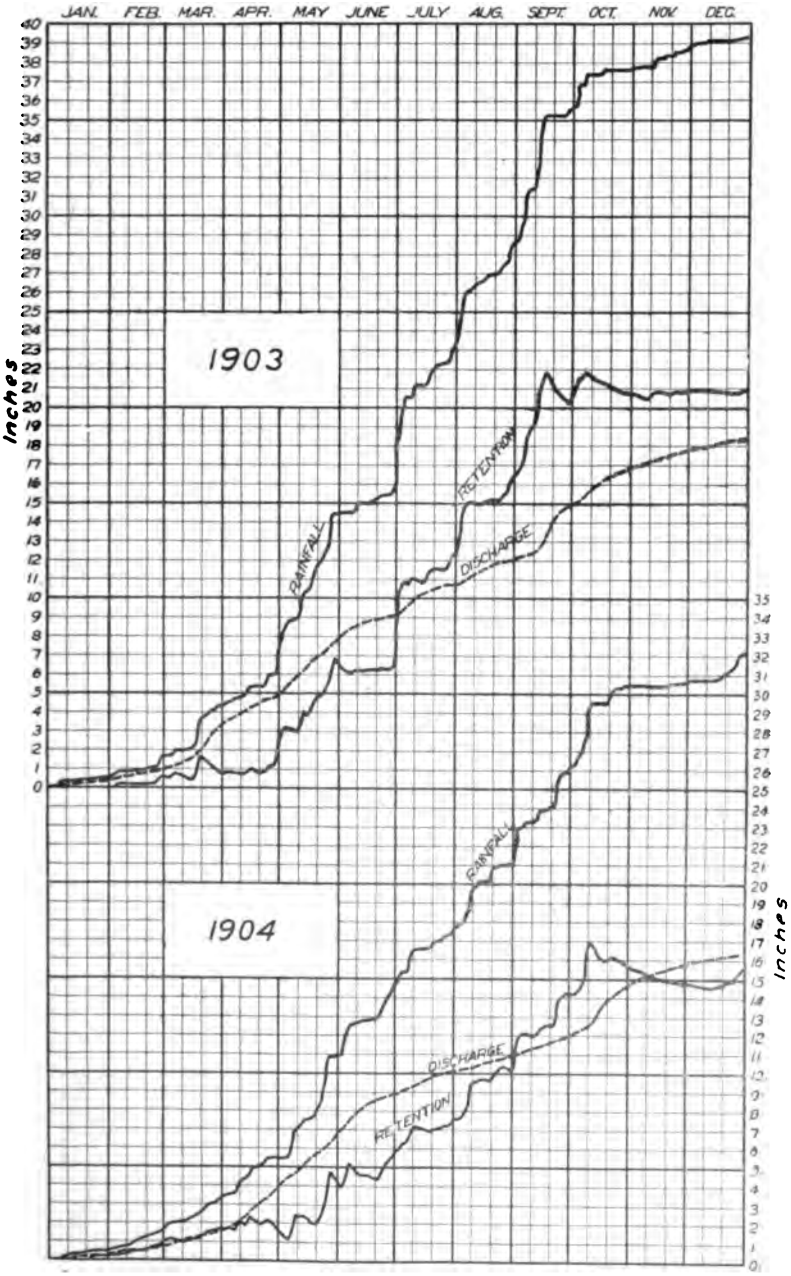


Fig. 43.—Cumulative Curves of Rainfall, Runoff and Retention on the Wisconsin River Above Necedah, Wisconsin (after Landa).

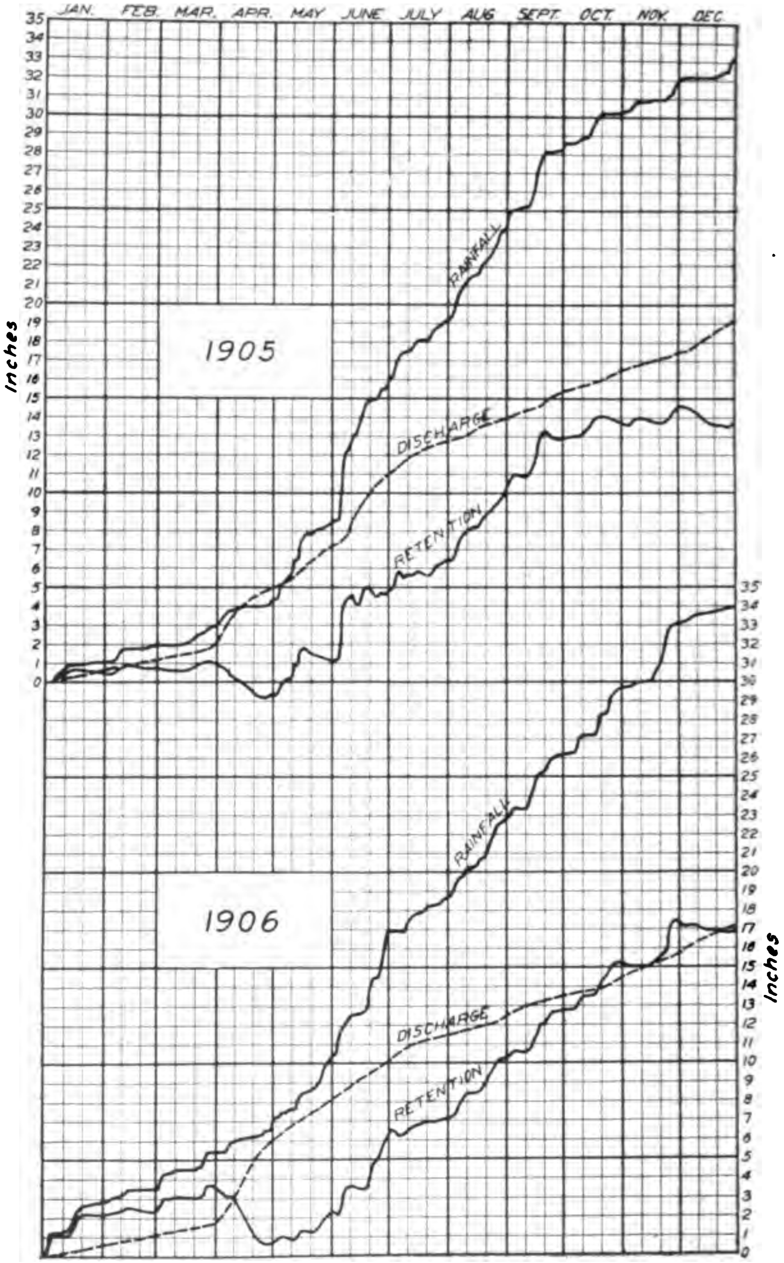


Fig. 44.—Cumulative Curves of Rainfall, Runoff and Retention on the Wisconsin River Above Necedah, Wisconsin (after Landa).

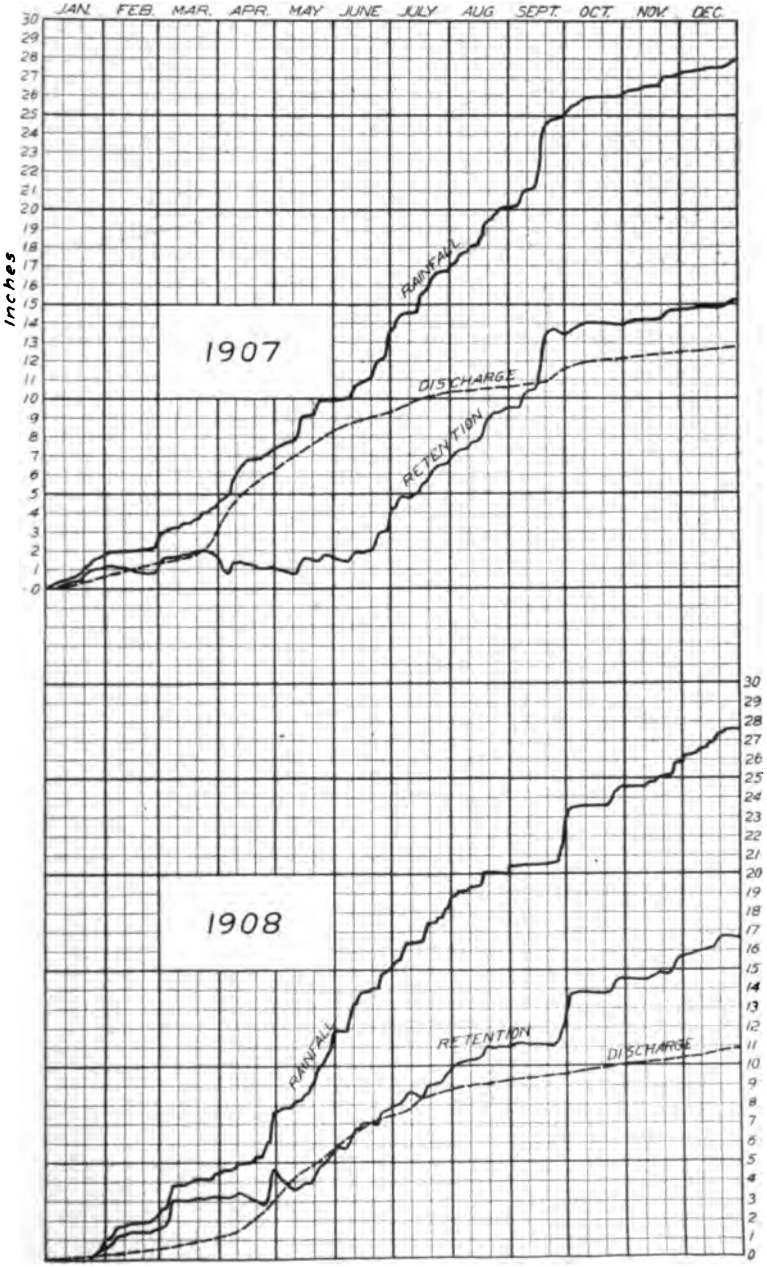


Fig. 45—Cumulative Curves of Rainfall, Runoff and Retention on the Wisconsin River Above Necedah, Wisconsin (after Landa).

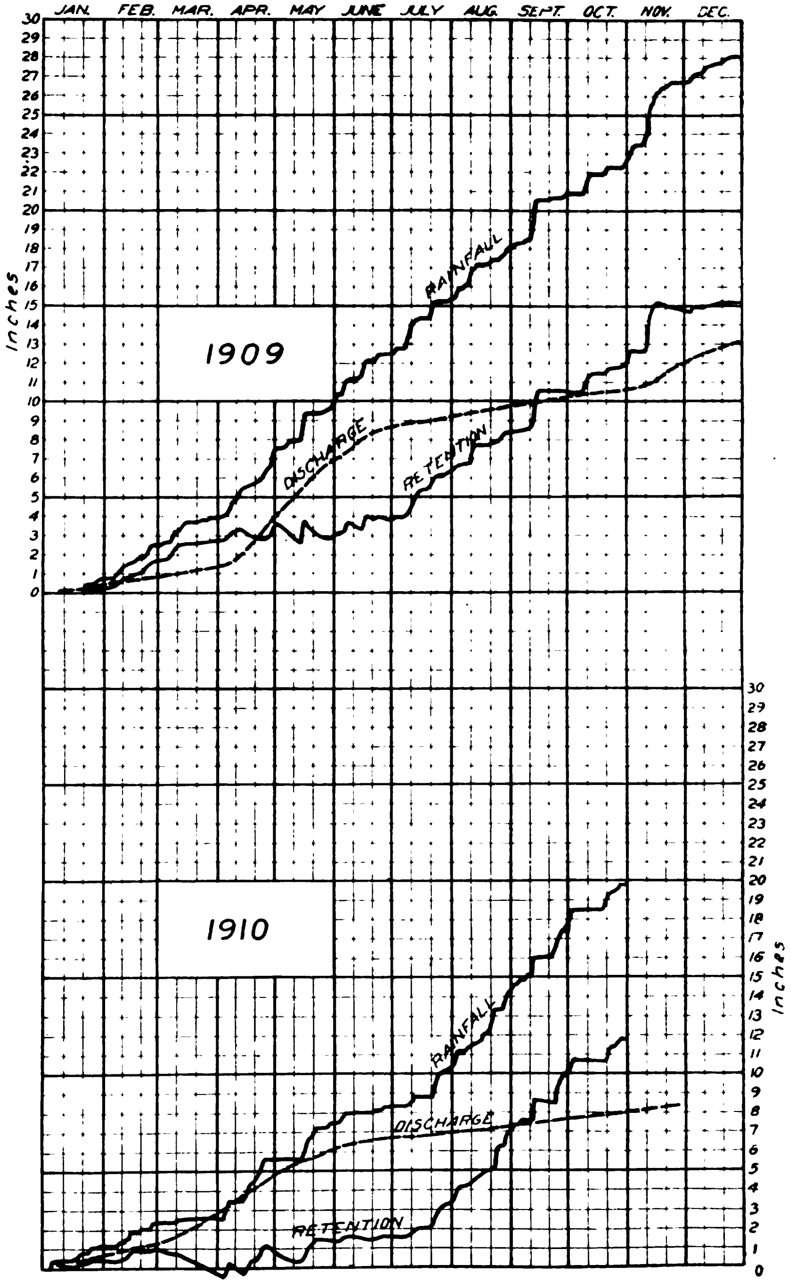


Fig. 46.—Cumulative Curves of Rainfall, Runoff and Retention on the Wisconsin River Above Necedah, Wisconsin (after Landa).

tion for the month or year and the mean conditions of flow during corresponding periods. In the consideration of the monthly relations,. attention is again called to the fact that the precipitation of the previous month may have a decided effect on the flow of the month considered; also that in the case of early months of the year the snowfall of the preceding months, and the temperature of the month, will frequently have more direct effect on the monthly runoff than the total monthly rainfall. The diagrams in Figures 47 and 48 show the monthly rainfall-runoff relations for three small drainage areas near Philadelphia. All of these data point to a close mutual relation of rainfall to runoff, modified, as has been pointed out, by certain concurrent conditions. Conclusions from the limited data of measured flow on the Wisconsin and other rivers of the state may be sustained or confirmed by deductions based on gage heights which have been observed on various rivers under the direction of the U. S. engineer at various points in or adjoining the state. Gage height is a function from which measured flow can be determined if the relations are stationary and well established or if established for each condition where conditions are changing.

The relations of gage height to flow are not direct but are nevertheless sufficiently determinate to afford a relative measure of flow which renders their consideration of much value.

Observations of daily gage heights have been made on the Wisconsin River at Portage since 1873. The early observations were made during the navigation season only, but usually cover the extreme conditions of flow and are fairly indicative of the conditions of the year. The diagrams in Figures 77 to 85 (see Appendix A, page 149) inclusive, show the relations of daily gage height throughout the year from 1873 to 1910 inclusive. thus giving the data for the detailed study of the flow conditions of this stream for thirty-seven years.

Similar data for the Wolf River for the years 1888 to 1910 are shown in Figures 87 to 91 (see Figure 86, and also Appendix B, page 164), and for the upper Mississippi River at Hastings, Minn., for the years 1880 to 1909, in Figures 92 to 99 (see Figure 32, also Appendix C, page 175). These data are

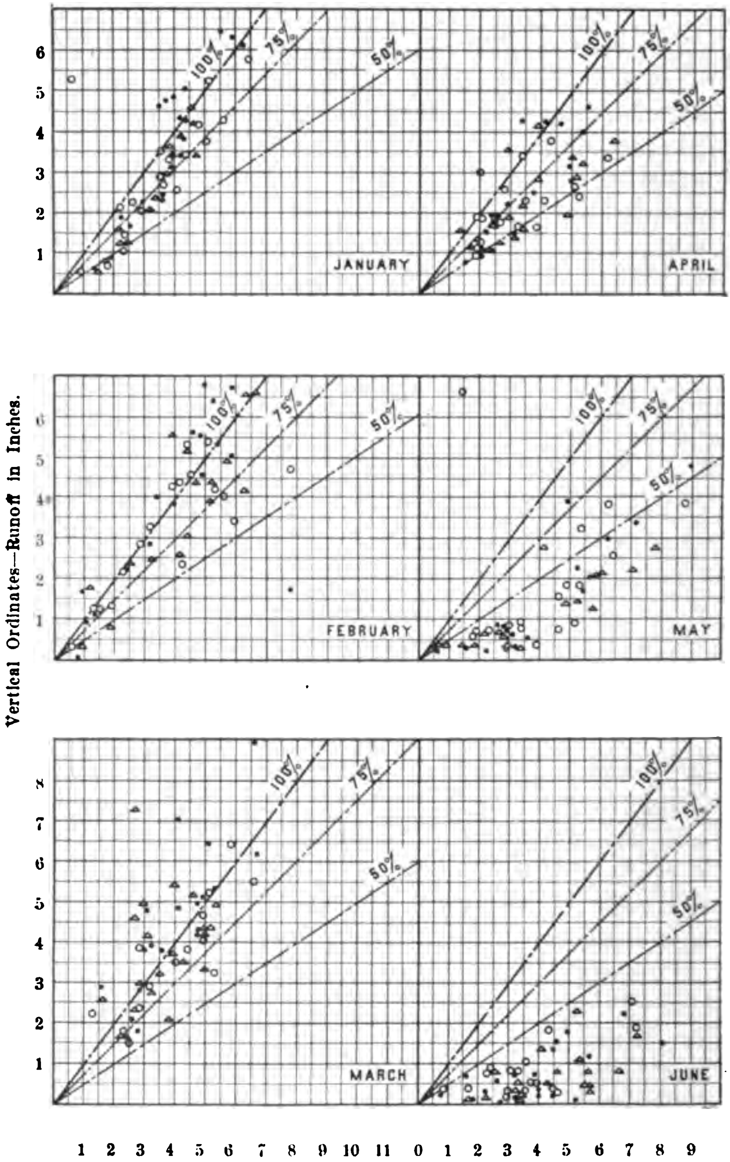


Fig. 47.—Rainfall and Runoff, Tohickson, Neshaminy and Perkiomen Creeks, Near Philadelphia, Pa.

Horizontal Ordinates—Rainfall in Inches.

· Tohickson Creek Watershed—Area 102.2 Sq. Miles.
△ Neshaminy " " — " 139.3 " "
○ Perkiomen " " — " 152.9 " "

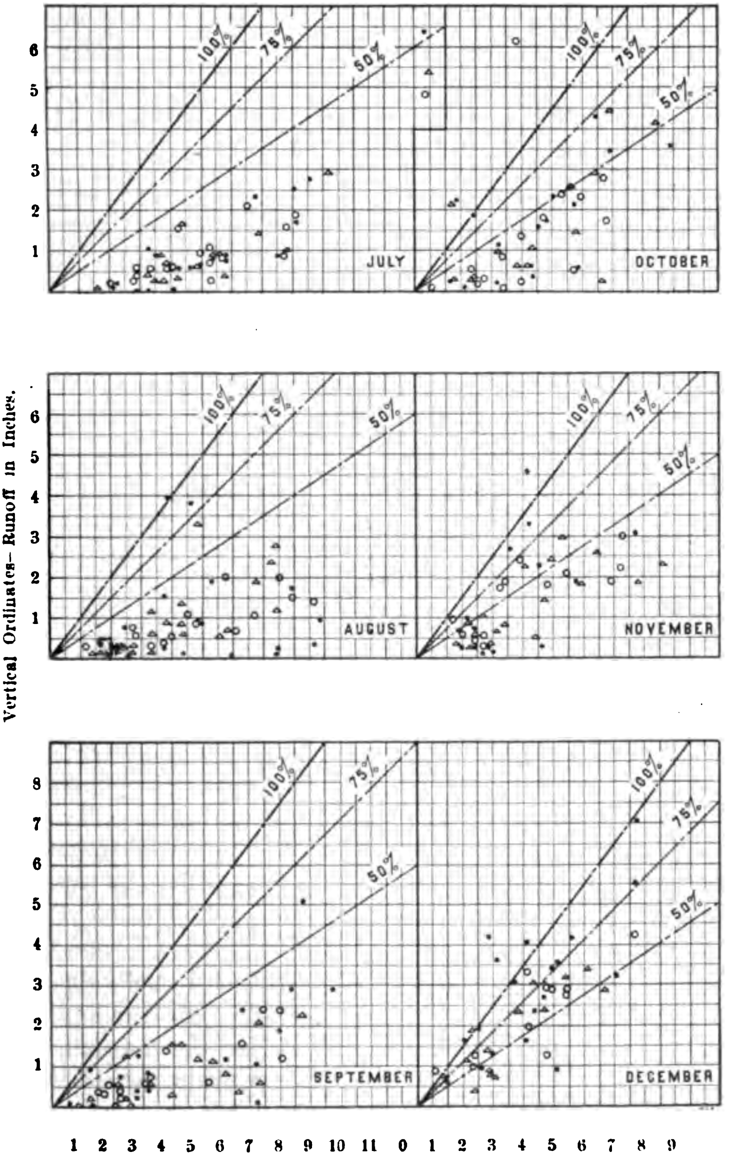


Fig. 48.—Rainfall and Runoff. Tohickson, Neshaminy and Perkiomen Creeks, Near Philadelphia, Pa.

Horizontal Ordinates—Rainfall in Inches.

• Tohickson Creek Watershed—Area 102.2 Sq. Miles.
△ Neshaminy " " — " 139.3 " "
○ Perkiomen " " — " 152.0 " "

given in detail together with tabulations of rainfall, etc., in order that the conclusions herein contained may be fully understood.

MONTHLY RELATIONS OF PRECIPITATION AND RUNOFF ON THE WISCONSIN RIVER

Figure 48 is a study of the monthly rainfall-runoff relations at Portage for the entire period. In this diagram the mean gage height for the month shows the relative runoff. If these relations be considered for any one month of various years, we find a more or less marked relation between increased rainfall and increased stream flow. The increase is not always proportional. as indicated by the inclined lines drawn on the diagrams, and the departure from the rule measures both the effect of other factors and the error in comparative rainfall measurements, due to our artificial divisions of time. The same month for each period—1873–1885, 1886–1897, and 1898–1904—is represented by a different symbol, so that the effects of changes on the drainage area, if any, may be noted.

The relations of precipitation to runoff for January, February, March, and April, as might be expected on account of the modifying effects of the winter's snow and ice deposits, are somewhat obscure. In May the effect of increased rainfall is quite obvious, although in some cases the winter's storage has, apparently, affected the flow. In June the effects of increased rainfall are quite apparent. In July and August while some effects of increased rainfall are apparent, the flow is in general independent of rainfall and is apparently most largely supplied from the ground water. In September and October the increased runoff due to increased rainfall is again more apparent. In some cases the flow is evidently still maintained by the ground water, while in other cases the exhausted ground water was evidently being replenished by the rainfall. In November and December the relations are again somewhat obscured by the condition of the ground water; the relative flow has, however, increased during the last three months as vegetation has decreased.

It is evident from these diagrams that no marked changes in the rainfall-runoff relations for these different periods have

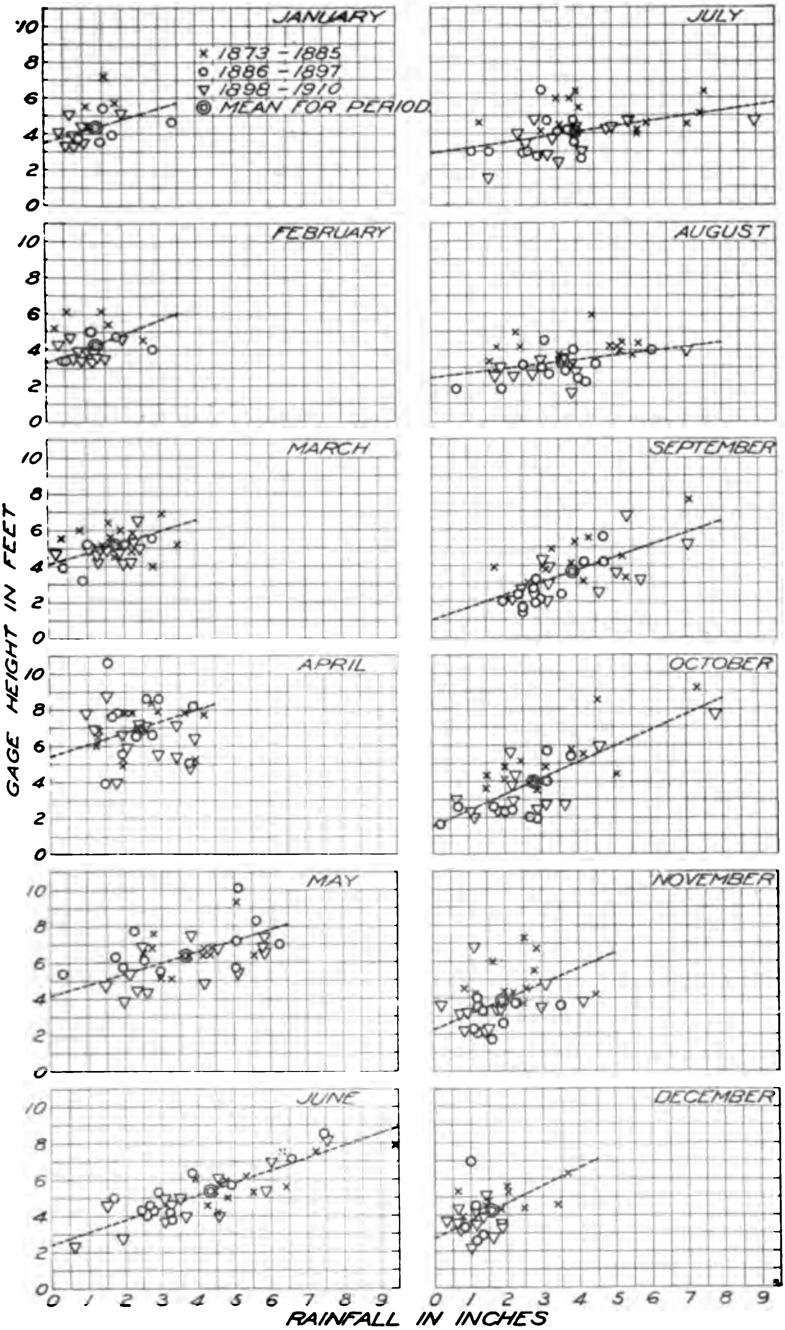


Fig. 49.—Comparative Monthly Precipitation and Runoff of the Wisconsin River at Portage, Wisconsin.

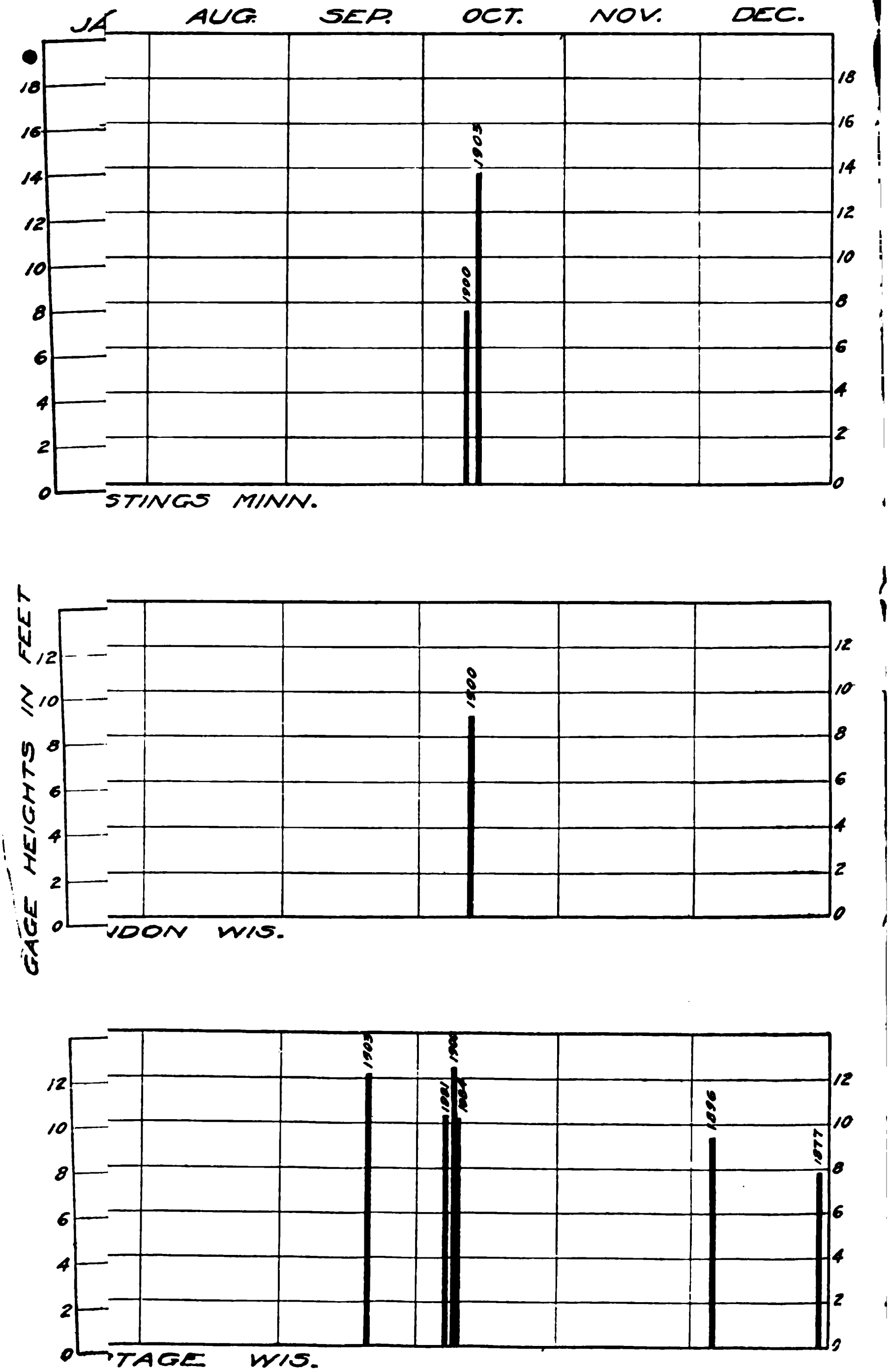
taken place. In some months the flow for the lighter rains seems to be better, and for the heavier rains, poorer in the earlier periods. Taking into account all conditions of rainfall as shown by these diagrams, if there are any changes in the flow relations from the earlier to the later
slight as to be immaterial.

Chronological View of Precipitation—Runoff Relations on the Wisconsin River

A chronological view is perhaps better shown by Figure 50, on which the mean monthly gage height is considered month by month with the corresponding rainfall conditions. The high water peak for each year is shown by an arrow, and the extreme low water is indicated by a point. It will be noted that the high water does not always occur in the same month, although as a rule the high water peak occurs in the spring when the stored snow and ice are melting and running off. The fall floods that occurred in 1881–1900 and 1903 were the result of most extraordinary rainfall conditions as is clearly indicated on the diagram. The extreme floods of early summer in the years 1880 and 1905 were also the result of exceptional maximum rainfall conditions. These flood conditions are clearly explained by the corresponding rainfall conditions and are clearly not the result of drainage work or deforestation.

The time of occurence of floods on the Wisconsin, Wolf, and Mississippi rivers is well shown by Figure 51, which shows the day of the flood peak and its gage height. Similar chronological monthly diagrams for the Wolf River above New London, and for the Mississippi River above Hastings, Minnesota, are shown in Figures 52 and 53. Figures 54 and 55 show a direct comparison of the actual monthly quantities of precipitation and runoff measured in inches on the Wisconsin and Menominee drainage areas at Merrill, Wisconsin, and Iron Mountain, Michigan, respectively, for the years 1903 to 1907. A careful study of these diagrams will lead to the same conclusion as of that for the Wisconsin River.

The relation of quantity of rainfall to actual quantity of stream flow on certain Wisconsin streams is shown by Figures



sippi, Wolf, and Wisconsin River.

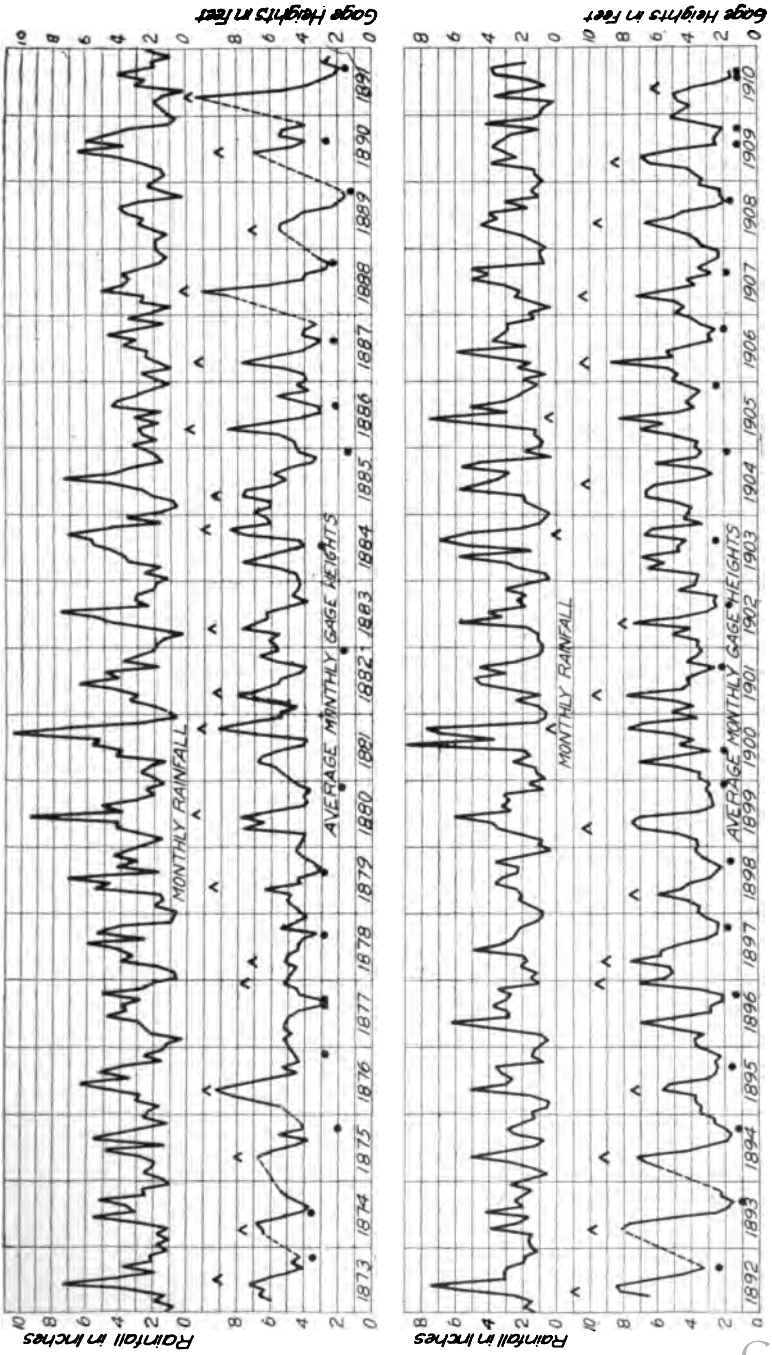


Fig. 50.—Chronological Diagram of Monthly Precipitation and Runoff of the Wisconsin River Above Portage, Wisconsin.

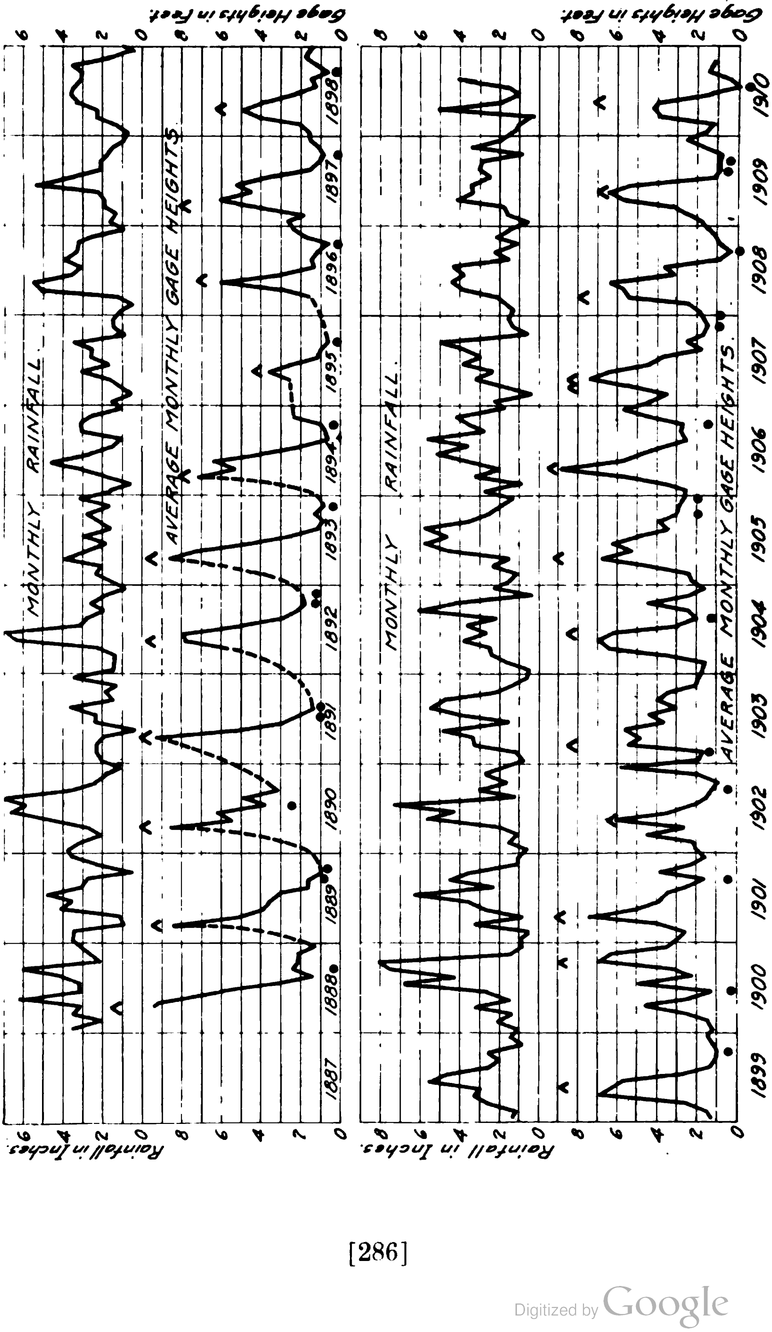


Fig. 52.—Chronological Diagram of Precipitation-Runoff Conditions on the Wolf River at New London, Wisconsin.

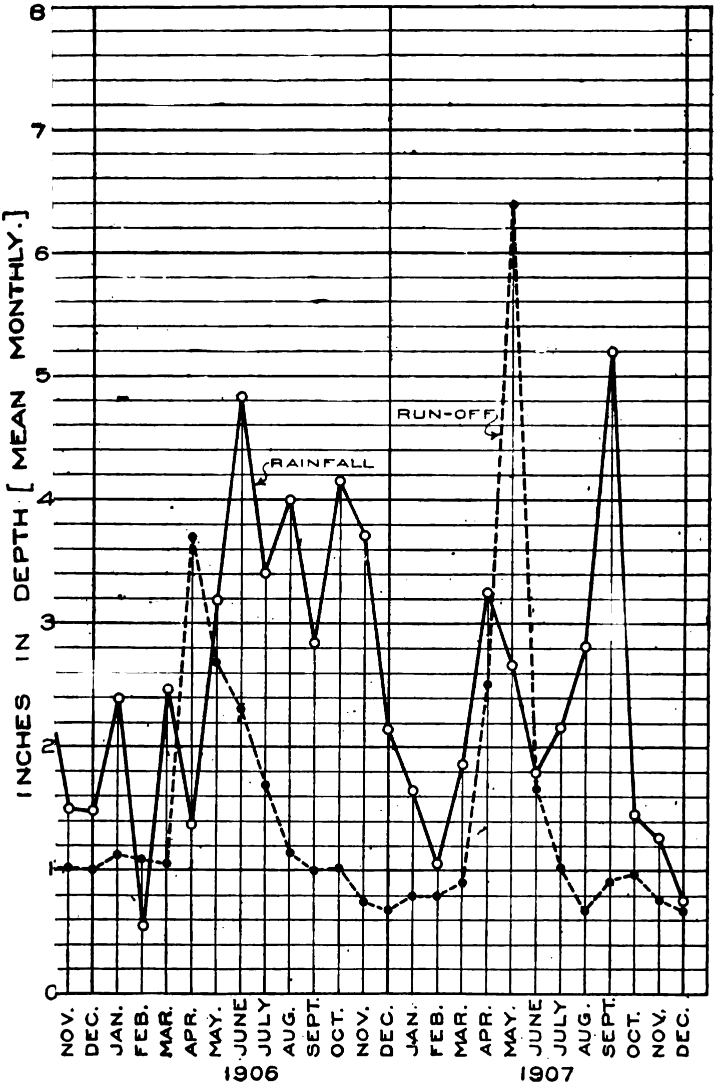


DIAGRAM
SHOWING RELATION OF
RAINFALL AND RUN-OFF
ON THE
MENOMINEE RIVER WATER-SHED

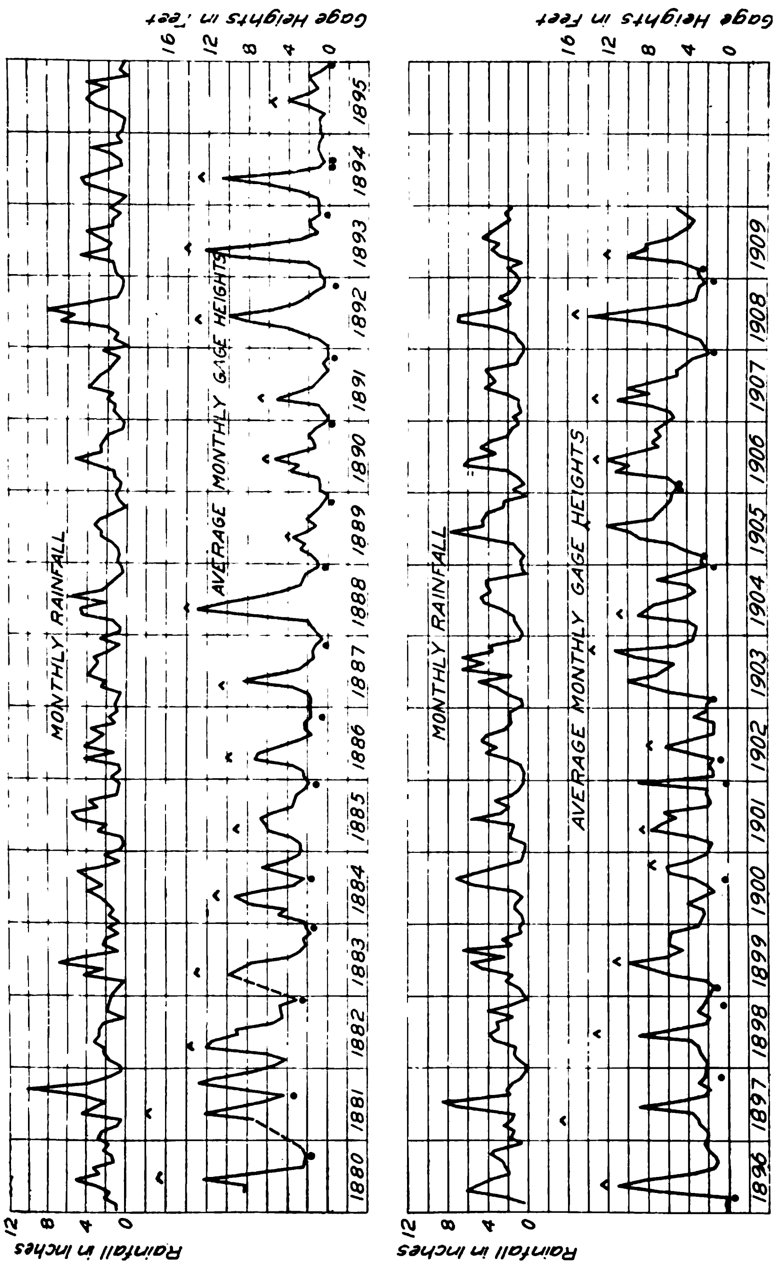


Fig. 53.—Chronological Diagram of Precipitation-Runoff Conditions of the Upper Mississippi River at Hastings, Minnesota.

56 to 59 inclusive, which give these relations for each month of the year. These diagrams are worthy of careful consideration. The diagonal lines indicate percentage relations of the monthly runoff to the monthly rainfall. If these diagrams be examined in detail, it will be noted that in January while the rainfall is low, not averaging more than an inch and a half, the runoff as a rule is fairly high, sometimes running up to over 100 per cent; such a condition is due to the fact that snow perhaps from the previous month, has melted and run off so that the current runoff was greater than the rainfall. The same is true for February, March, and April. As the conditions for the later months are examined, it will be noted that the rainfall has increased somewhat and that the runoff is also increased. The April runoff is often much above the rainfall because during March and April the winter snows are melted and run away, and the actual runoff of the stream is frequently much above the rainfall, especially where the rainfall for the month is low. In some cases the actual runoff for certain drainage areas may be about three times the rainfall for particular years.

On account of the conditions in January and February, due to the frozen ground, the lowest stream flow of the year occurs in these months. In March and April, and sometimes in May, a very high stream flow results because of the stored waters in the form of ice and snow from the preceding winter.

In the later months—May, June, July, and August—the runoff begins to grow less. In June a decided reduction in flow will be noted. The flow will perhaps average over 50% in May; the average is less than 50%, probably not more than 40% in June; and in July it is still less. It will be noted that in many cases the point falls below the 25% line in July; in August it drops still farther, and, in some cases the runoff is not more than 5% of the rainfall; in September it is still low but is beginning to rise again. The rainfall is heavier in May, June July, and August than earlier in the season, yet the stream flow is much less. The reason for this drop in the percentage of runoff is quite evident.

An examination of the conditions prevailing on most drain-

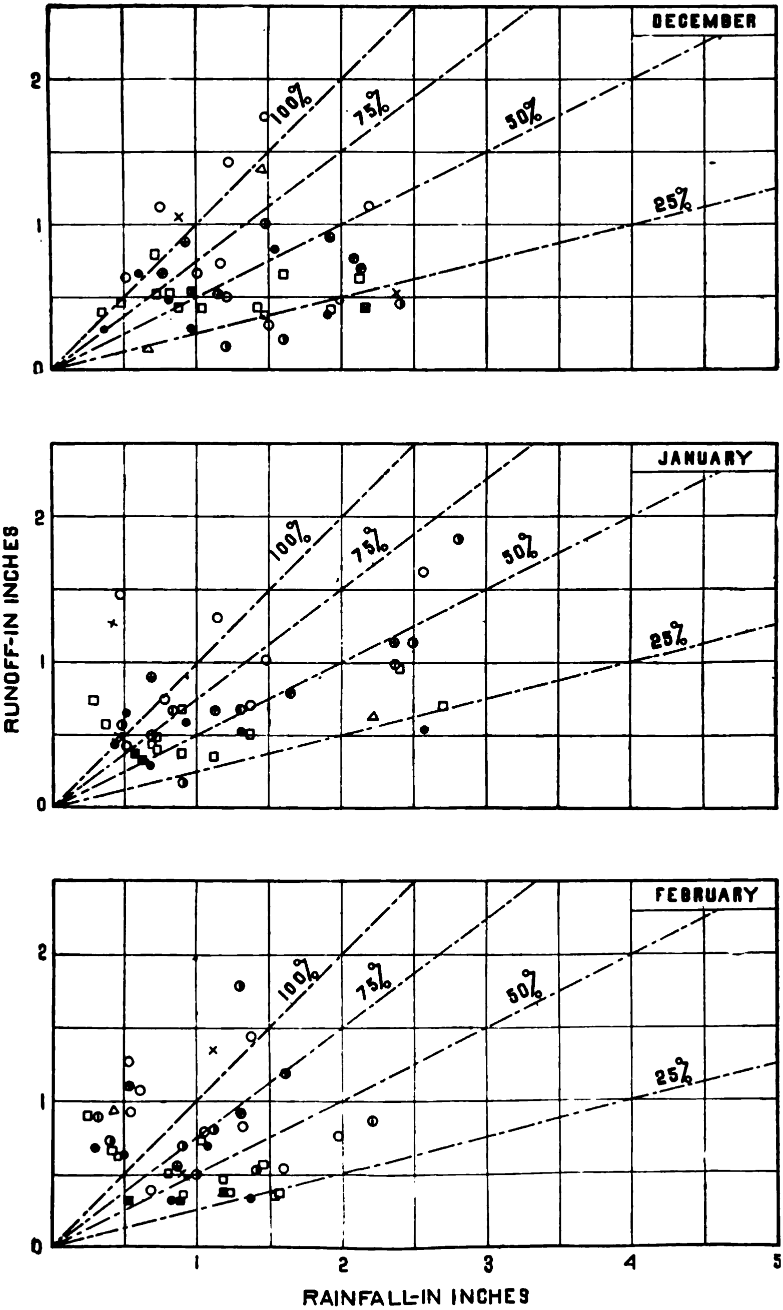


Fig. 56.—Monthly Rainfall and Runoff Relations on Wisconsin Streams.

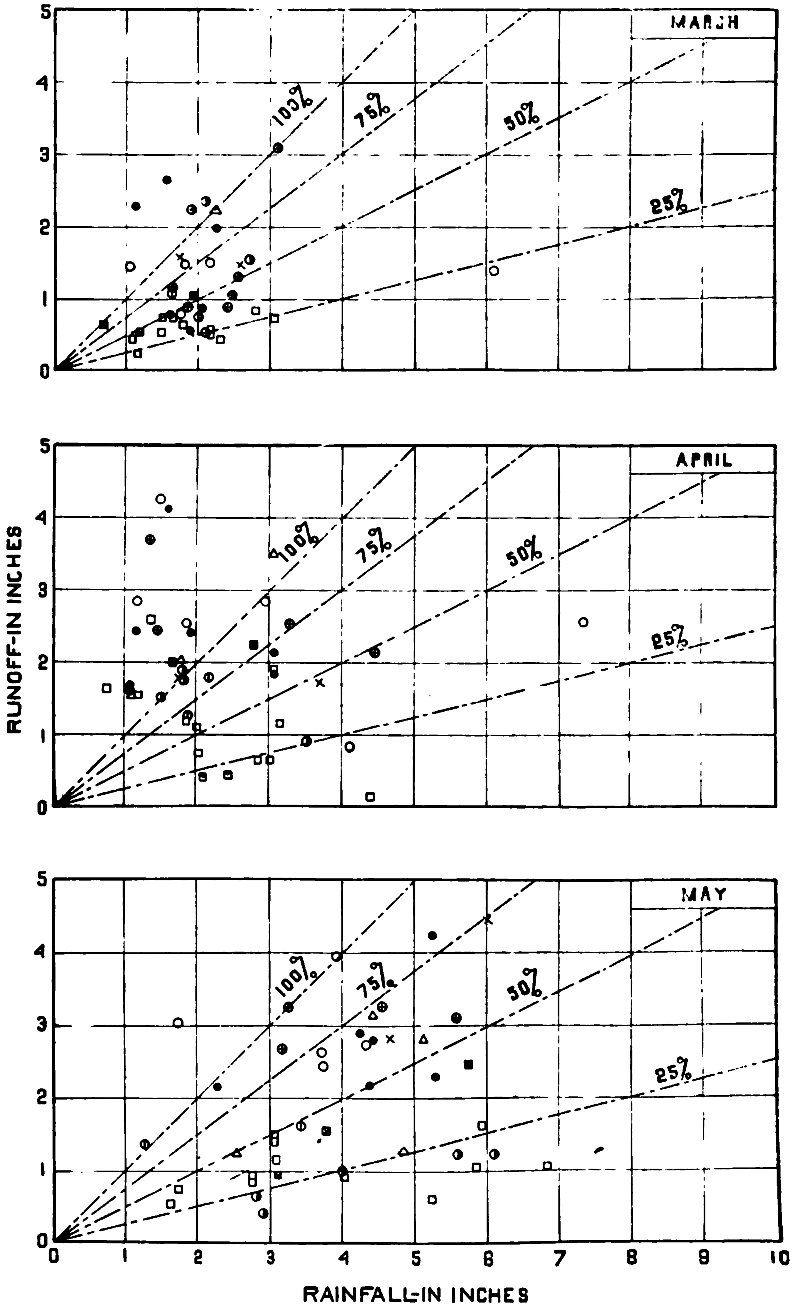


Fig. 57.—Monthly Rainfall and Runoff Relation on Wisconsin Streams.

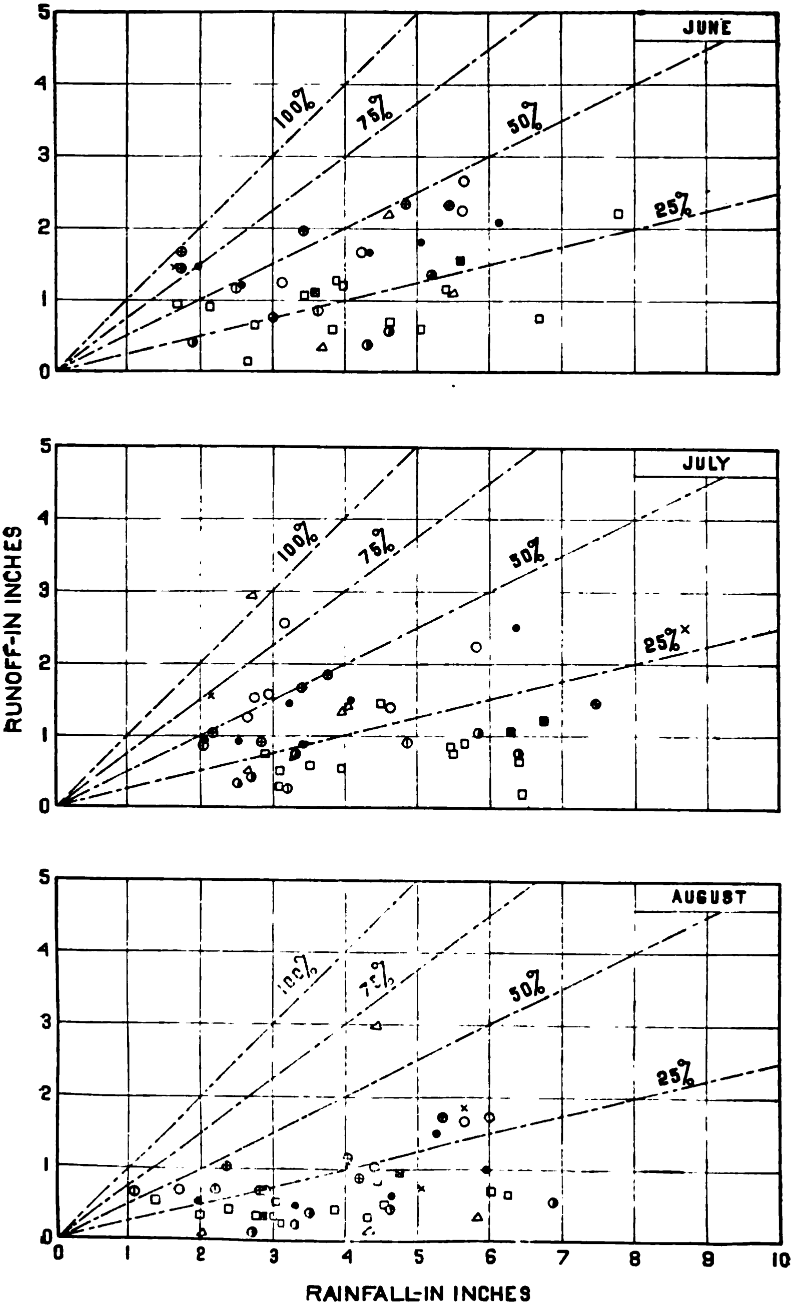


Fig. 58.—Monthly Rainfall and Runoff Relation on Wisconsin Streams.

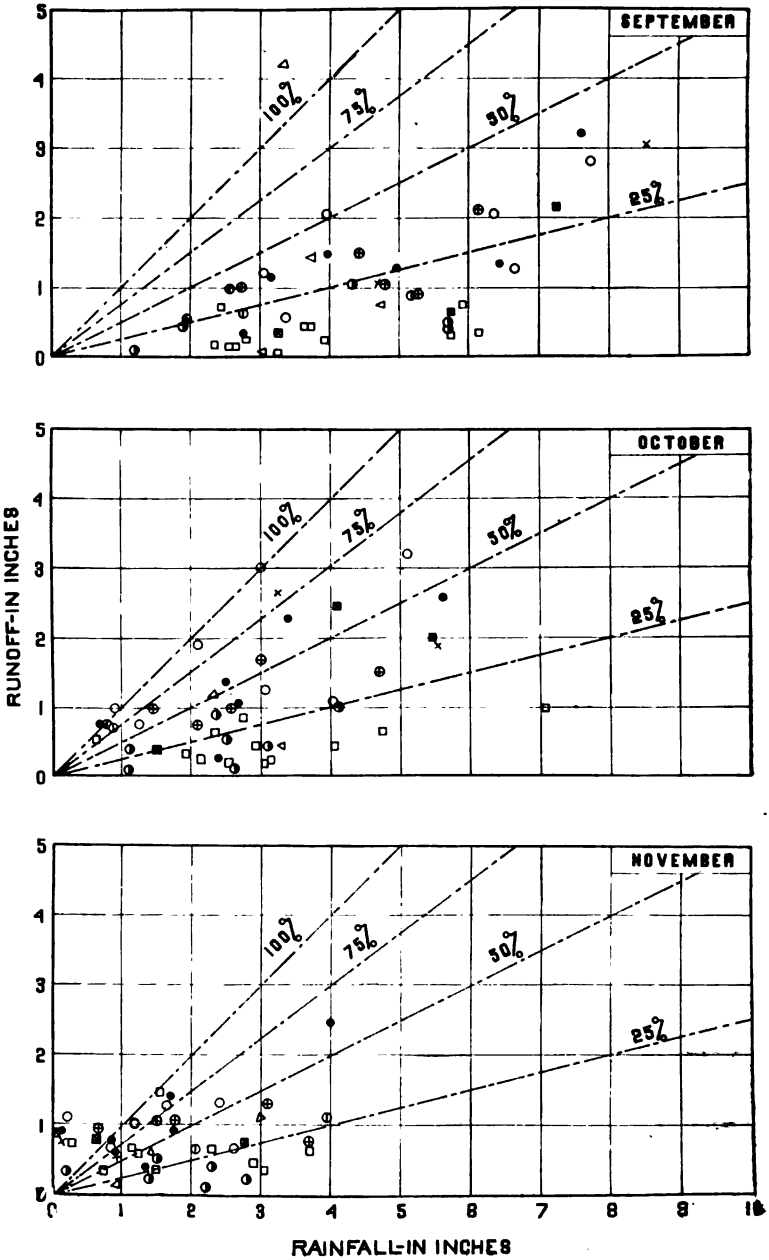


Fig. [illegible].--Monthly Rainfall and Runoff Relation on Wisconsin Streams.

age areas makes it apparent that while many conditions modify the relation between rainfall and runoff for any monthly period, yet, the presence or absence of vegetation is usually one of the most important factors in this variation. During the summer months the vegetation on a drainage area takes up and utilizes in plant growth large volumes of water which in many cases would otherwise flow away or sink into pervious strata and augment the flow of streams. Evaporation is also a factor which may or may not be important according as the water is held near the surface by a more or less impervious soil or absorbed and removed from such influence by deep porous beds The flow of streams is kept up during the summer season most largely from waters that are stored in the ground and that gradually flow out and make a perennial stream possible. Very heavy rainfall may occur and sometimes does occur during the summer months. A rainfall of 6 or 7 inches during the month of March or earlier in the season. would cause a great flood, but in the summer time, with vegetation at its best, such a rainfall might cause no increase in the flow of a stream. It is all taken up by evaporation or by vegetation and is lost, so far as stream flow is concerned.

As the relations are traced later in the season, it will be seen that the stream flow begins to increase as vegetation decreases. In November the comparative flow has increased materially, and in December it is usually still higher.

COMPARATIVE HYDROGRAPHS OF VARIOUS DRAINAGE AREAS ON THE WISCONSIN RIVER

Figures 60 and 61 show the comparative hydrographs of the drainage areas above Rhinelander, Merrill, Necedah. and Kilbourn on the Wisconsin River. The discharges from day to day on these hydrographs are expressed in cubic feet per second per square mile of drainage area.

The drainage areas of the upper river are on the Archean deposits, and the relative discharges are somewhat higher than on the lower areas where there is less surface storage and where more or less of the rainfall is lost in seepage into the Potsdam sandstone. The comparative rainfalls on the three lower areas

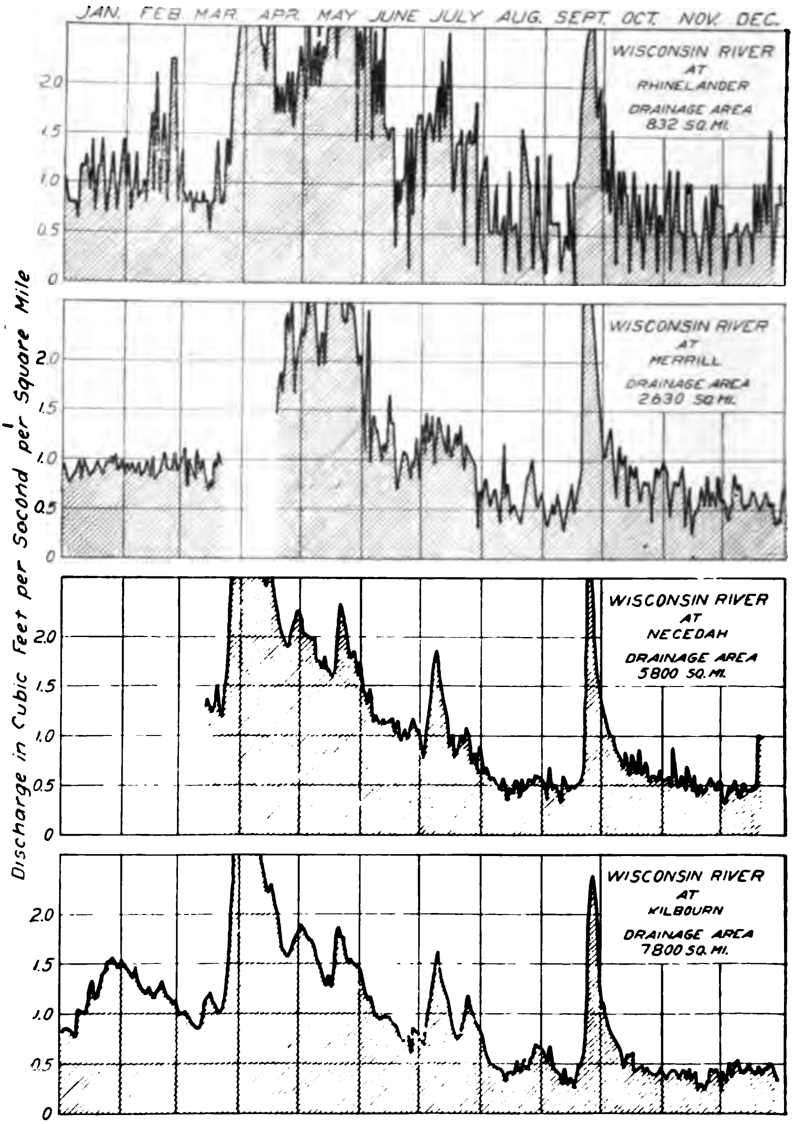


Fig. 00.—Comparative Hydrographs of the Flow of the Wisconsin River at Various Points on its Drainage Area for the Year, 1906.

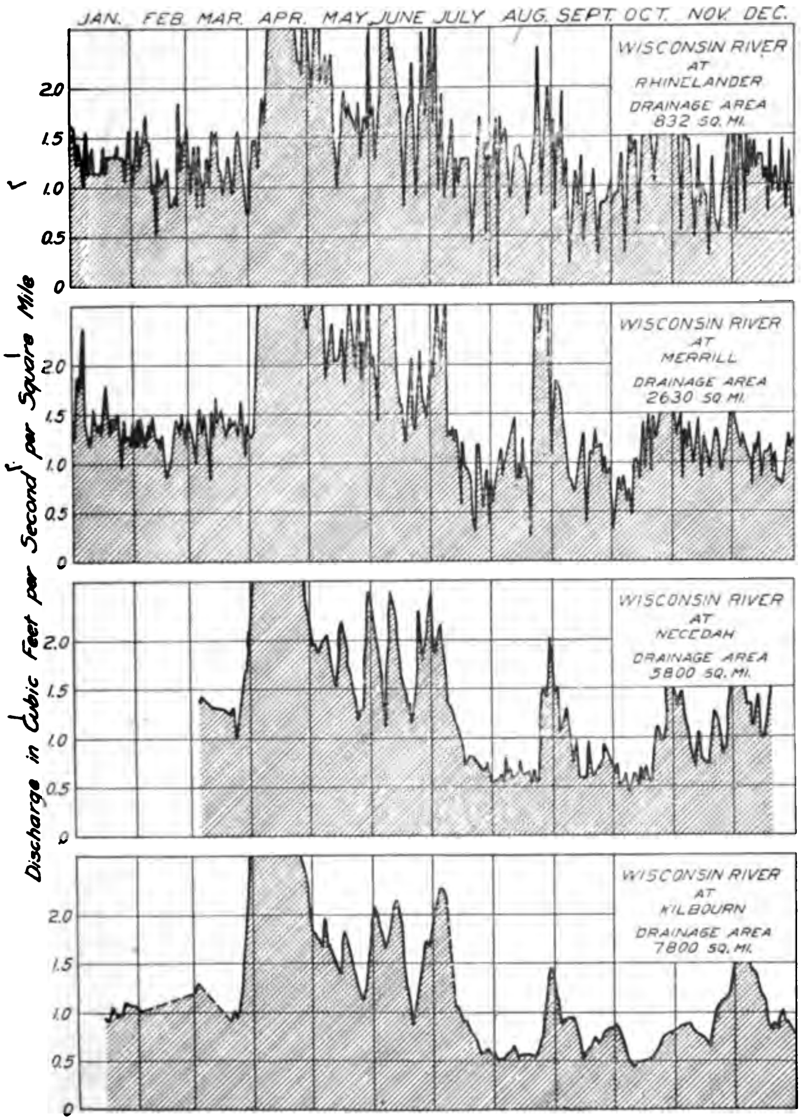


Fig. 61.—Comparative Hydrographs of the Flow of the Wisconsin River at Various Points on its Drainage Area for the Year 1907.

are shown in Figure 13. The hydrographs of the two upper areas show, in a marked way, the effect of the water power developments on the upper river. The Sunday shutdown of these powers is shown by the reduced flow which occurs on the hydrograph with great regularity each seven days. This drop is sometimes obscured by the concurrence of changes in flow due to other causes.

Comparative Hydrographs of Different Rivers

Figures 62 and 63 show comparative hydrographs of the Menominee, Peshtigo, Wisconsin, and Chippewa rivers for 1907 and 1908, at points about in the same latitude. While the rainfall on these areas differs somewhat, there is a general similarity in the same that results in a corresponding similarity of flow. The geological and topographical conditions on these areas are somewhat similar, but it is believed by the writer (from such examinations as he has been able to make) that the Menominee and Peshtigo drainage areas contain a larger proportion of sand deposits which greatly adds to the storage and results in the larger and more uniform flow shown by the hydrographs. These hydrographs are introduced to show the value of the comparison of various stream flows where physical conditions are similar.

ANNUAL RELATIONS

A study of monthly relations is essential to a comprehensive understanding of stream flow variations. However, for a clear understanding of some of the more general relations of rainfall and runoff, and of the possible modifying effect of other factors, the annual relations are important.

Table 4 gives the average rainfall, runoff, and ratio between runoff and rainfall on various drainage areas in the United States. While the increase of the runoff with the rainfall is shown by this table, it is also evident that in some cases other factors have also had an important effect.

Considering the rainfall-runoff relations of a single stream

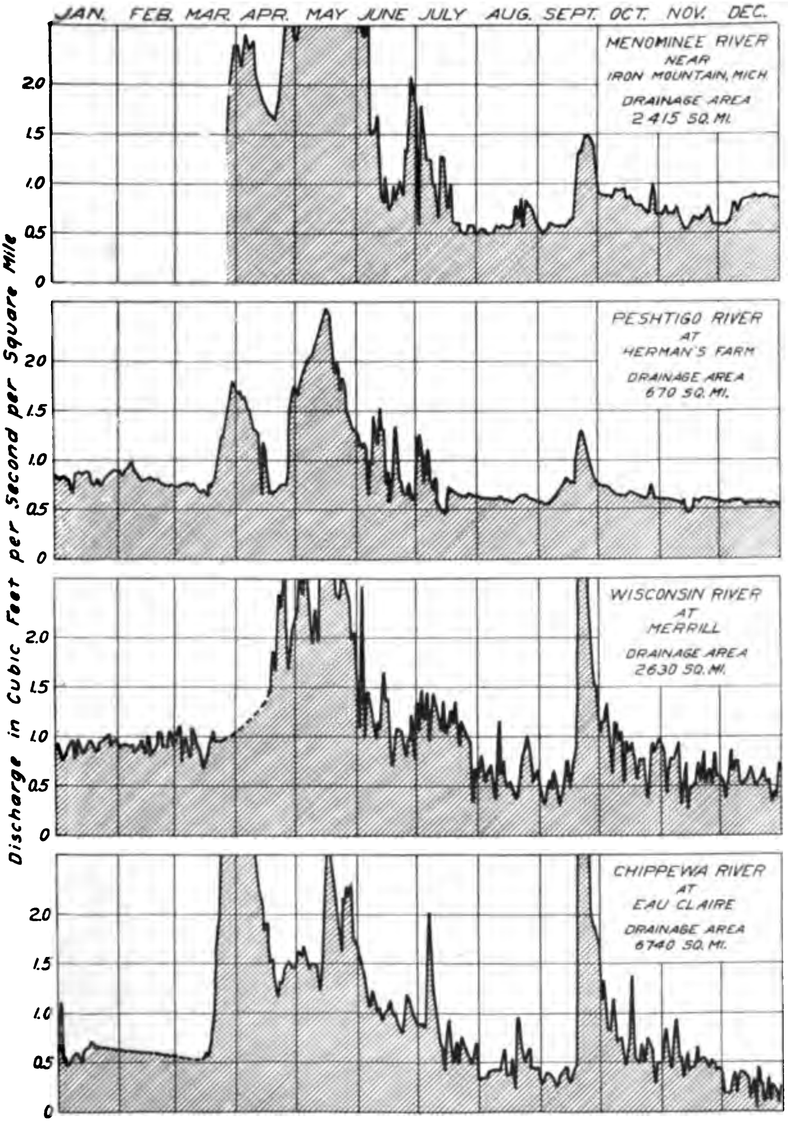


Fig. 62.—Comparative Hydrographs of the Flow of Four Rivers in Wisconsin for the Year 1907.

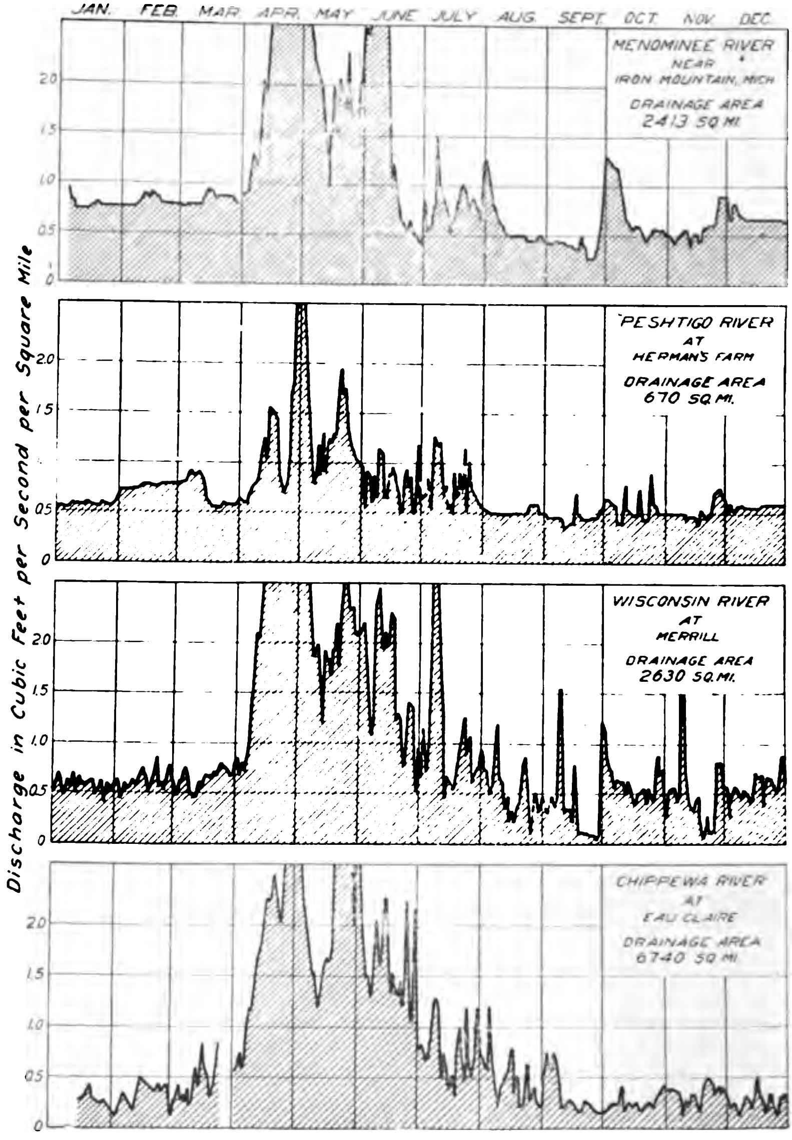


Fig. 63.—Comparative Hydrographs of the Flow of Four Rivers in Wisconsin for the Year 1908.

during different years, the same law is also found in general to apply (see Table 5). It will be seen, however, that even on a single drainage area an increase in total rainfall is not always followed by an increase in the total annual runoff, for other factors, such as distribution of rainfall, temperature, vegetation, evaporation, etc., may and do have an important modifying effect.

ANNUAL PRECIPITATION RUNOFF RELATIONS OF VARIOUS WISCONSIN RIVERS

Figure 64 gives a graphical expression of the annual relations of rainfall-runoff on various Wisconsin streams. Both rainfall and runoff are expressed in inches in depth on the drainage area above the point where the stream flow was measured. The location of each symbol on the diagram shows the amount of rainfall and of runoff, and the resulting percentage relations for a particular stream for a particular year.

From this diagram it will be noted that on Wisconsin streams the percentage of runoff to rainfall is sometimes as low as 16% and sometimes as high as 70%. As a general rule, the rivers of the state may be divided into two groups—one group, which includes the Menominee, Chippewa, Flambeau, Peshtigo, and Wisconsin (rivers which rise in the granitic highlands of the state), averages about 55% runoff; the other group, which includes the Rock, St. Croix, and Fox rivers—which originate on territory underlaid by later geological deposits—averages about 27%. The departure in each group from the average conditions may be regarded as the measure of the effect of other factors besides quantity of precipitation in affecting runoff.

Table 4.—Showing Mean Annual Rainfall and Runoff, and the Percentage of Runoff

River.	Years.	Mean Rainfall Inches.	Mean Run-off Inches.	Runoff in % of Rainfall.
Muskingum	1875-1888	39.7	13.1	33.0
Geneseo, N. Y.	1890-1898	40.3	14.2	35.1
Croton, N. Y.	1877-1899	49.4	22.8	46.2
Lake Cochituate, Mass	1863-1900	47.1	20.3	43.0
Sudbury, Mass	1875-1900	46.1	22.6	48.9
Mystic Lake, Mass	1878-1895	44.1	20.0	45.2
Neshaminy Creek, Pa.	1884-1899	47.6	23.1	47.4
Hudson, N. Y.	1888-1901	44.2	23.3	52.5
Pequannock	1891-1899	46.0	26.8	57.2
Connecticut	1872-1885	43.0	22.0	51.1
Upper Mississippi	1892-1895	24.6	3.61	14.7
Des Plaines, Ill	1896	39.61	6.7	16.8
Republican, Neb	1898	27.0	.39	1.4
Platte, Neb	1894	12.8	1.00	7.8
South Platte, Neb	1898	17.00	.9	5.3
Wisconsin, Wis	1903-1908	33.71	21.82	64.8
Peshtigo, Wis	1907-1908	24.96	11.02	43.2
Chippewa, Wis	1904-1908	32.54	16.16	49.6
Flambeau, Wis	1903-1904	39.29	19.07	48.5
St. Croix, Wis	1902-1904	51.38	17.11	33.3
Menominee, Wis	1903-1907	32.83	17.54	53.4
Rock, Wis	1904-1908	32.20	6.84	21.2
Fox, Wis	1897-1906	32.58	7.97	24.5

Table 5.—Annual Relation of Rainfall and Runoff on the Fox River

Year.	Rainfall inches.	Runoff inches.	Runoff in % of rainfall.
1897	26.08	6.04	23.2
1898	26.15	5.47	20.9
1899	29.74	6.26	21.1
1900	36.76	6.70	18.2
1901	30.27	8.07	26.7
1902	30.50	6.66	21.8
1903	38.75	9.00	24.1
1904	25.31	9.27	26.2
1905	36.87	11.97	32.3
1906	36.36	10.81	29.8
1907	32.62	10.48	32.1
Annual Relation of Rainfall and Runoff on the Chippewa River			
1903	37.97	21.07	55.6
1904	35.86	16.83	47.0
1905	36.95	17.62	47.7
1906	31.80	16.55	52.0
1907	23.36	14.27	61.0
1908	29.31	10.63	36.3

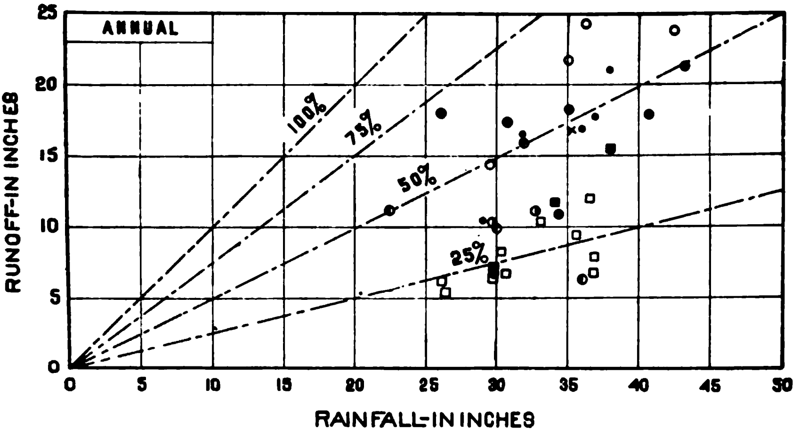


Fig. 64.—Relation of Annual Precipitation and Runoff on Various Rivers of Wisconsin.

Legend for Rainfall Charts

Black River	△
Chippewa River	●
Flambeau River	×
Fox River	□
Menominee River	⊕
Peshtigo River	⦶
Rock River	◑
St. Croix River	⊠
Wisconsin River	○

ANNUAL PRECIPITATION–GAGE HEIGHT RELATIONS ON CERTAIN WISCONSIN RIVERS

The relation between the annual precipitation and the mean annual gage height is shown by Figure 65. On this diagram the relation has been shown by certain characters to which the corresponding year has been indicated. A different character has been used to represent the periods 1873–1885, 1886–1898, and 1890–1909, in order that any marked variation in the relations would be made apparent. A diagonal line has been drawn from 0 through the mean relation for the entire series which shows by its inclination the mean ratio of precipitation to gage height. Two heavy lines have been drawn to mark the extreme variations from the mean. It will be noted that most of the points lie within plus or minus 15% of the mean line. and that the extreme conditions have been +25% and —21%. This diagram shows conclusively that for the last thirty-seven years the mean annual runoff on the Wisconsin River at Portage has varied directly with the total quantity of precipitation within a very limited range, and that all other causes, including distribution of precipitation, temperature, drainage, agricultural development, forest conditions, etc., have not changed this relation usually more than 15% of the mean or in one extreme case 25%. Figure 66 shows similar relations on the Wolf River at New London, Wisconsin.

CHRONOLOGICAL VIEW OF RELATIVE ANNUAL PRECIPITATION AND STREAM FLOW RELATIONS

Figure 67 is a diagram, essentially as prepared by Mr. W. C. Devereaux,* showing some of the annual conditions that have prevailed in the upper Wisconsin valley for the period, as far as available data would permit. On this diagram are shown:

1. Annual rainfall for the fall period.
2. Mean annual runoff at Portage 1873–1909.
3. Extreme low water at Portage 1873–1909.

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*Monthly Weather Review, 1910.

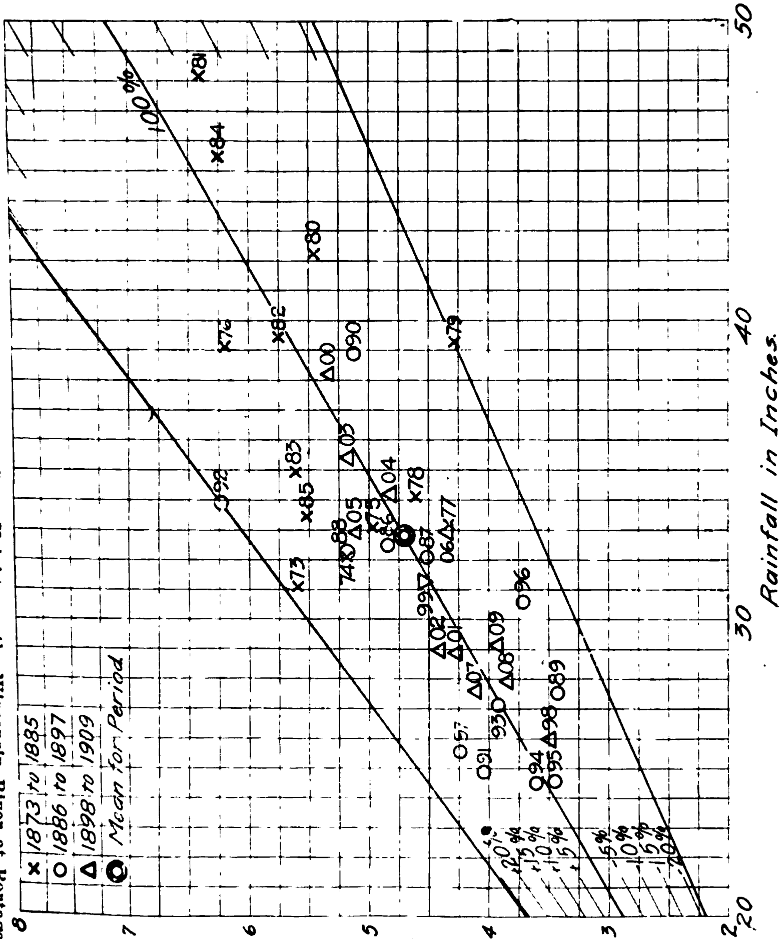


Fig. 65.—Relation of Annual Rainfall to Mean Gage Heights on the Wisconsin River at Portage.

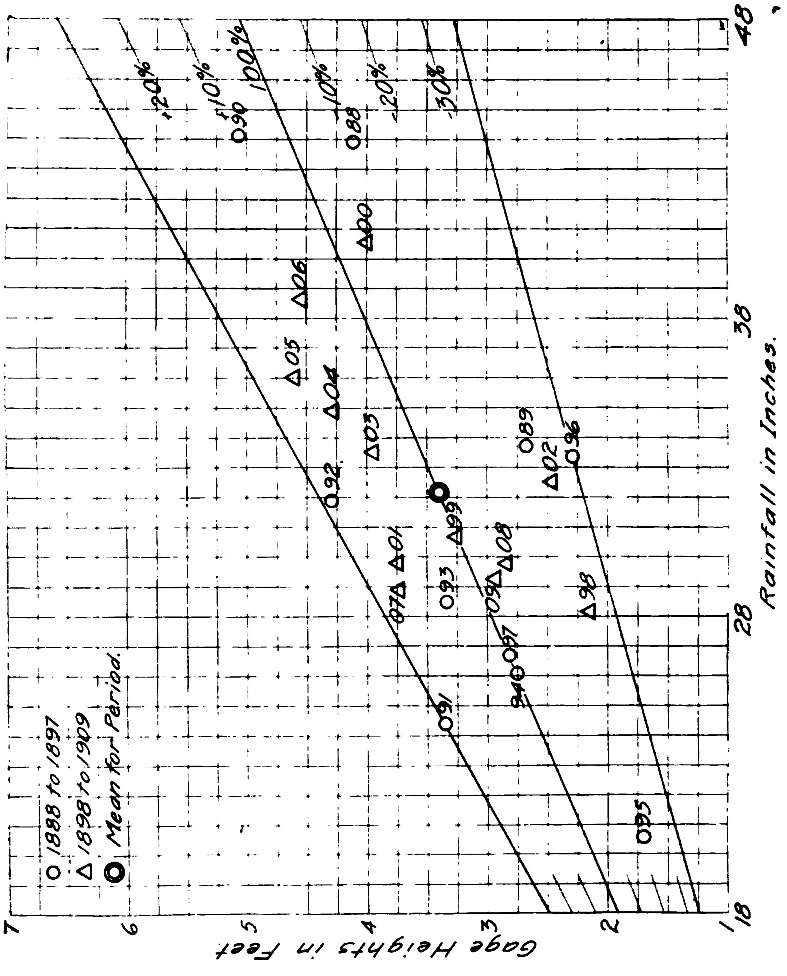
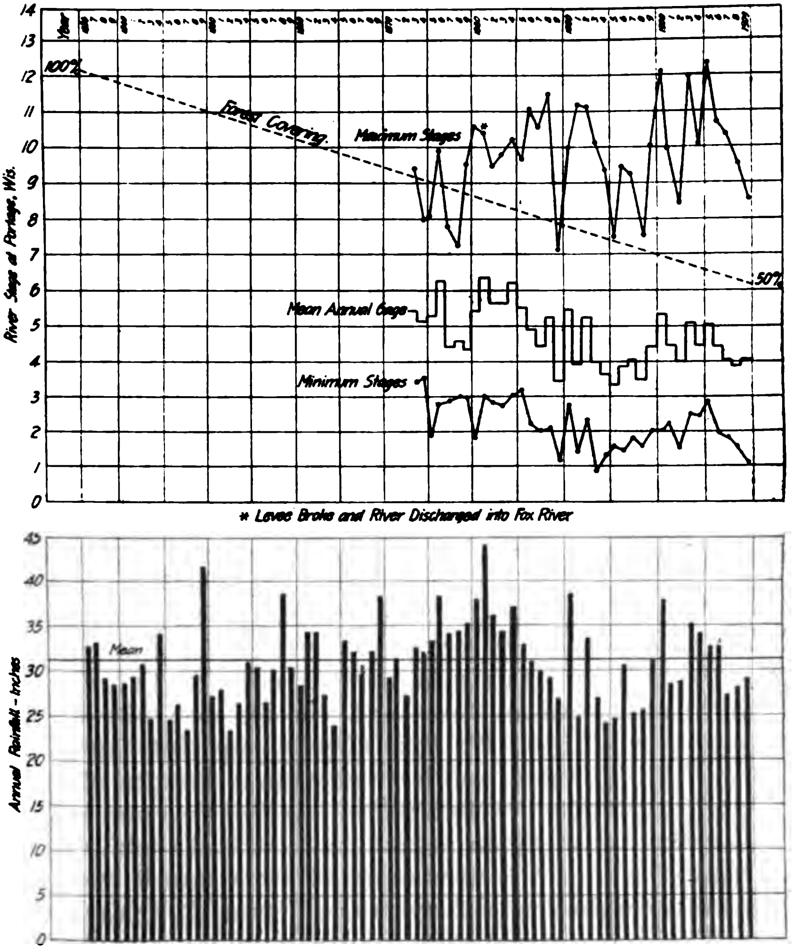


Fig. 68.—Relation of Annual Rainfall to Mean Gage Height on the Wolf River at New London.



* Levee Broke and River Discharged into Fox River

Annual precipitation and progress of deforestation, upper Wisconsin River Valley, and river gage record at Portage between April 1 and November 30

FIG. 67.

4. Extreme high water at Portage 1873–1909.
5. Approximate removal of the forests.

The flow data on this diagram are derived from the same source as are the series of hydrographs shown in Figures 77 to 85 and Tables 9 and 10. On this diagram it will be noted that the change in the average stage of water follows closely the change in the annual rainfall. In general, the change in rainfall has been a decrease since gagings were first taken at Portage, and as lumbering operations have caused a decrease in the forests, the question might probably be raised on the basis of this diagram as to whether the decrease in the average river stage has not been due partially at least to deforestation.

A Study of Relative Precipitation and Stream Flow Conditions

The effect, if any, of human agencies can be made evident in various ways. While lumbering and agricultural improvement were already developed in 1873, they have progressed to a much greater extent in the years that have since elapsed. Their effect, therefore, if any, should be shown by a change in the rainfall-runoff relations which should be manifest by a comparison of the earlier and later periods. It is often claimed that on account of these various changes due to human agencies that the extreme high water is increasing and that the low water is constantly becoming lower, and that the regularity of the stream is much less. We have already seen that these conditions of high, mean, and low water are a function of total precipitation and of its distribution, and that the mean annual flow varies, except within narrow limits, with the total annual precipitation. As a further study of these annual relations, the writer has prepared a diagram (see Figure 68) which shows:

1. The annual precipitation for each year, the mean for the period, and the progressive means during the same period.
2. The annual maximum gage height, the mean of the maximum annual gage height for the period, and the corresponding progressive means of maximum gage heights.
3. The mean annual gage height, the mean of the mean annual gage heights for the period, and their progressive means.

4. The minimum annual gage height, the mean of the annual gage heights, and their progressive means.

From this diagram it will be noted that the progressive means of the mean annual gage height and of the extreme low water, which conditions of flow are the result of continuous conditions during the year, follow closely the trend of the progressive average of the total annual rainfall. The extreme high water conditions on the contrary, being due to immediate conditions favorable to such results, are very irregular and do not follow so closely the variation in annual rainfall conditions. It is also evident that the relations during the earlier period are not markedly different from the later period. Figure 69 is a similar diagram for the Wolf River, and Figure 70 for the upper Mississippi River, both of which bear out the same relations shown by the Wisconsin River diagram.

A STUDY OF PRECIPITATION–GAGE HEIGHT RATIOS

Figure 71 is a chronological charting of the departure of the ratio of $\frac{\text{Rainfall}}{\text{Gage Height}}$ for each year. In this diagram the lower ratios show the better flow conditions (see Table 6). For example, the ratio of 30 inches of rainfall to 5 feet or 6 feet gage height would be 6 and 5 respectively. The ratio 5 here represents the mean annual gage height of 6 feet which consequently represents a higher flow than gage height 5 and ratio 6.

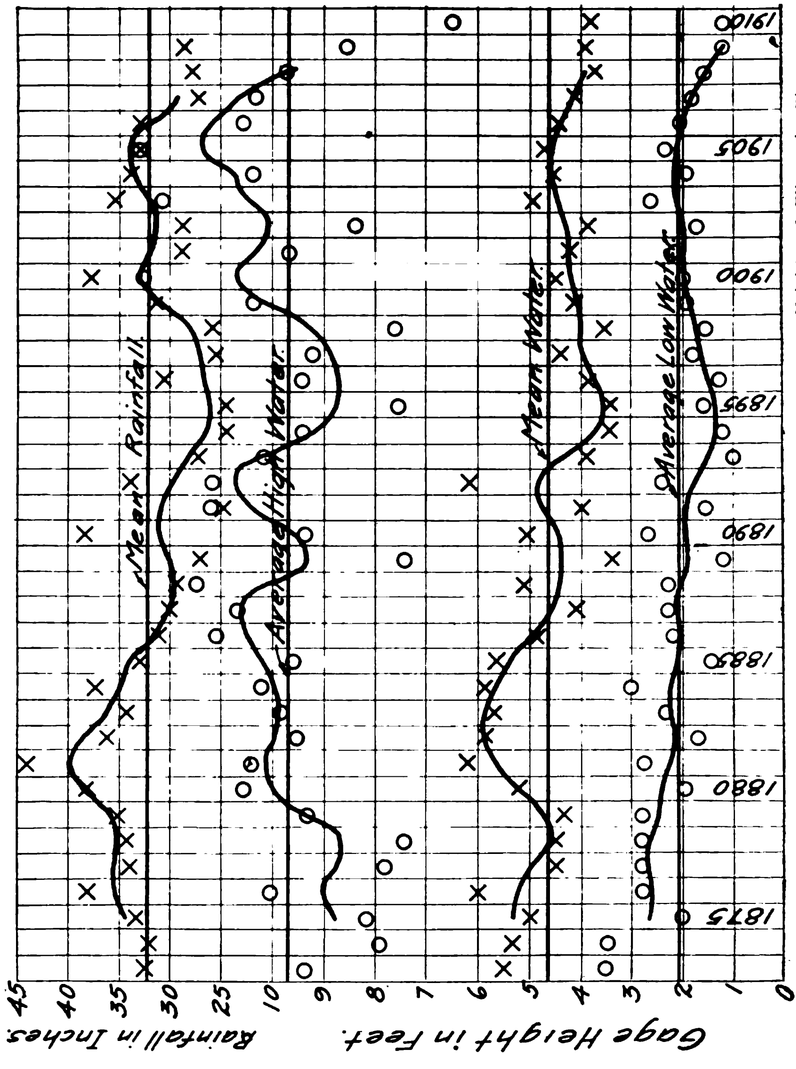


Fig. 68.—Comparison of Mean Annual Rainfall with Mean and Extreme Gage Heights of Wisconsin River at Portage, Wisconsin.

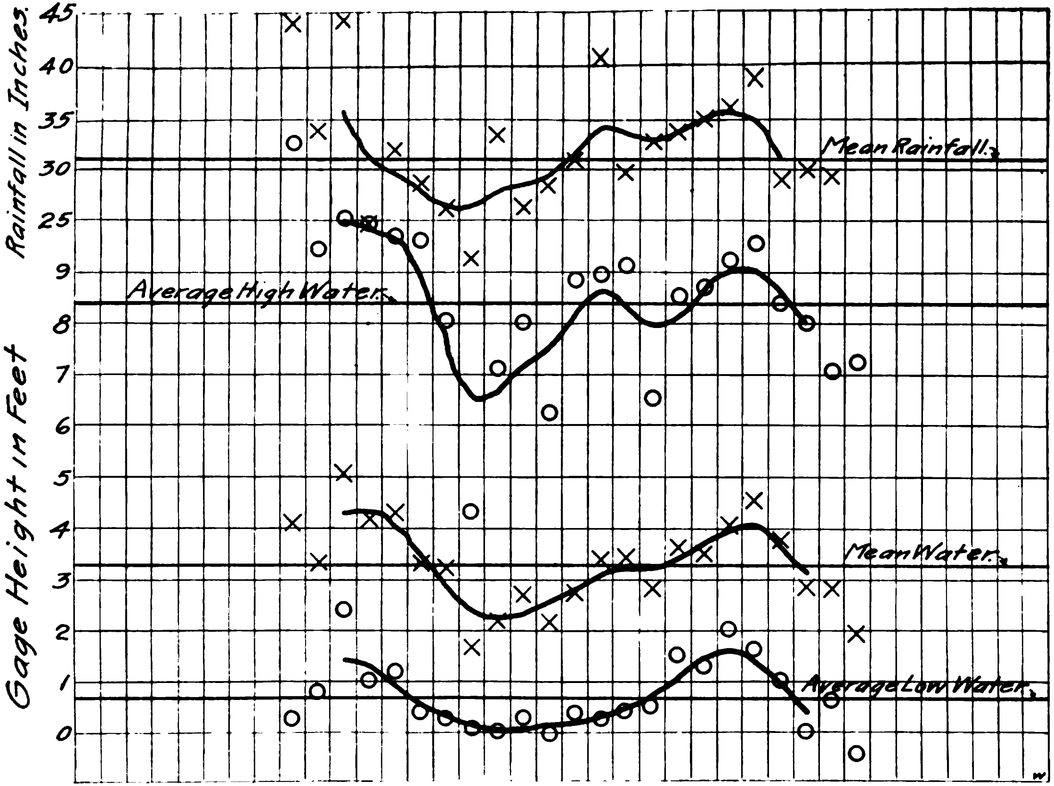


Fig. (8).—Chronological Comparisons of Mean Annual Precipitation with Mean and Extreme Gage Heights at New London, Wisconsin, on the Wolf River.

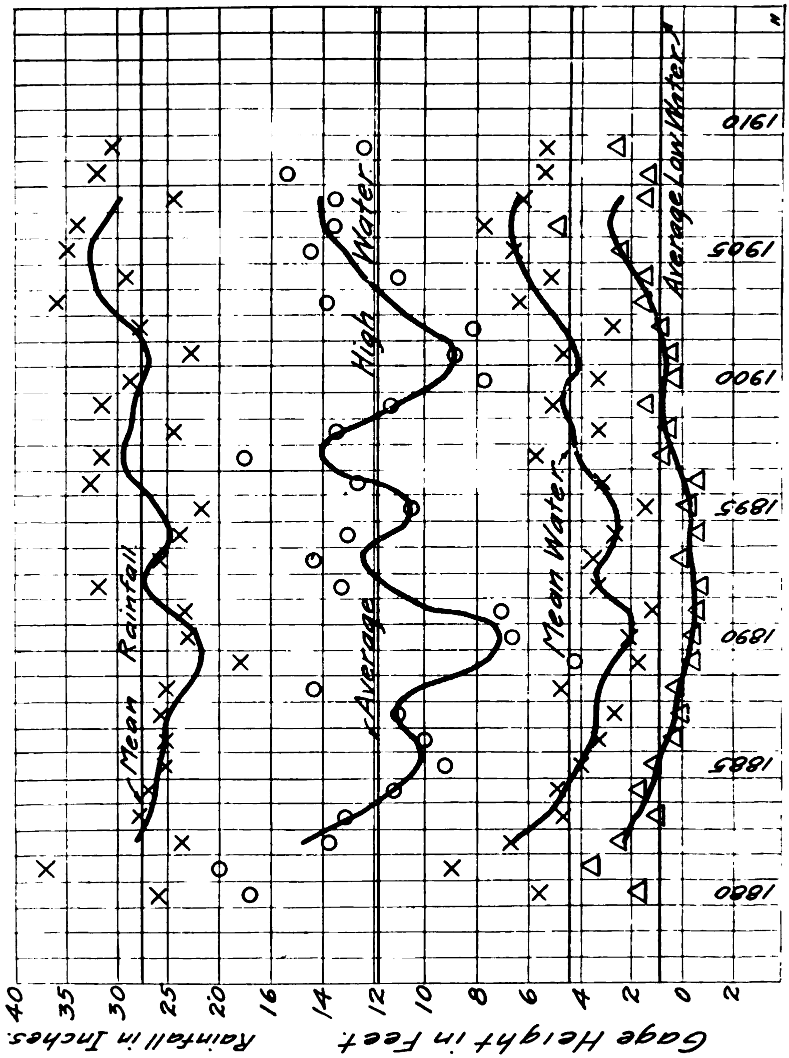


Fig. 70.—Comparison of Mean Annual Precipitation on the Mississippi Drainage Area Above Hastings, Minnesota, with Gage Heights at Hastings.

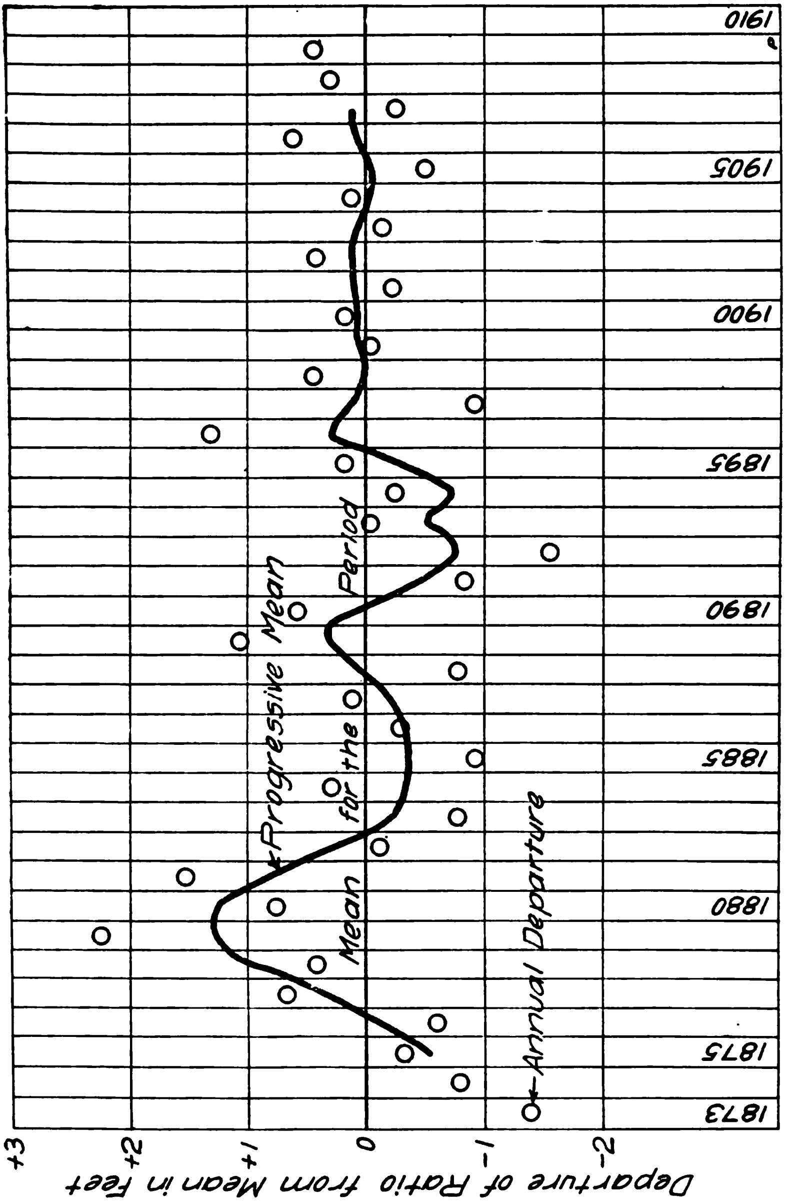


Fig. 71.—Chronological Comparison of Ratios of Annual Precipitations to Mean Annual Gage Heights on the Wisconsin River at Portage.

TABLE 6—RAINFALL AND GAGE HEIGHTS ON THE WISCONSIN RIVER ABOVE PORTAGE, WITH RATIOS—DEPARTURE AND PROGRESSIVE MEAN.

Year.	Rainfall.	Gage height.	Ratio.	Departure.	Progressive Mean.
1873	31.11	5.60	5.57	—1.41	
1874	32.08	5.20	6.18	— .80	
1875	33.06	4.97	6.66	— .32	—0.53
1876	39.14	6.20	6.32	— .66	—0.18
1877	33.20	4.35	7.65	+ .67	+0.31
1878	34.10	4.61	7.40	+ .42	+0.90
1879	39.28	4.26	9.25	+2.27	+1.29
1880	42.26	5.46	7.76	+ .78	+1.27
1881	48.15	6.40	7.53	+1.55	+0.84
1882	39.44	5.74	6.87	— .11	+0.22
1883	34.90	5.62	6.21	— .77	—0.21
1884	45.48	6.25	7.26	+ .28	—0.34
1885	33.47	5.52	6.05	— .93	—0.39
1886	32.51	4.86	6.68	— .30	—0.31
1887	32.09	4.52	7.10	+ .12	—0.22
1888	32.24	5.20	6.20	— .78	+0.02
1889	27.58	3.43	8.04	+1.06	+0.31
1890	38.82	5.13	7.56	+ .58	+0.13
1891	24.89	4.05	6.14	— .84	—0.49
1892	33.96	6.23	5.44	—1.54	—0.78
1893	27.16	3.92	6.92	— .06	—0.51
1894	24.51	3.62	6.75	— .23	—0.75
1895	24.60	3.43	7.16	+ .18	—0.75
1896	30.64	3.70	8.29	+1.31	—0.75
1897	25.58	4.23	6.04	— .94	+0.29
1898	25.92	3.48	7.44	+ .46	+0.31
1899	31.31	4.53	6.92	— .06	+0.10
1900	38.22	5.33	7.17	+ .19	+0.02
1901	28.84	4.27	6.76	— .22	+0.07
1902	28.95	3.88	7.44	+ .46	+0.06
1903	35.42	5.17	6.86	— .12	+0.07
1904	34.18	4.80	7.11	+ .13	+0.11
1905	32.98	5.08	6.47	— .51	+0.06
1906	32.98	4.33	7.61	+ .63	—0.04
1907	27.67	4.12	6.72	— .26	—0.01
1908	27.93	3.83	7.28	+ .30	+0.07
1909	29.12	3.92	7.42	+ .44	+0.13

The departure from the mean of these ratios is plotted for the period 1873–1910 on this diagram, and the progressive mean during the period is shown by the irregular heavy line. The position of this line below or above the mean line for the period shows a corresponding better or poorer condition of flow. If the forests or other early and normal conditions were favorable to increased stream flow then the $\frac{\text{Rainfall}}{\text{Gage Height}}$ ratio should be smaller in the earlier years and the line of the progressive mean should lie below the "mean line for the period." If deforestation has decreased this ratio, then the progressive mean of the ratio should lie above the "mean for the period" during the later years. If the forests or other early normal conditions served to regulate stream flow and in consequence of the change, the regulation has been injuriously affected, then the progressive mean should agree more closely with the "mean for the period" during the earlier, than during the later years.

The diagram shows clearly that the greatest extreme in these ratios took place in the first half of the period, the poorest ratio being in 1874 and the best in 1892. There are no indications that the ratios as a whole were uniformly better during the earlier years. The uniformity of flow during the last fifteen years is much greater than at any time during the twenty-two years preceding. Similar diagrams for the Wolf River at New London (Figure 72) and for the upper Mississippi River at Hastings (Figure 73) show that in neither of these cases was the ratio of precipitation to stream flow better in the earlier condition than at the present time.

Another view of this problem is shown in Figure 74. Here are shown the annual departures from the mean for the year of the annual ratios of maximum to mean, and of mean to minimum gage heights on the Wisconsin River at Portage.

This diagram indicates that the ratios of the flood peaks have been slightly higher on the Wisconsin River during the later years, but that the low water ratio has remained essentially constant for the entire period. Similar diagrams for the Wolf River (Fig. 75) and for the upper Mississippi River (Fig. 76), show considerable variation in the ratio of the high water peaks and no tendency toward an increase. On the Wolf River, the ratio of both high and low water conditions is now about at the average. On the Mississippi River, the ratio of high water condition has been less during the later years, and the low water conditions have been more favorable. This may be partially due to the reservoir systems on the upper Mississippi, yet the conditions during the years 1885–1900, when the reservoir system was also in use, do not indicate that this is the case.

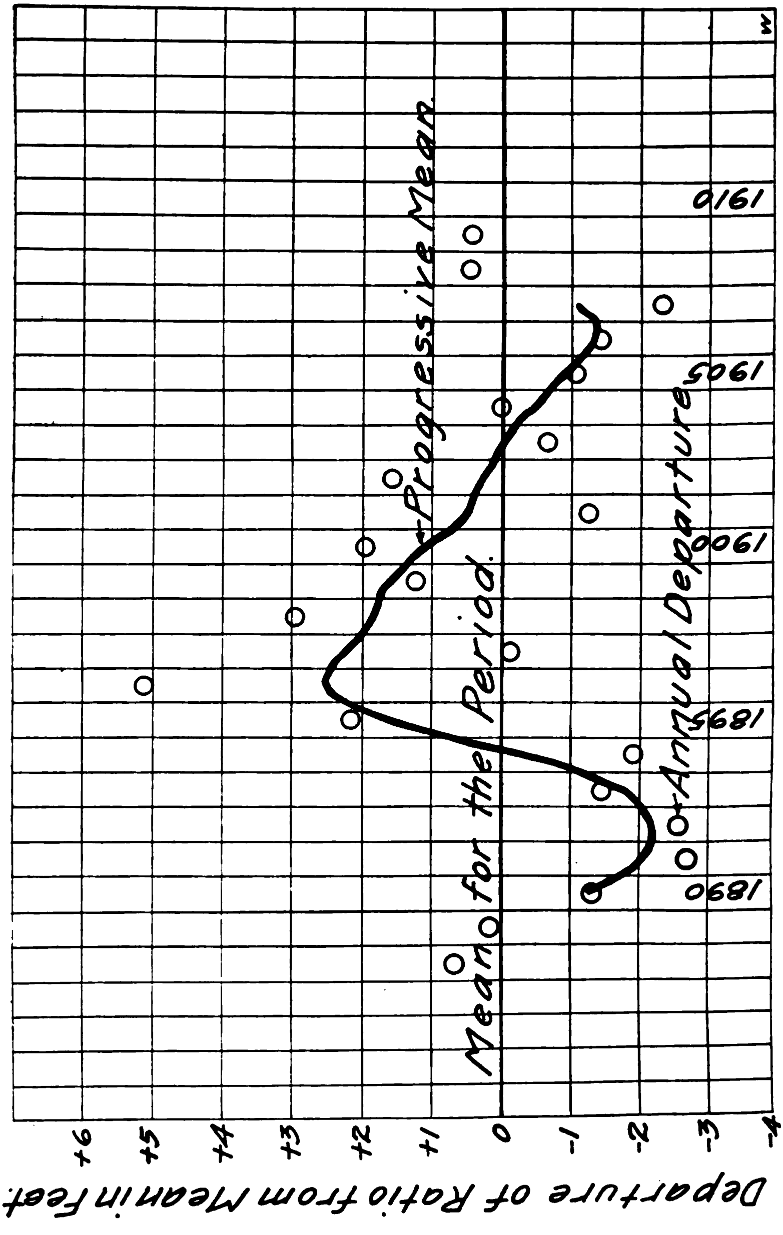


Fig. 72.—Chronological Comparisons of Ratios of Annual Precipitation to the Mean Annual Gage Height on the Wolf River at New London, Wisconsin.

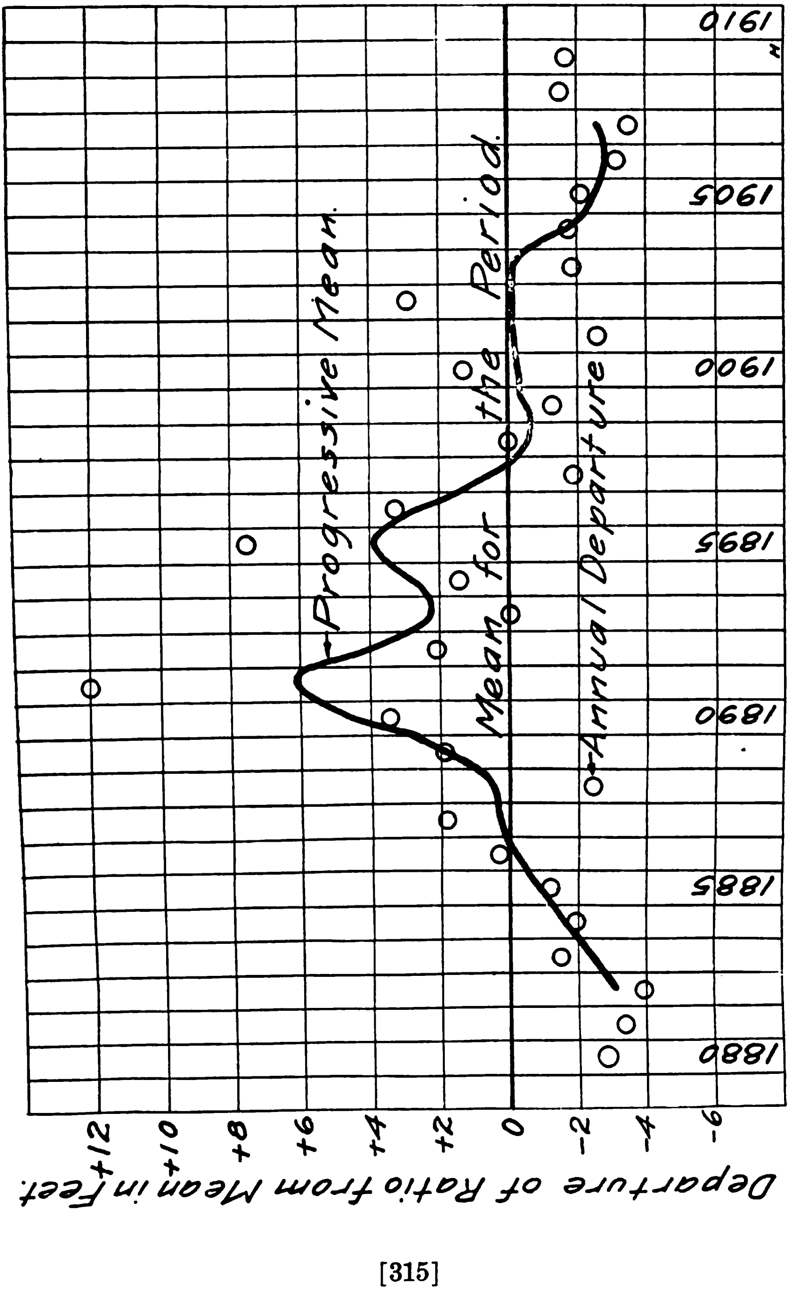


Fig. 78.—Chronological Comparison of Ratios of Annual Precipitation on the Upper Mississippi River Drainage Area Above Hastings, Minnesota, to the Mean Annual Gage Height at Hastings.

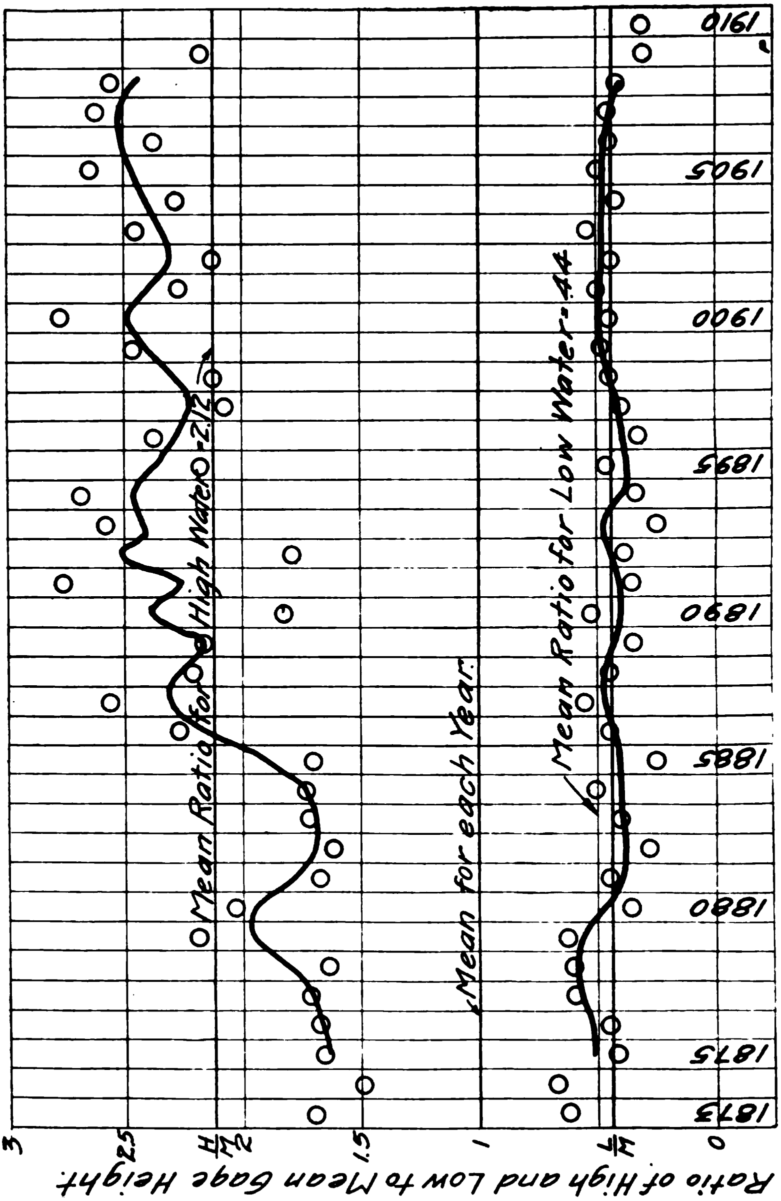


Fig. 74.—Chronological Comparisons of Annual Ratios of Extreme and Mean Gage Heights on the Wisconsin River at Portage.

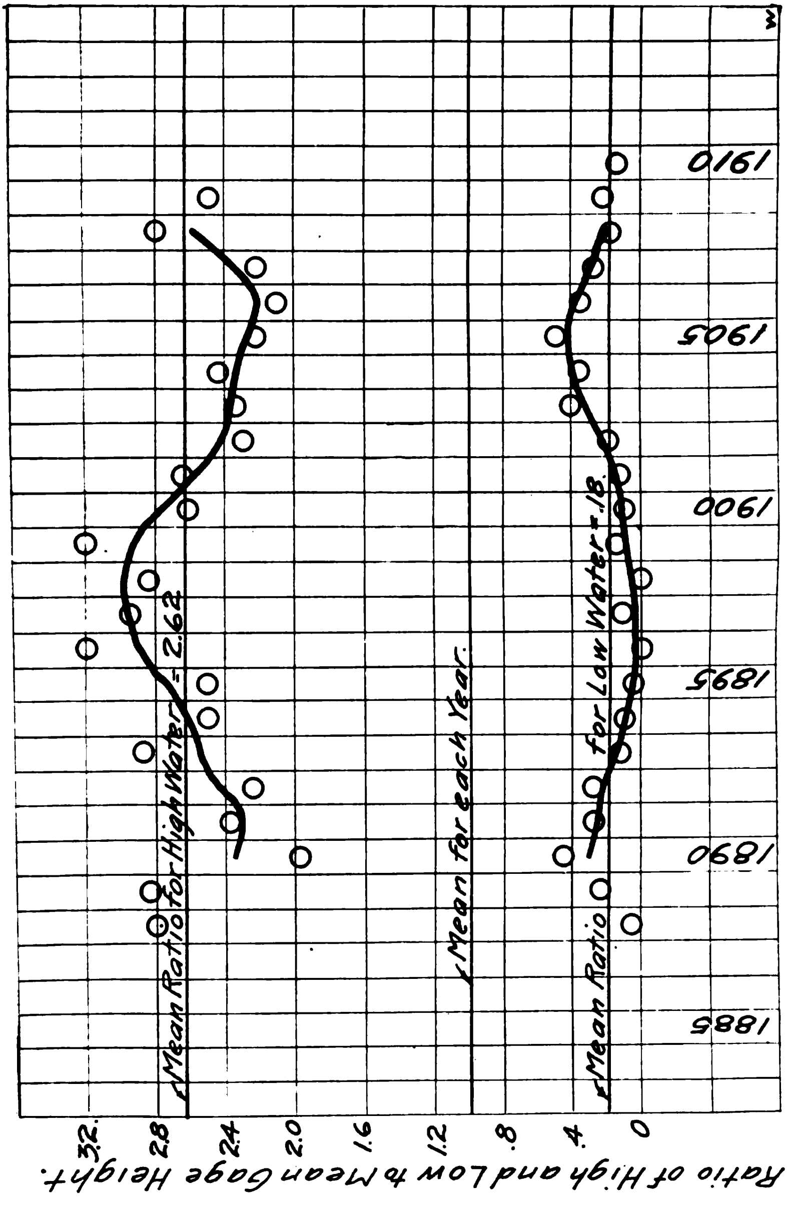


Fig. 75.—Chronological Comparisons of Annual Ratios of Extreme and Mean Gage Heights on the Wolf River at New London.

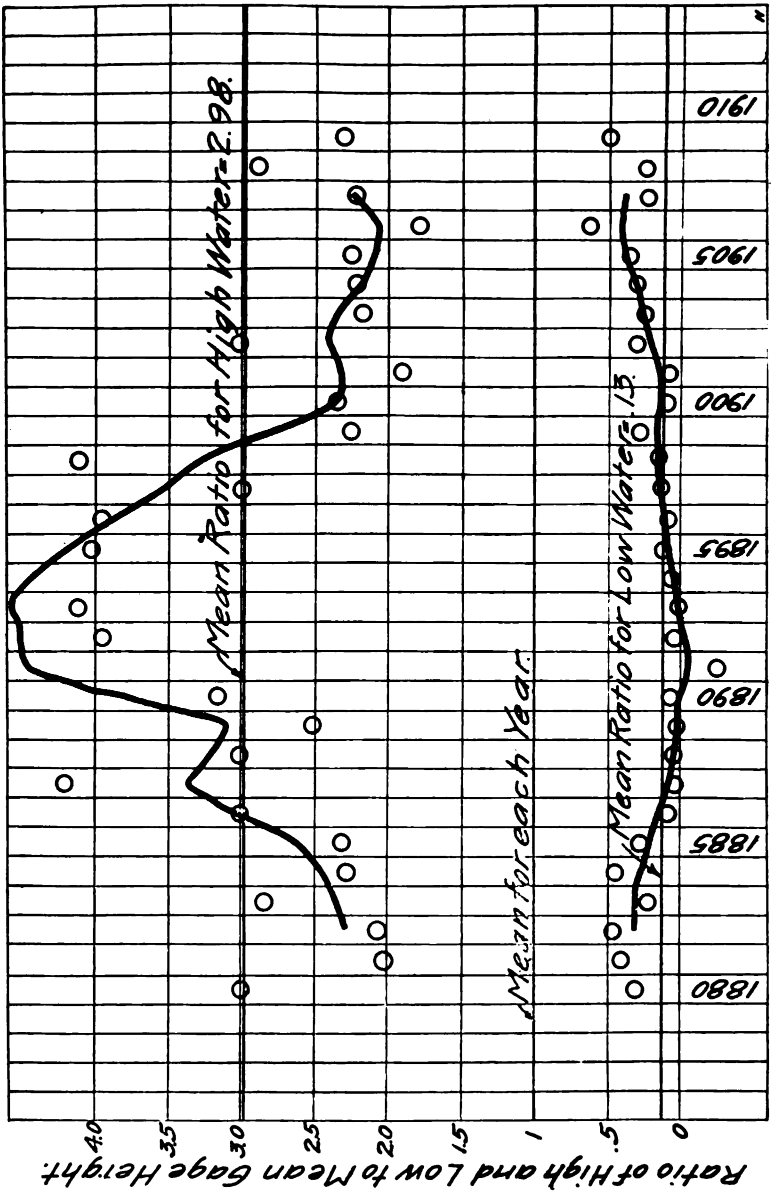


Fig. 76.—Chronological Comparisons of Annual Ratios of Extreme and Mean Gage Heights on the Mississippi River at Hastings, Minnesota.

CONCLUSIONS

From the preceding somewhat detailed study of the general relations to runoff or stream flow conditions in Wisconsin, the following conclusions can be clearly drawn

Relating to stream flow in general:

1st. That in general the quantities and variations of stream flow are due to a large number of concomitant factors, some fixed and some variable, each having an influence in proportion to its own character and intensity and the relative character and intensity of every other factor. The combinations of these factors are so numerous, and differ so widely even on a single drainage area, that no problem of flow can be determined even approximately from a consideration of the factors themselves except by means of a comparison with the actual relative effects under circumstances more or less similar to those under investigation.

2nd. That the difference between the flows of Wisconsin streams having essentially the same quantity and distribution of precipitation, depends primarily on geological and topographical conditions, including the relative amounts of natural storage resulting therefrom.

3rd. That the mean annual stream flow on any stream, and the variation in the stream flow throughout the year, depend mainly on the quantity and distribution of the precipitation, modified by temperature conditions.

4th. That extreme floods are due to the simultaneous occurrence of temperature and precipitation conditions, favorable to excessive runoff, and that no practicable works of man, such as drainage works, reservoirs, or reforestation, can materially affect the flood peaks on the larger Wisconsin rivers.

Relating to the regulation of stream flow by reservoirs:

5th. That the ideal regulation as indicated by the Hudson River hydrograph (Figure 31) can never be realized on Wisconsin streams, and seldom be approximated except on small streams of exceedingly limited area.

6th. That the topography of Wisconsin is entirely unsuited to the construction of extensive reservoir systems that will materially affect flood peaks or largely equalize the high and low water flow of the larger streams of the state.

7th. That the uses for which reservoir systems may be constructed are essentially antagonistic and that the specific use to which they should be applied must depend on the local conditions which may modify the relative values of the services which they can fulfill.

8th. That on many drainage areas of Wisconsin where water powers have been developed, reservoir systems, which will materially increase the low water flow of the stream, are now financially practicable, but that every stream constitutes a problem in itself which needs independent study and investigation, and no general conclusion can be safely drawn in regard to the practicability of such reservoir systems on Wisconsin streams.

Relating to the effect of forests on stream flow:

9th. That if forests have any effect on stream flow that can be assigned a practical value, such effects must be manifest by an actual change in the conditions of flow when deforestation occurs, and that if deforestation is not accompanied by an actual change in the stream conditions, the effects are entirely theoretical and practically valueless.

10th. That in general the deforestation or cutting of timber in Wisconsin has had no material effect, either favorable or adverse, on the high water, mean water, or low water flow of the streams, or on the regularity of such flow.

11th. That on the Wisconsin River the changes in the original conditions due to drainage, deforestation, etc., have not affected the mean and low water flow but may have slightly improved the regularity of the ratio of precipitation to runoff, and may have slightly increased the ratio of high water peak to mean water; but that such effects have been insignificant and of no material advantage or disadvantage to the flow of the stream, and that the flow of the Wolf River and of the Upper Mississippi River shows no such changes in flood peaks or betterment in regularity.

12th. That while theoretically forests will decrease the amount of precipation reaching the earth's surface—thus decreasing the water supply of streams—and, under certain circumstances may afford a limited storage which may aid in the regulation of stream flow, yet the examination of the actual flow of the streams of Wisconsin shows no indication of such effects.

13th. That if any effects on stream flow have resulted from deforestation they have been entirely counteracted and obscured by the drainage of marshes, the clearing of farm lands, the second growth of timber and brush, or other similar occurrences.

14th. That so far as all evidence examined is concerned, there is nothing to show that the planting of forests in Wisconsin would in any way add to or take away from the quantity or regularity of stream flow, or decrease the flood heights.

15th. That therefore the reforestation of the headwaters of Wisconsin streams cannot be expected to add to the quantity or regularity of the power which may be developed on any stream, or to change or improve the flow of the stream for any other useful purpose.

Finally:

16th. That the changes and variations in the flow ratio in Wisconsin rivers are due to undetermined factors which modify the precipitation-runoff relations as similar indeterminate factors modify the amount and distribution of the annual rainfall. Such phenomena occur in cycles and are not found to be continuously favorable or unfavorable to the quantity or regularity of stream flow.

17th: That the flows of Wisconsin streams as shown by their total annual mean annual, and maximum and minimum flows, have neither increased or decreased during the last thirty-five or forty years except as such increase or decrease has been caused by the annual and seasonal variation in the quantity and distribution of the rainfall, together with the modifying effects of the accompanying temperature.

NOTE 1: The following formula for progressive means has been used:

$$\frac{a + 4b + 6c + 4d + e}{16} = c'$$

where c' is the progressive average of the middle year of the group. As far as procurable rainfall records of three or more stations for each area have been averaged in plotting the progressive curves. The data for Southeastern New England, Upper Ohio Valley, and Middle Mississippi Valley have been taken from the Report of the Chief of the Weather Bureau, 1896–97. The records for the above sections from 1896 to date, and for Wisconsin, Eastern South Carolina, and the Great Valley, California, have been compiled from the Summaries of the Climatological Data, U. S. Department of Agriculture, and from The Monthly Weather Reviews. Records for the last section have been tabulated on pages 159–160. The stations from which such records were procured are given below:

(A) Southeastern New England—New Bedford (1817–1909). Boston (1820–1909). Providence (1834–1909).

(B) Upper Ohio Valley—Marietta (1830–1909). Portsmouth (1832–1909). Cincinnati (1837–1909).

(C) Wisconsin—(a) Fort Winnebago (Portage) (1836–1845). Janesville (1854–1858). Rockrun (1861–1868). Madison (1869–1909). (See Table 7.)
- (b) Fort Crawford (Prairie du Chien) (1836–1844). Dubuque (1854–1875). McGregor (1876–1890). Prairie du Chien (1891–1909).
- (c) Fort Howard (Green Bay) (1836–1840). Embarrass (1864–1866).
- (d) Fort Snelling (1837–1857). St. Paul (1859–1909).
- (e) Milwaukee (1844–1909).
- (f) Superior City (1860–1866). Duluth (1871–1909).

(D) Middle Mississippi Valley—Muscatine (1849–1909). Monticello (1857–1909). Marengo (1858–1909). Peoria (1858–1909).

(E) Great Valley, California — Sacramento (1850–1909). Stockton (1850–1909).

(F) Eastern South Carolina—Charleston (1738–1765), (1785–1791), (1807–1811), (1832–1909).

NOTE 2:—Fig. 4 has been produced from the rainfall tables included in Table 7 which were prepared by Mr. W. C. Devereaux, Local Forecaster U. S. Weather Bureau, Milwaukee, Wisconsin. The progressive means were calculated by use of the formulæ given in Note 1.

TABLE 7.—ANNUAL PRECIPITATION.

Year.	(a) Madison.	(b) Prairie du Chien.	(c) Embarrass (Green Bay).	(d) St. Paul.	Milwaukee.	(e) Duluth.	Manitowoc.	La Crosse.	Means.
1836	34.1	27.2	37.7	......	......	......	......	......	33.0
1837	31.4	34.0	40.5	24.1	......	......	......	......	32.5
1838	28.0	22.0	37.6	27.7	......	......	......	......	28.8
1839	28.9	31.9	31.3	21.2	......	......	......	......	28.3
1840	27.1	31.3	30.5	23.1	......	......	......	......	28.0
1841	28.4	31.1	......	21.6	34.2	......	......	......	28.8
1842	24.3	38.4	......	26.7	......	......	......	......	29.8
1843	22.8	25.5	......	23.8	......	......	......	......	24.0
1844	33.1	30.2	......	30.4	32.5	......	......	......	33.8
1845	25.9	......	......	25.5	20.5	......	......	......	21.0
1846	......	......	......	26.0	25.2	......	......	......	25.6
1847	......	......	......	21.9	22.4	......	......	......	22.2
1848	......	......	......	23.2	33.4	......	......	......	28.3
1849	......	......	......	49.8	31.0	......	......	......	40.4
1850	......	......	......	25.5	26.3	......	......	......	25.9
1851	......	......	......	23.4	30.4	......	......	......	26.9
1852	......	......	......	15.0	29.4	......	......	......	22.2
1853	......	......	......	20.4	*30.0	......	......	......	25.2
1854	36.4	27.7	......	26.5	31.7	......	......	......	30.6
1855	31.1	29.1	......	24.9	36.0	......	......	......	30.3
1856	22.1	*32.0	......	22.7	29.0	......	......	......	26.4
1857	27.4	29.0	......	32.1	30.8	......	......	......	29.8
1858	42.6	47.3	......	*20.1	45.1	......	......	......	38.8
1859	*30.0	*32.4	......	29.1	28.9	......	......	......	30.1
1860	*29.7	26.9	......	34.2	22.0	27.7	......	......	28.1
1861	36.5	35.8	......	30.5	31.9	35.4	......	......	34.0
1862	38.8	34.9	......	34.5	38.8	22.8	......	......	34.0
1863	31.3	33.9	......	15.7	31.7	17.8	33.9	......	27.4
1864	20.5	25.0	28.9	14.9	27.8	20.9	26.8	......	23.5
1865	31.6	36.2	36.3	38.1	30.1	30.0	31.6	......	33.4
1866	37.7	36.4	35.4	27.9	34.0	25.7	27.2	......	32.0
1867	29.9	34.8	29.0	33.6	24.5	*30.4	24.3	......	29.5
1868	27.5	33.4	38.8	30.7	29.2	*32.3	30.5	......	31.8
1869	46.8	39.0	39.3	32.2	37.8	*32.0	38.4	......	37.9
1870	28.1	25.1	*31.9	32.1	26.5	*29.0	28.6	......	29.0
1871	29.4	*31.1	37.7	30.7	32.0	31.2	28.4	......	31.5
1872	22.4	*28.4	28.5	29.6	26.3	30.1	23.8	......	27.0
1873	27.5	*31.7	35.0	34.6	30.6	38.7	34.3	......	32.0
1874	29.0	30.8	31.0	35.5	30.8	36.4	28.1	31.9	31.7
1875	22.8	35.5	43.9	30.7	35.6	26.9	30.3	38.7	33.0
1876	36.3	40.1	48.9	23.6	50.4	32.3	32.6	39.4	38.0
1877	27.7	31.0	34.4	28.7	46.2	34.3	28.2	36.6	33.4
1878	38.9	39.9	37.6	22.6	38.3	28.1	33.0	31.4	33.7
1879	35.2	33.1	41.6	32.5	24.9	45.3	27.2	34.9	34.4
1880	46.7	39.2	49.8	29.8	30.0	38.1	30.7	34.3	37.3
1881	52.9	46.6	*47.4	30.2	39.1	37.6	37.6	44.7	43.4
1882	42.9	36.6	49.0	23.1	28.4	38.0	40.1	22.9	35.8
1883	39.9	39.0	42.2	26.5	29.5	23.2	38.1	33.7	34.0
1884	49.2	36.2	*42.1	26.1	30.6	35.4	40.5	35.3	36.9
1885	40.6	34.0	42.6	25.3	32.6	20.0	31.8	30.7	32.2
1886	28.8	26.2	45.4	22.9	31.5	33.4	34.0	22.5	30.6
1887	38.9	33.5	32.6	25.9	30.5	28.6	26.5	17.4	29.2
1888	22.9	29.4	35.5	25.8	23.5	27.3	28.2	34.8	28.4
1889	20.2	23.9	32.6	17.1	31.7	32.0	28.5	24.4	26.3
1890	37.0	39.8	36.2	23.5	30.1	24.1	32.9	34.8	32.3
1891	24.3	22.4	26.0	21.8	29.8	29.5	24.7	27.0	25.7
1892	37.1	41.9	33.0	32.6	35.0	28.5	32.8	36.2	34.6
1893	31.4	29.4	33.0	26.0	32.9	23.3	31.0	24.6	29.0
1894	23.8	20.5	35.9	25.8	27.8	31.7	25.5	22.9	26.7
1895	13.1	18.4	21.0	24.3	24.9	22.3	20.2	23.0	20.9
1896	30.0	33.5	33.1	34.7	29.0	27.3	31.2	29.2	31.0
1897	22.6	29.4	34.1	30.5	31.0	30.9	26.6	21.3	28.3
1898	31.9	24.2	29.1	25.3	32.4	19.7	25.7	23.5	26.5

TABLE 7.—ANNUAL PRECIPITATION.—Continued.

Year.	(a) Madison.	(b) Prairie du Chien.	(c) Embarrass (Green Bay).	(d) St. Paul.	Milwaukee.	(e) Duluth.	Manitowoc.	La Crosse.	Means.
1899	27.0	28.0	25.8	27.5	22.8	30.6	21.3	35.4	27.3
1900	29.6	30.5	32.8	34.2	30.1	23.1	29.3	41.8	31.4
1901	20.2	20.7	26.5	25.8	18.7	26.7	25.9	31.9	24.9
1902	31.2	44.6	27.6	31.8	28.6	26.1	32.9	30.3	31.6
1903	34.8	37.0	28.9	37.9	33.4	28.0	32.2	34.4	33.3
1904	31.2	29.3	30.6	33.8	29.9	24.4	30.3	25.4	29.4
1905	25.5	38.5	31.9	30.8	32.2	35.8	31.8	35.3	32.7
1906	32.4	36.8	37.9	33.2	30.2	28.8	30.5	27.8	32.2
1907	30.3	32.1	27.3	23.1	34.9	23.9	29.7	28.4	28.7
1908	25.7	36.1	21.9	31.6	28.3	31.0	28.5	31.6	29.3
1909	30.8	33.1	26.8	31.8	31.5	33.6	30.9	38.3	32.1
Means	31.1	32.5	34.8	27.7	30.9	29.4	30.2	31.0	30.9

Note:
(a) At Fort Winnebago (Portage) 1836-1845; at Janesville 1854-1858; at Rockrun 1861-1868; at Madison 1869-1909.
(b) At Fort Crawford (Prairie du Chien) 1836-1844; at Dubuque 1854-1875; at McGregor 1876-1890; at Prairie du Chien 1891-1909.
(c) At Fort Howard (Green Bay) 1836-1840; at Embarrass 1864-1866; at Green Bay 1887-1909.
(d) At Fort Snelling 1837-1857, at St. Paul 1859-1909.
(e) At Superior City 1860-1866; at Duluth 1871-1909.
(*) Amount estimated.

APPENDIX A.

THE WISCONSIN RIVER.

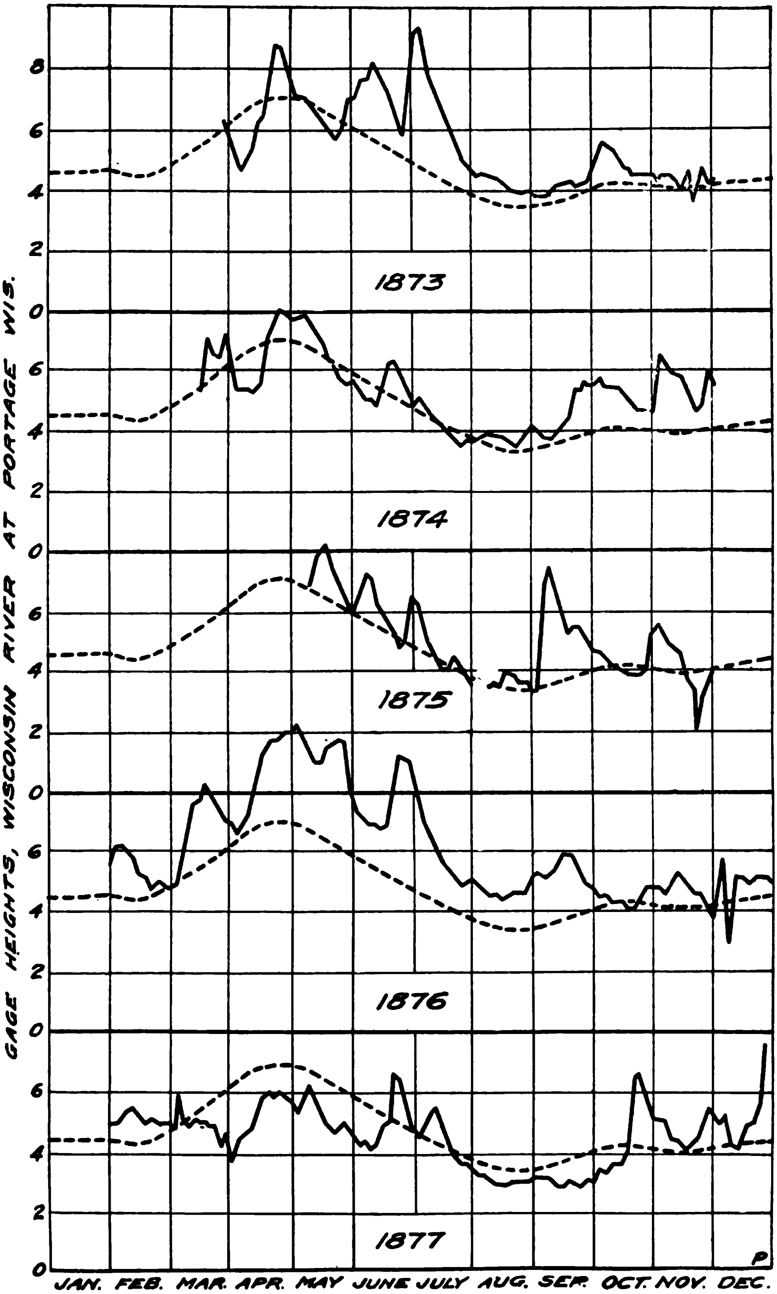


Fig. 77.—Hydrographs of Daily Flow of the Wisconsin River at Portage.

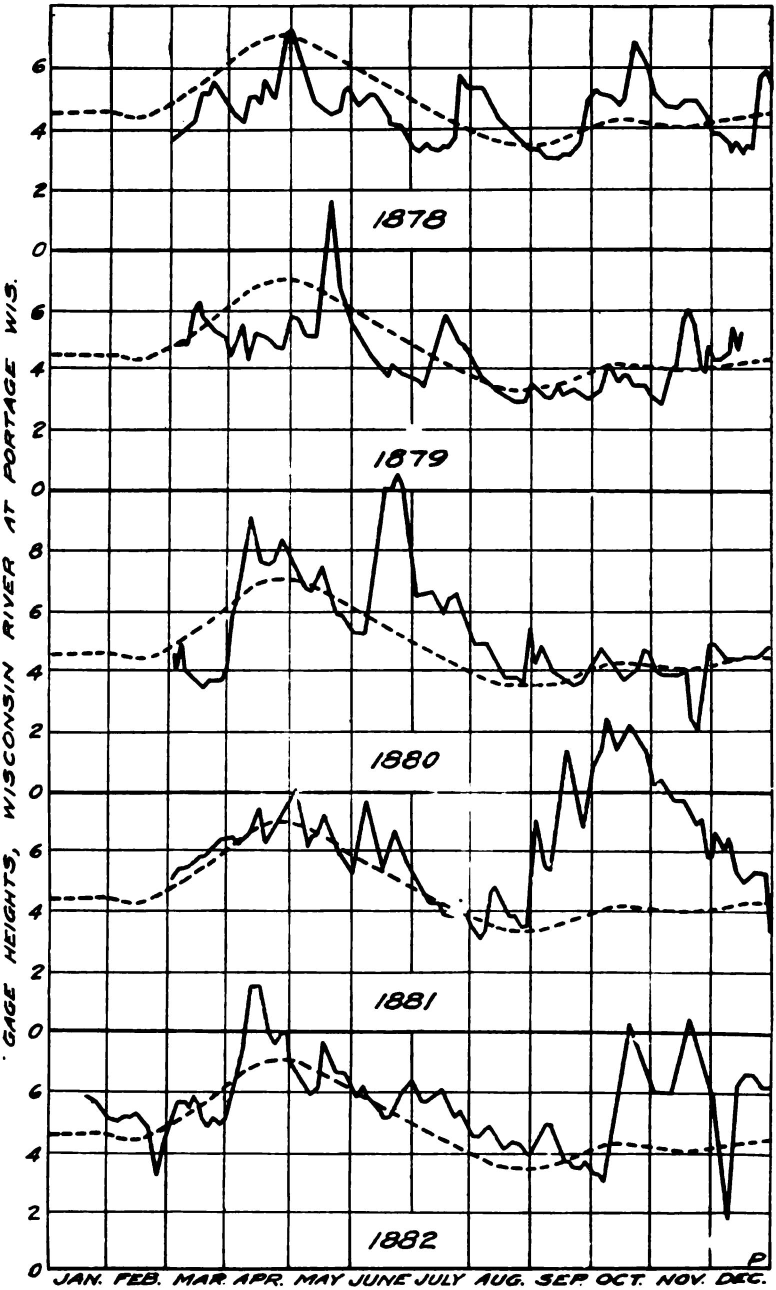


Fig. 78.—Hydrographs of Daily Flow of the Wisconsin River at Portage.

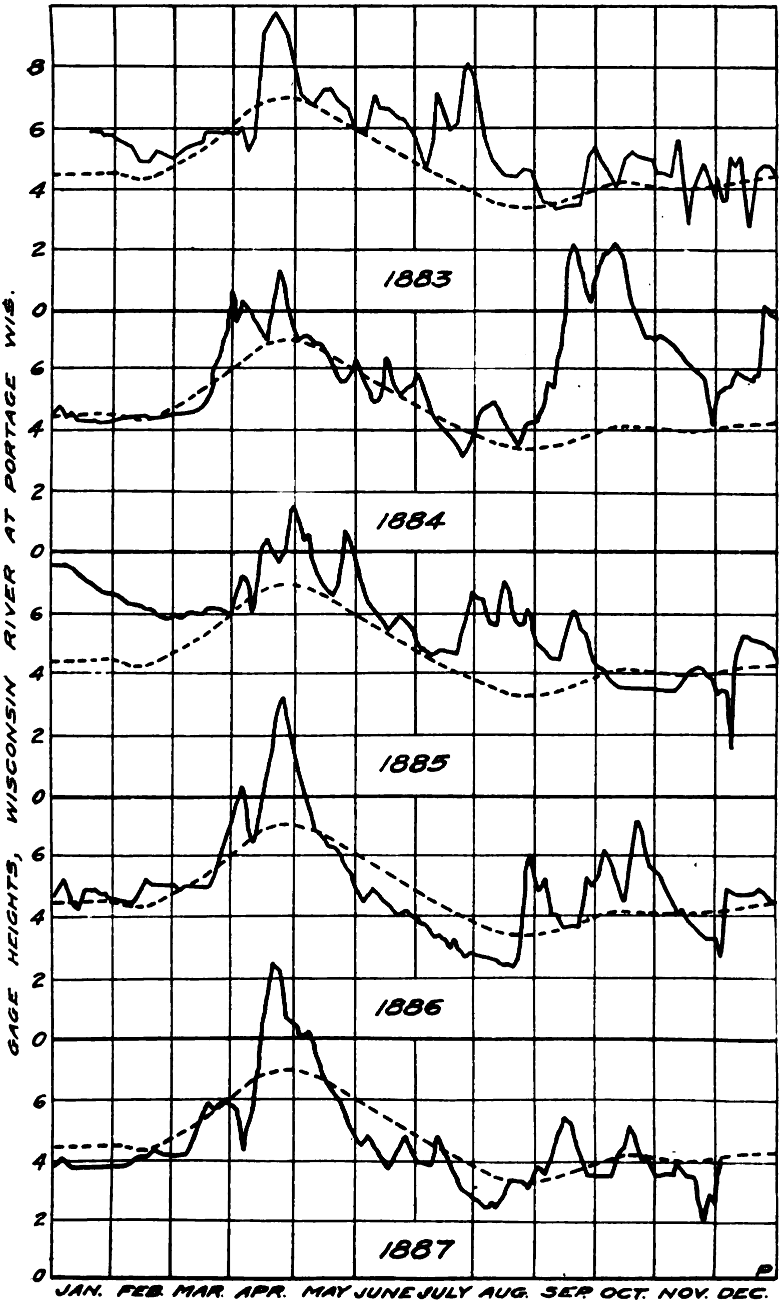


Fig. 79.—Hydrographs of Daily Flow of the Wisconsin River at Portage.

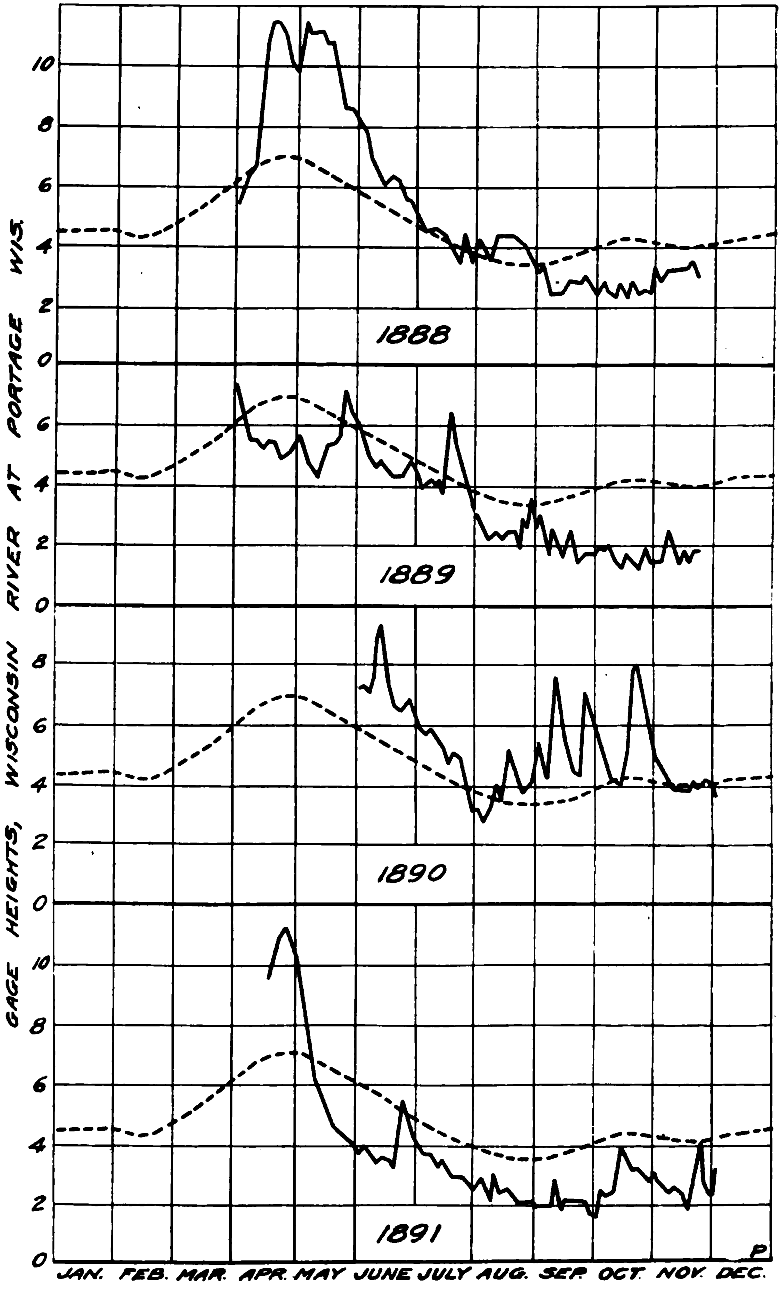


Fig. 80.—Hydrographs of Daily Flow of the Wisconsin River at Portage.

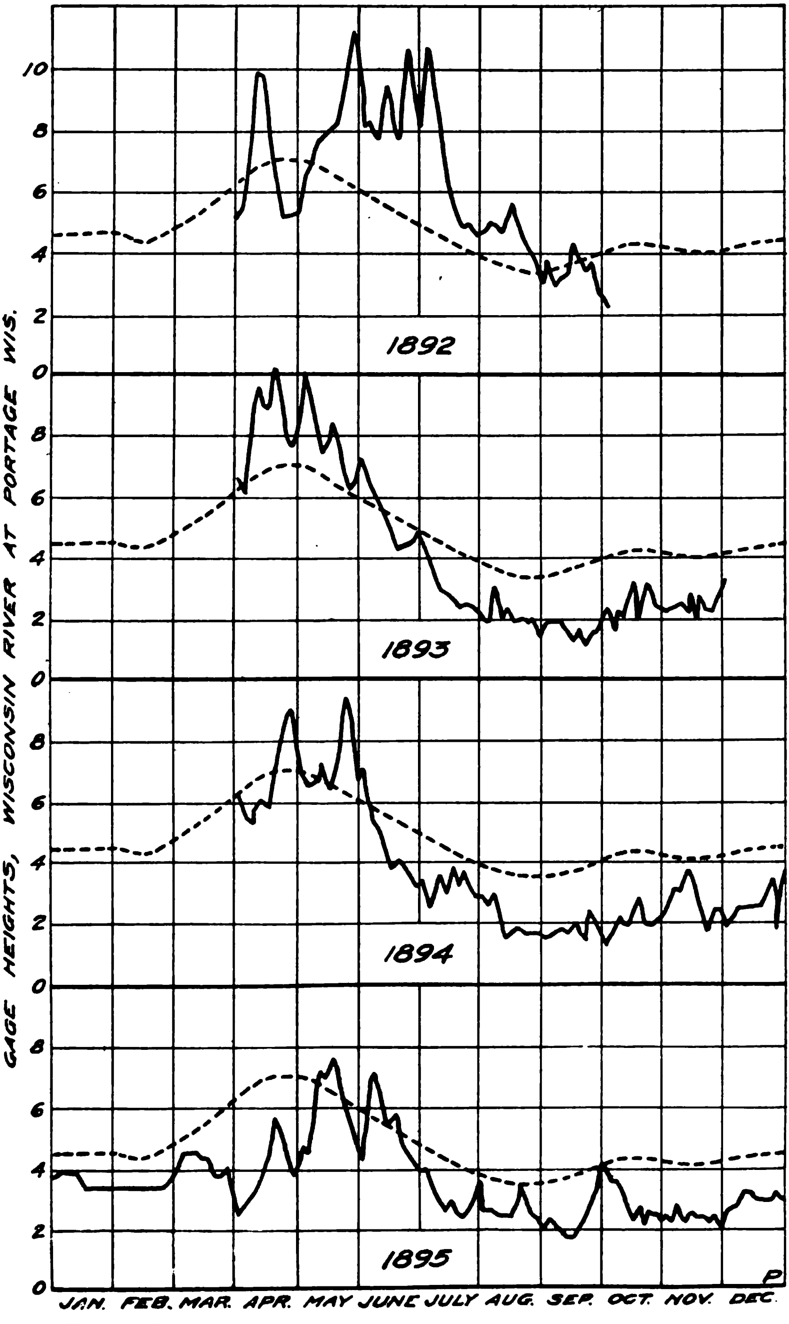


Fig. 81.—Hydrographs of Daily Flow of the Wisconsin River at Portage.

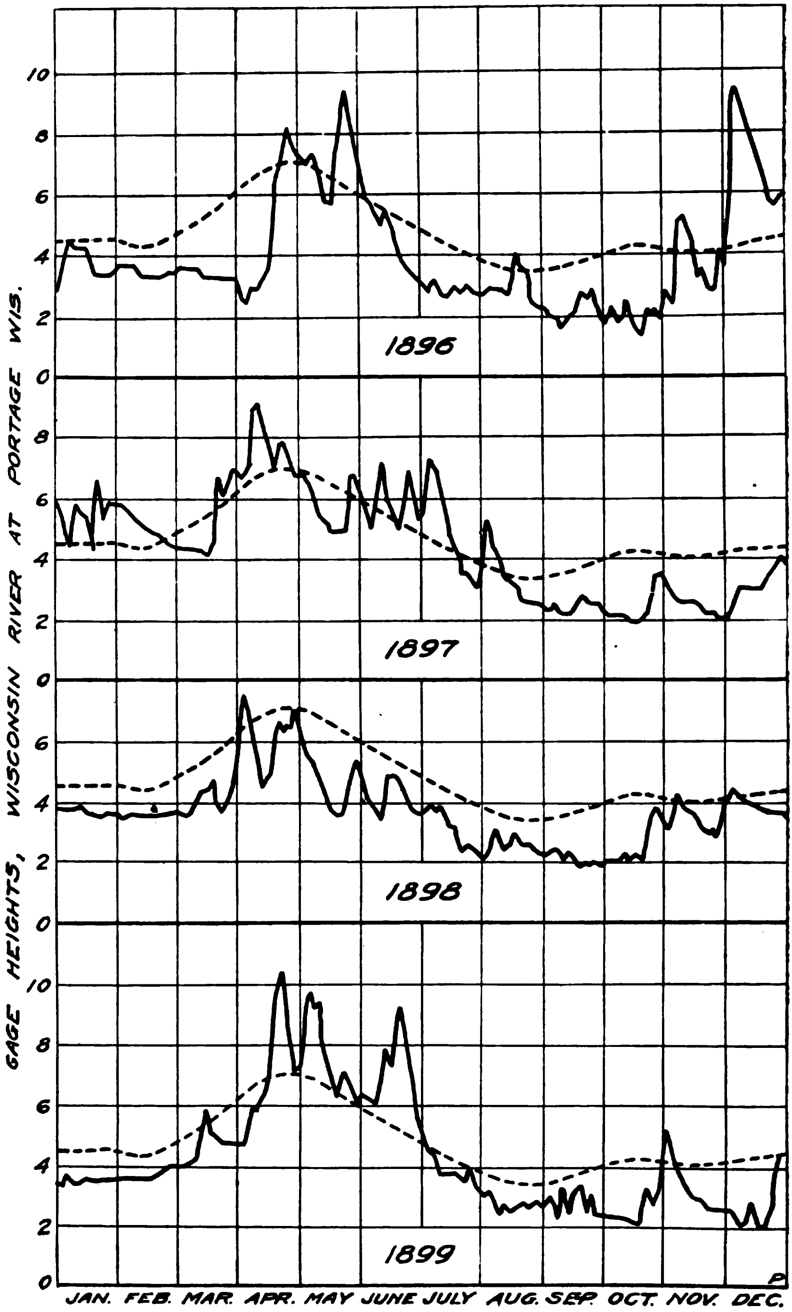


Fig. 82.—Hydrographs of Daily Flow of the Wisconsin River at Portage.

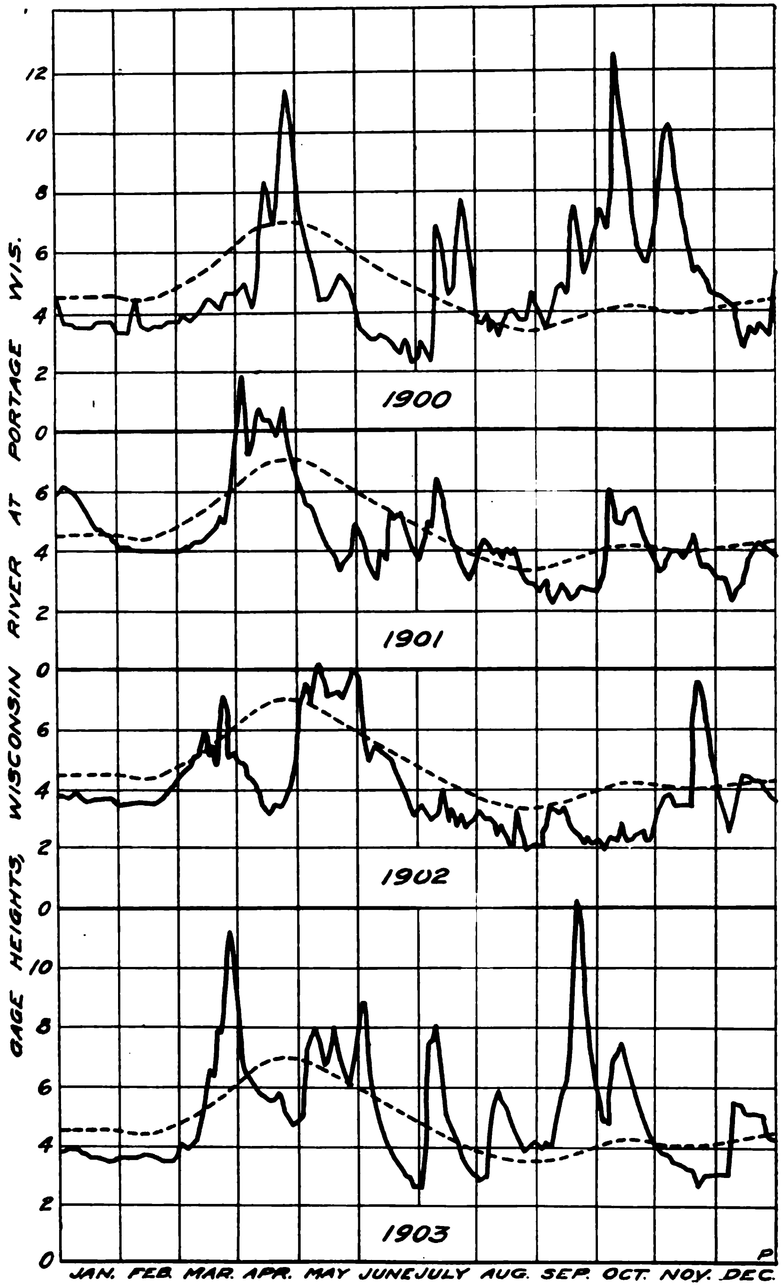


Fig. 88.—Hydrographs of Daily Flow of the Wisconsin River at Portage.

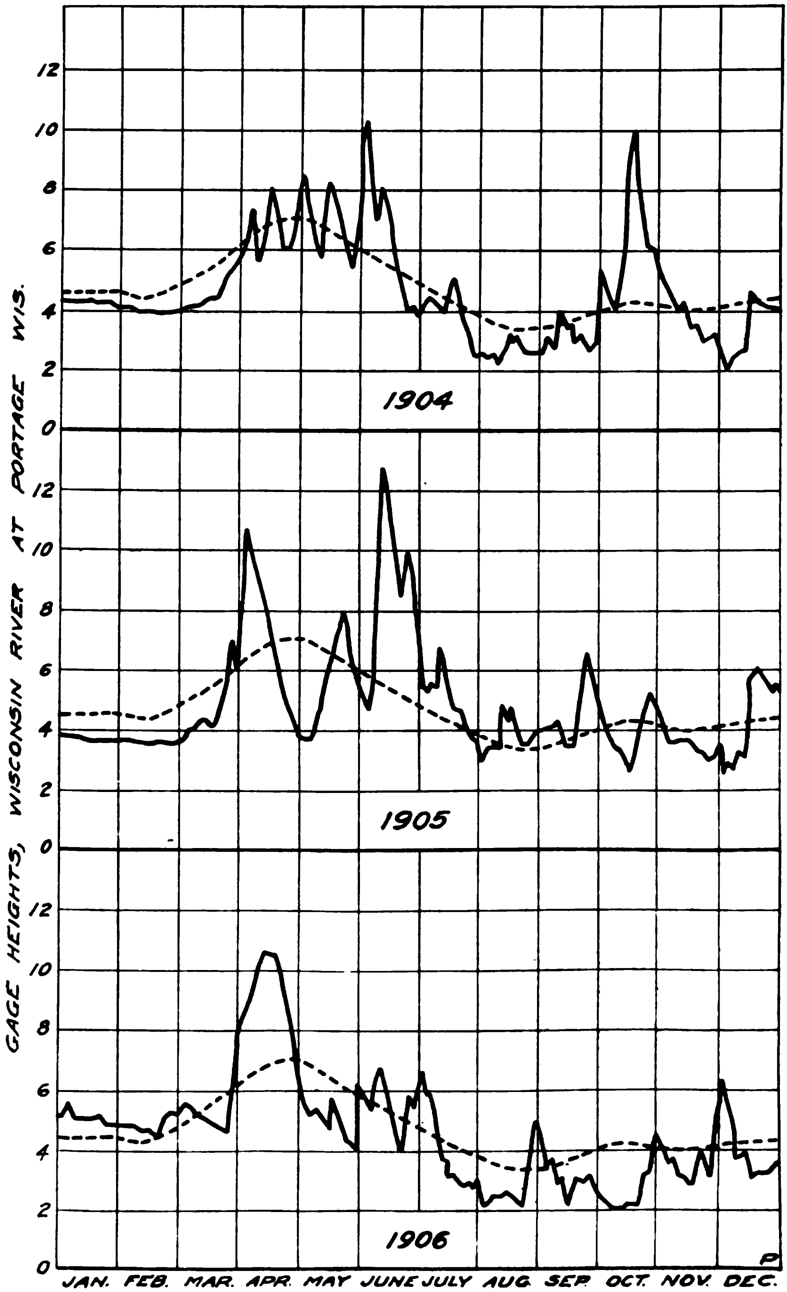


Fig. 84.—Hydrographs of Daily Flow of the Wisconsin River at Portage.

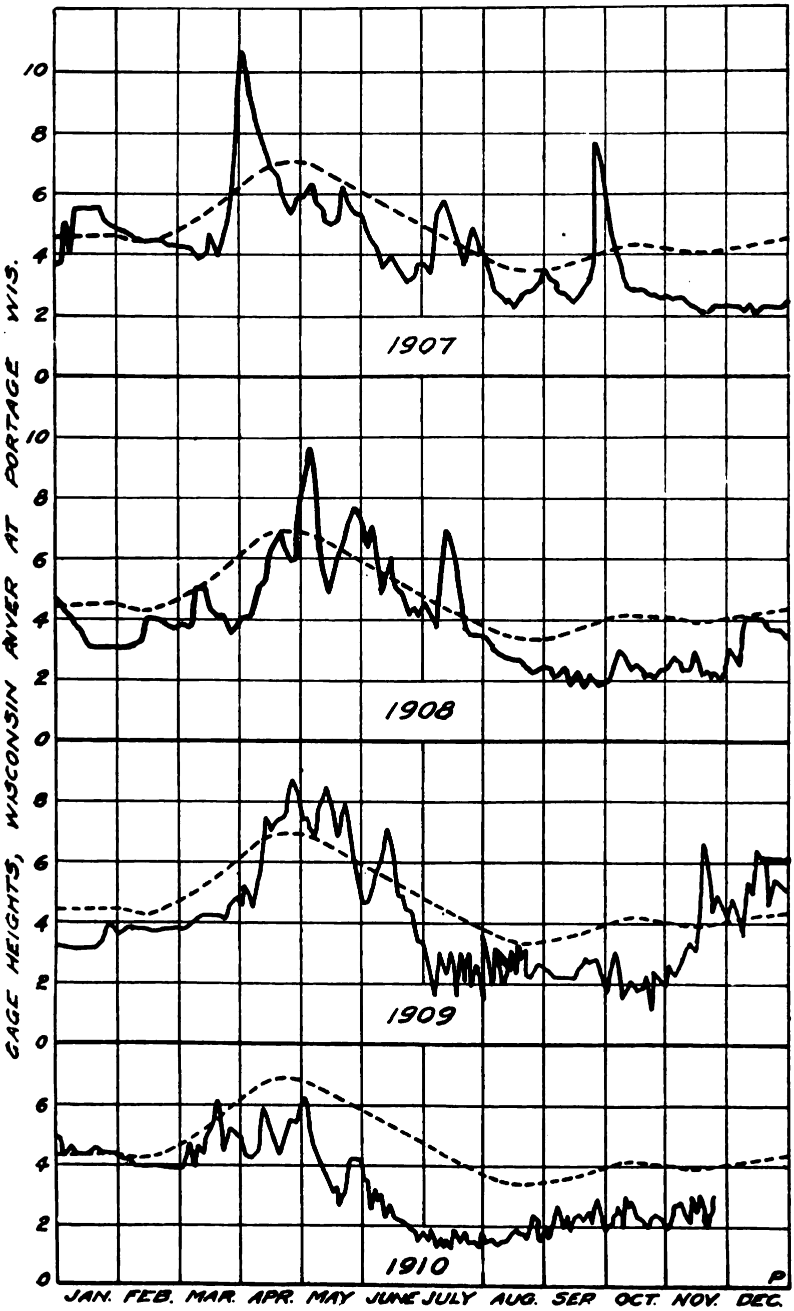


Fig. 85.—Hydrographs of Daily Flow of the Wisconsin River at Portage.

TABLE 8.—MEAN MONTHLY AND ANNUAL PRECIPITATION IN WISCONSIN RIVER VALLEY ABOVE PORTAGE.

Year.	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total annual.
1873....	1.28	0.74	1.90	1.25	3.54	7.25	3.98	1.81	3.84	2.00	2.19	1.33	31.11
1874....	1.78	0.99	1.56	1.28	2.48	5.50	3.04	3.53	5.26	2.44	2.77	0.97	32.08
1875....	1.44	2.53	2.10	1.86	2.72	4.79	1.33	5.55	4.31	2.72	1.15	2.56	33.06
1876....	2.23	1.61	3.03	2.74	5.08	6.31	3.45	5.26	3.94	1.48	2.56	1.47	39.14
1877....	1.23	0.22	2.20	1.90	2.99	4.81	3.52	3.92	2.76	5.07	2.58	2.02	33.20
1878....	0.62	0.76	1.73	3.93	3.26	4.25	5.95	2.46	5.36	4.16	0.86	0.76	34.10
1879....	0.59	1.99	1.39	1.59	5.52	4.54	7.08	1.62	4.19	2.88	4.46	3.38	39.23
1880....	2.49	1.40	2.77	4.19	4.13	9.43	3.82	5.14	3.07	1.96	2.48	1.38	42.26
1881....	1.87	2.80	2.23	1.30	4.29	3.83	5.68	5.19	10.54	7.30	2.50	0.62	48.15
1882....	1.09	1.98	3.48	2.91	4.23	6.42	4.06	4.92	1.71	3.83	2.83	1.98	39.44
1883....	1.82	1.44	0.34	2.01	4.36	5.32	7.55	2.33	3.12	3.02	1.94	1.81	35.06
1884...	1.12	2.54	1.62	3.66	3.74	4.47	5.68	5.71	7.09	4.56	1.63	3.66	45.48
1885....	1.55	0.55	0.82	2.21	2.77	4.62	7.43	4.43	3.31	1.46	1.84	2.46	33.47
1886....	3.35	1.81	2.76	2.92	1.72	3.20	1.60	4.54	4.19	3.06	2.25	1.11	32.51
1887....	1.75	2.80	0.99	1.80	2.51	2.42	3.96	3.04	4.75	3.16	1.33	3.58	32.09
1888....	2.07	0.99	2.77	2.59	5.10	3.84	3.49	3.93	2.78	1.67	1.28	1.76	32.27
1889....	1.97	1.90	1.20	1.94	2.95	2.66	3.77	4.06	2.87	0.20	1.59	2.46	27.58
1890....	1.84	1.35	1.52	2.51	4.52	6.57	3.94	6.09	4.73	3.82	1.19	0.74	38.82
1891....	1.42	1.71	2.14	1.53	0.33	3.26	2.66	4.26	1.95	2.24	1.04	2.35	24.89
1892....	1.20	1.96	1.45	2.31	5.62	7.45	3.04	3.12	2.87	1.97	1.85	1.12	33.96
1893....	1.54	1.55	1.44	3.89	2.25	1.67	4.17	1.94	2.50	2.00	1. 4	2.77	27.16
1894....	1.53	0.50	1.49	2.77	5.04	2.76	1.21	0.69	2.49	2.88	1.91	1.24	24.51
1895....	1.43	0.51	0.38	1.48	5.02	2.88	2.58	3.29	3.58	0.70	1.44	1.31	24.60
1896....	0.89	0.43	0.86	3.77	6.24	2.71	2.93	3.71	2.98	2.68	3.48	0.96	30.64
1897....	1.53	1.15	2.10	1.66	1.93	4.90	3.20	2.54	2.37	2.20	1.20	0.80	25.58
1898....	0.78	1.49	2.19	2.07	2.32	3.66	3.22	2.26	2.22	3.63	1.77	0.31	25.92
1899...	1.01	0.86	2.13	3.41	3.80	6.01	2.62	3.26	2.75	3.12	0.73	1.61	31.31
1900....	0.64	1.19	1.24	2.58	1.49	1.94	8.91	3.65	7.04	7.78	1.11	0.62	38.22
1901 ...	0.67	1.05	2.40	0.96	2.60	4.56	4.85	2.99	4.59	2.29	1.09	0.79	28.84
1902....	0.75	1.11	1.06	1.79	5.82	3.24	4.17	1.89	2.41	1.79	3.08	1.84	28.95
1903....	0.42	0.83	2.38	2.91	5.80	1.50	5.46	7.04	5.36	2.15	0.93	0.64	35.42
1904....	0.38	1.08	1.85	1.93	5.85	4.54	3.35	2.78	5.74	4.60	0.22	1.86	34.18
1905...	0.85	0.83	1.34	1.20	5.08	7.55	2.85	5.10	3.04	2.18	1.82	1.14	32.98
1906....	1.98	0.62	2.27	1.50	4.19	5.88	2.39	3.78	3.20	2.88	2.96	1.33	32.98
1907....	1.67	0.31	1.53	2.41	2.14	3.10	4.99	3.96	5.06	0.65	0.84	1.01	27.67
1908....	0.60	1.29	1.97	3.42	4.55	3.48	4.02	1.75	3.15	1.05	1.50	1.15	27.93
1909....	0.76	1.25	1.29	3.92	2.47	3.08	3.52	3.25	2.82	1.14	4.21	1.41	29.12
1910....	0.99	0.82	0.19	3.79	1.98	0.65	1.58	3.78	3.86	1.82	......	......	......
Means.	1.35	1.29	1.74	2.42	3.69	4.34	3.97	3.65	3.89	2.75	1.89	1.58	32.81

TABLE 9.—GAGE HEIGHTS AT PORTAGE ON THE WISCONSIN RIVER, SHOWING COMPARATIVE AVERAGE AND EXTREME CONDITIONS OF FLOW FOR VARIOUS YEARS.

Year	Extreme High Water.	Extreme Low Water.	Avg. for the Year.	No. of Days Above 10 ft.	No. of Days Above 8 ft.	No. of Days Above 6 ft.	No. of Days Below 3 ft.	No. of Days Below 2 ft.	No. of Days Below 1.5 ft.	Remarks.
1873	9.4	3.5	5.55	0	14	101	0	0	0	Jan., Feb., March, Dec., missing.
1874	8.0	3.5	5.33	0	0	79	0	0	0	Jan., Feb., March, Dec.. missing.
1875	8.2	2.0	4.97	0	1	63	3	0	0	Missing Jan., Feb., Mch., April, Dec.
1876	10.1	2.8	6.06	2	63	132	1	0	0	Missing, Jan.
1877	7.8	2.8	4.55	0	0	19	23	0	0	Missing, Jan.
1878	7.4	2.8	4.53	0	0	22	0	0	0	Missing, Jan., Feb.
1879	9.7	2.8	4.38	0	8	15	12	0	0	Missing, Jan., Feb., 15 days of Dec.
1880	10.6	1.9	5.22	5	23	85	6	1	0	Missing, Jan., Feb.
1881	10.4	2.8	6.23	6	46	169	1	0	0	Missing, Jan., Feb.
1882	9.5	1.7	5.91	0	16	113	2	1	0	Missing, 19 days of Jan.
1883	9.8	2.3	5.72	0	16	90	3	0	0	Missing, 19 days of Jan.
1884	10.2	3.0	5.90	3	45	140	0	0	0	
1885	9.6	1.4	5.65	0	20	146	2	2	1	
1886	11.1	2.2	4.86	6	20	68	40	0	0	
1887	10.6	2.3	4.10	6	21	38	31	0	0	Missing, Dec.
1888	11.5	2.3	5.20	34	52	76	57	0	0	Missing, Jan., Feb., Mch., Dec., 10 days Nov.
1889	7.4	1.2	3.42	0	0	12	110	68	19	Missing, Jan., Feb., Mch., Dec., 10 days Nov.
1890	9.4	2.7	5.13	0	6	49	2	0	0	Missing, Jan., Feb., Mch., Apr., May, Dec.
1891	11.2	1.5	4.05	12	17	23	120	23	0	Missing, Jan., Feb., Mch., Dec., 15 days Apr.
1892	11.2	2.4	6.23	9	50	86	9	0	0	Missing, Jan., Feb., Mch., Oct., Nov., Dec.
1893	10.1	1.0	3.92	1	27	64	141	69	13	Missing, Jan., Feb., Mch., Dec.
1894	9.4	1.2	3.50	0	11	49	149	74	10	Missing, Jan., Feb., Mch.
1895	7.5	1.6	3.43	0	0	21	158	14	0	
1896	9.4	1.3	3.94	0	15	54	137	31	1	
1897	9.2	1.8	4.40	0	8	73	107	7	0	
1898	7.6	1.6	3.58	0	0	19	101	9	0	
1899	10.4	2.0	4.22	2	23	72	114	0	0	
1900	12.5	2.0	4.50	11	30	75	24	0	0	
1901	9.8	2.1	4.30	0	18	44	51	0	0	
1902	8.3	1.7	3.90	0	5	41	94	4	0	
1903	12.2	2.6	4.95	8	20	85	21	0	0	
1904	10.4	1.9	4.54	1	17	66	70	1	0	
1905	12.6	2.4	4.78	10	30	63	16	0	0	
1906	10.6	2.0	4.48	11	23	44	79	0	0	
1907	10.6	1.8	4.04	5	8	29	120	2	0	
1908	9.7	1.6	3.78	0	6	40	122	15	0	
1909	8.7	1.2	3.98	0	8	53	115	20	2	
1910	6.3	1.2	3.90	0	0	6	86	61	24	To Sept. 1st.

TABLE 10.—GAGE HEIGHTS AT PORTAGE ON THE WISCONSIN RIVER SHOWING AVERAGE MONTHLY AND ANNUAL GAGE HEIGHTS FOR VARIOUS YEARS.

Year.	Average.												
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
1873.....	......	......	6.0	6.7	6.4	7.3	6.3	4.9	4.1	4.8	4.3	......	5.55
1874.....	......	......	6.4	6.5	6.9	5.3	4.1	3.7	4.5	5.1	5.5	......	5.33
1875.....	......	......	......	......	6.8	5.9	4.6	3.7	5.5	4.1	4.2	......	4.97
1876.....	......	5.4	6.9	8.4	9.3	7.5	5.9	4.4	5.3	4.3	4.5	4.8	6.06
1877.....	......	5.2	4.8	5.3	5.2	5.0	4.4	3.1	2.9	4.4	4.5	5.3	4.55
1878.....	......	......	4.5	5.2	5.1	4.6	4.6	4.1	3.3	5.5	4.5	3.9	4.53
1879.....	......	......	5.1	4.9	6.4	4.2	4.5	3.3	3.1	3.5	4.2	4.6	4.38
1880.....	......	......	4.0	7.7	6.4	7.9	5.9	4.1	3.9	4.1	3.7	4.5	5.22
1881.....	......	......	5.8	6.8	6.5	6.2	4.1	3.9	7.2	9.2	7.3	5.3	6.23
1882.....	5.5	4.6	5.2	7.9	6.5	5.6	5.4	4.2	3.9	5.7	6.7	5.6	5.91
1883.....	5.7	6.1	5.5	7.8	6.8	6.2	6.3	4.9	3.8	4.8	4.4	4.3	5.72
1884.....	4.4	4.5	5.6	7.8	6.4	5.4	4.0	4.3	7.6	8.5	6.0	6.2	5.90
1885.....	7.1	6.1	6.0	7.8	7.6	5.8	5.1	5.9	4.9	3.6	3.5	4.4	5.65
1886.....	4.6	4.7	5.5	8.6	6.3	4.2	3.0	3.2	4.2	5.7	3.7	4.5	4.86
1887.....	3.9	4.0	5.2	7.8	6.1	4.3	3.5	3.0	4.2	4.0	3.3	......	4.10
1888.....	......	......	......	8.6	10.1	6.3	4.1	4.0	2.7	2.6	3.2	......	5.20
1889.....	......	......	......	5.5	5.5	4.6	4.2	2.4	1.9	1.6	1.7	......	3.42
1890.....	......	......	......	......	......	7.1	4.7	4.0	5.6	5.4	4.0	......	5.13
1891.....	......	......	......	10.6	5.4	3.8	3.0	2.2	2.0	2.9	2.5	......	4.05
1892.....	..	......	......	6.5	8.3	8.5	6.4	4.5	3.2	......	......	......	6.23
1893.....	......	......	......	8.2	7.7	5.0	2.6	1.8	1.5	2.3	2.3	......	3.92
1894.....	......	......	......	6.6	7.2	4.3	3.0	1.8	1.6	1.9	2.6	2.6	3.50
1895.....	3.5	3.4	3.9	3.9	5.7	5.3	2.9	2.5	2.4	2.6	2.2	2.9	3.43
1896.....	3.7	3.8	3.2	5.0	7.0	4.4	2.7	2.8	2.1	2.0	3.6	7.0	3.94
1897.....	5.4	5.0	5.2	7.6	5.7	5.7	4.7	3.1	2.4	2.4	2.3	3.2	3.40
1898.....	3.6	3.5	4.2	5.9	4.5	4.0	2.8	2.5	2.1	2.7	3.4	3.7	4.58
1899.....	3.5	3.6	4.6	7.1	7.5	7.0	3.4	2.6	2.7	2.7	3.1	2.8	4.22
1900.....	3.5	3.4	4.6	7.1	4.8	2.8	4.7	3.7	5.1	7.7	6.8	3.6	4.50
1901.....	5.1	3.9	5.0	7.8	4.4	4.0	4.2	3.4	2.5	4.3	3.6	3.3	4.30
1902.....	3.6	3.5	5.0	4.0	7.4	4.6	3.0	2.4	2.6	2.4	4.7	3.7	3.90
1903.....	3.6	3.5	6.5	5.5	6.9	4.6	4.7	4.2	6.7	5.6	3.2	4.4	4.95
1904.....	4.1	3.9	4.7	6.6	6.6	6.1	3.7	2.6	3.2	6.0	3.6	3.3	4.54
1905.....	3.7	3.5	4.9	6.9	5.6	8.2	4.8	3.7	4.3	3.8	3.4	4.3	4.78
1906.....	5.1	4.7	5.3	8.7	4.9	5.4	4.0	2.7	3.0	2.5	3.5	3.9	4.48
1907.....	4.9	4.3	4.9	7.2	5.4	3.7	4.3	2.7	3.6	3.0	2.2	2.2	4.04
1908.....	3.4	3.6	4.1	5.4	6.8	5.0	4.4	2.5	2.0	2.3	2.3	3.5	3.78
1909.....	3.3	3.7	4.2	6.4	6.9	5.0	2.4	2.6	2.3	2.0	3.8	5.7	3.98
1910.....	4.4	3.9	4.7	4.9	3.9	2.4	1.5	1.6	......	......	......	......	3.90
Means..	4.35	4.24	5.14	7.08	6.40	5.36	4.15	3.34	3.62	4.05	3.84	4.26	4.66

APPENDIX B.

THE WOLF RIVER.

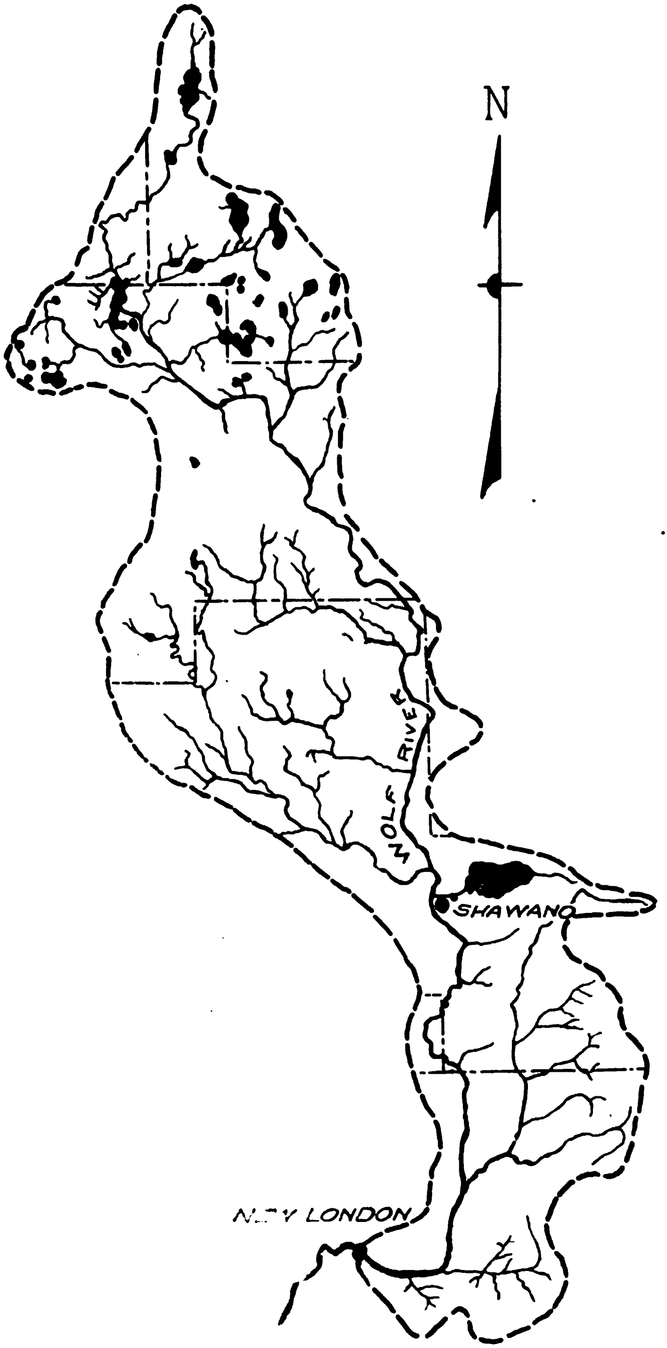


Fig. 86.—Map of Drainage Area of the Wolf River above New London, Wisconsin.

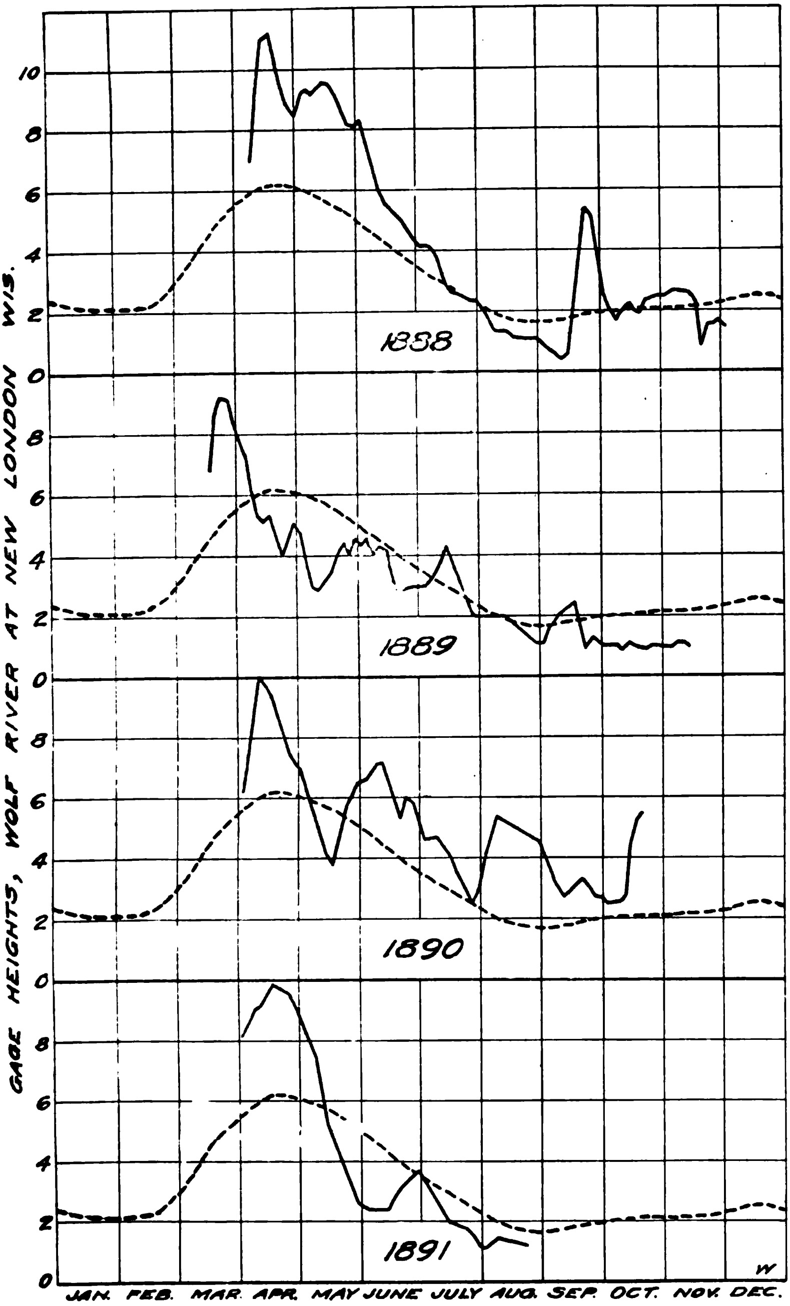


Fig. 87.—Hydrographs of the Daily Flow of the Wolf River at New London.

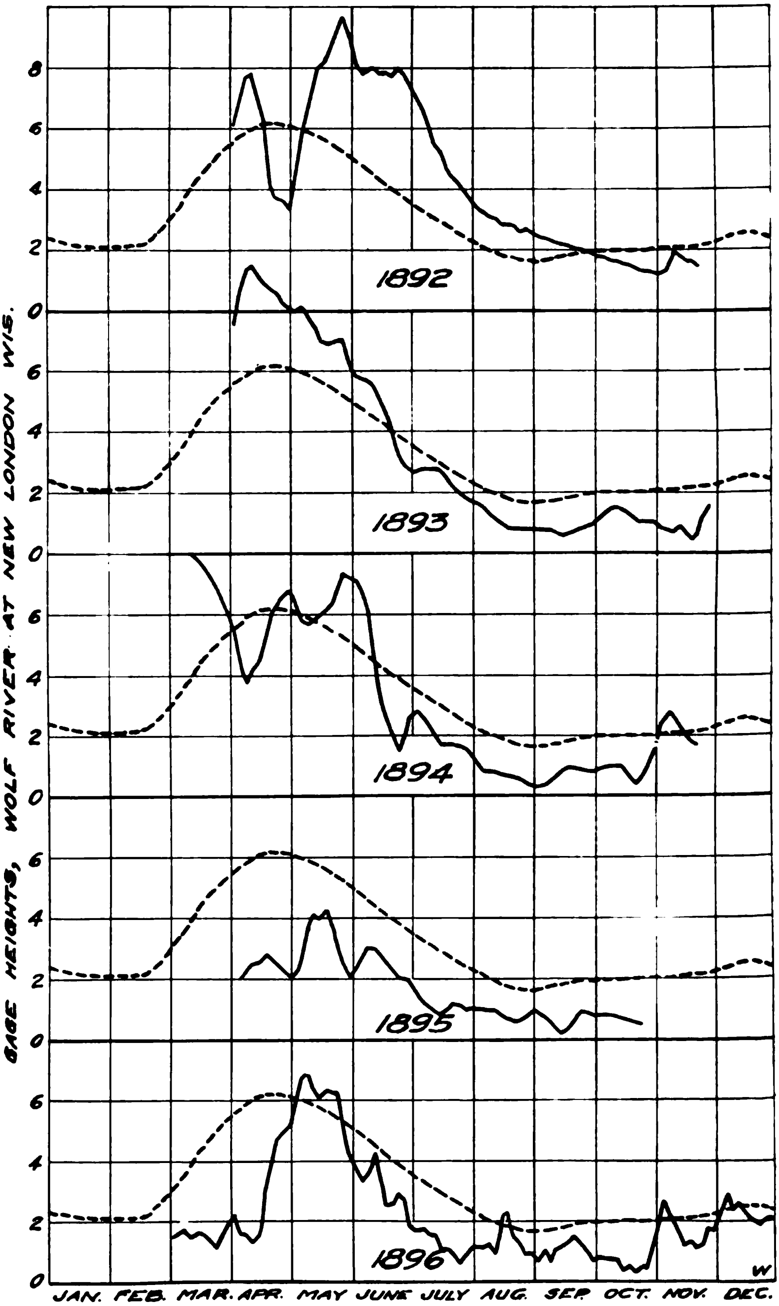


Fig. 88.—Hydrographs of the Daily Flow of the Wolf River at New London.

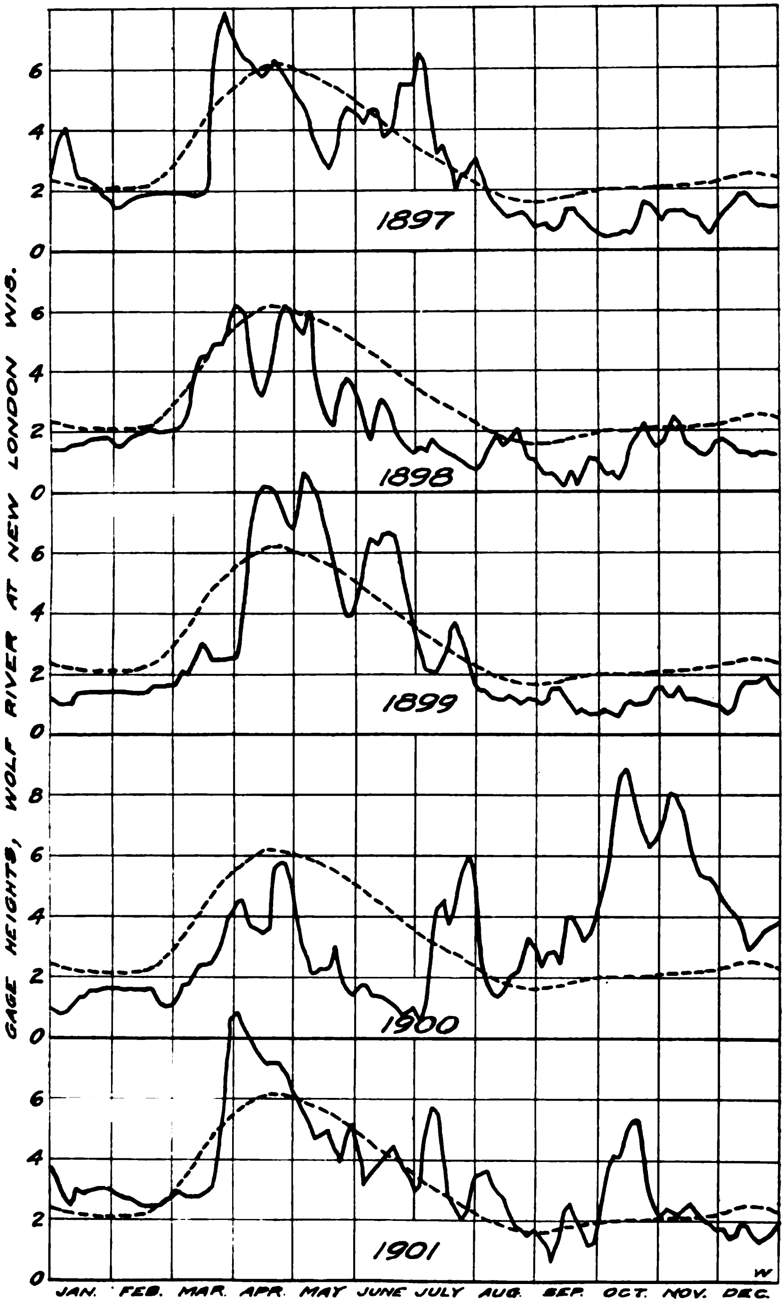


Fig. 89.—Hydrographs of the Daily Flow of the Wolf River at New London.

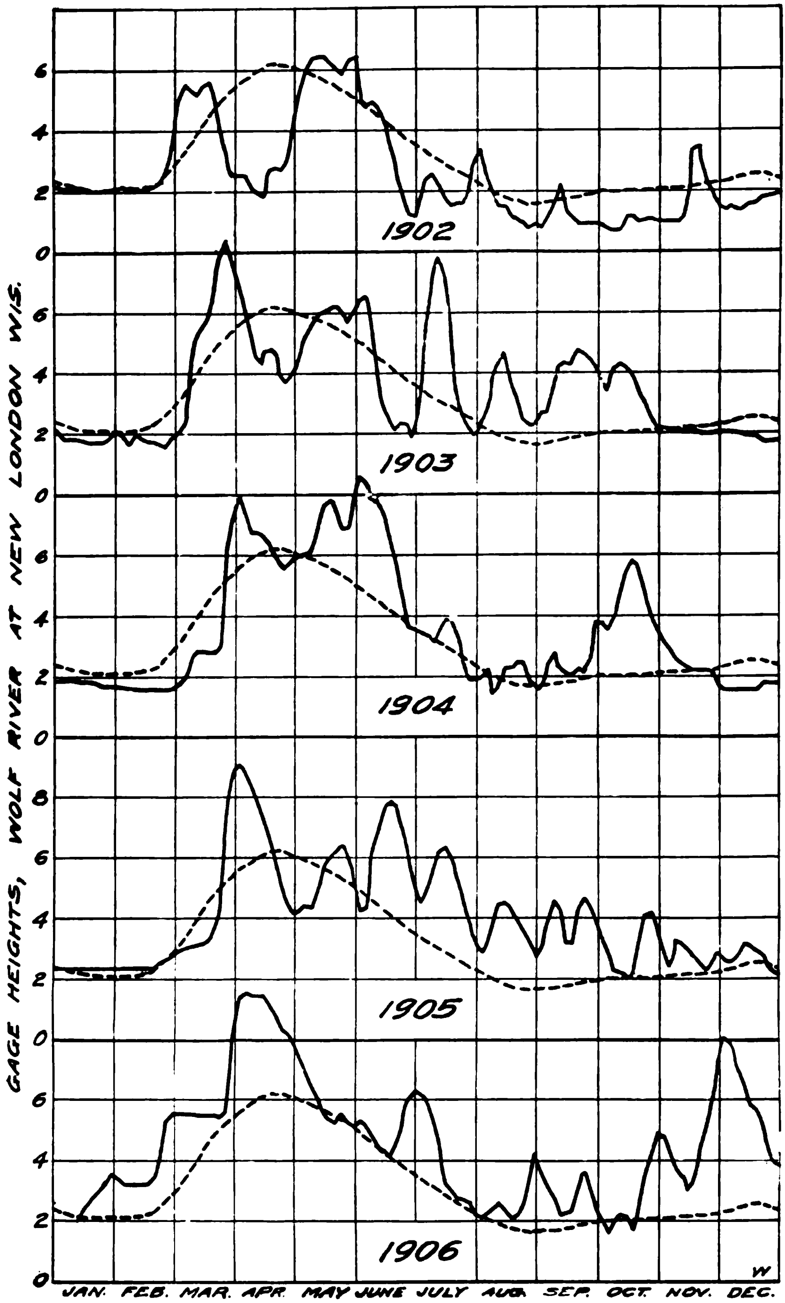


Fig. 90.—Hydrographs of the Daily Flow of the Wolf River at New London.

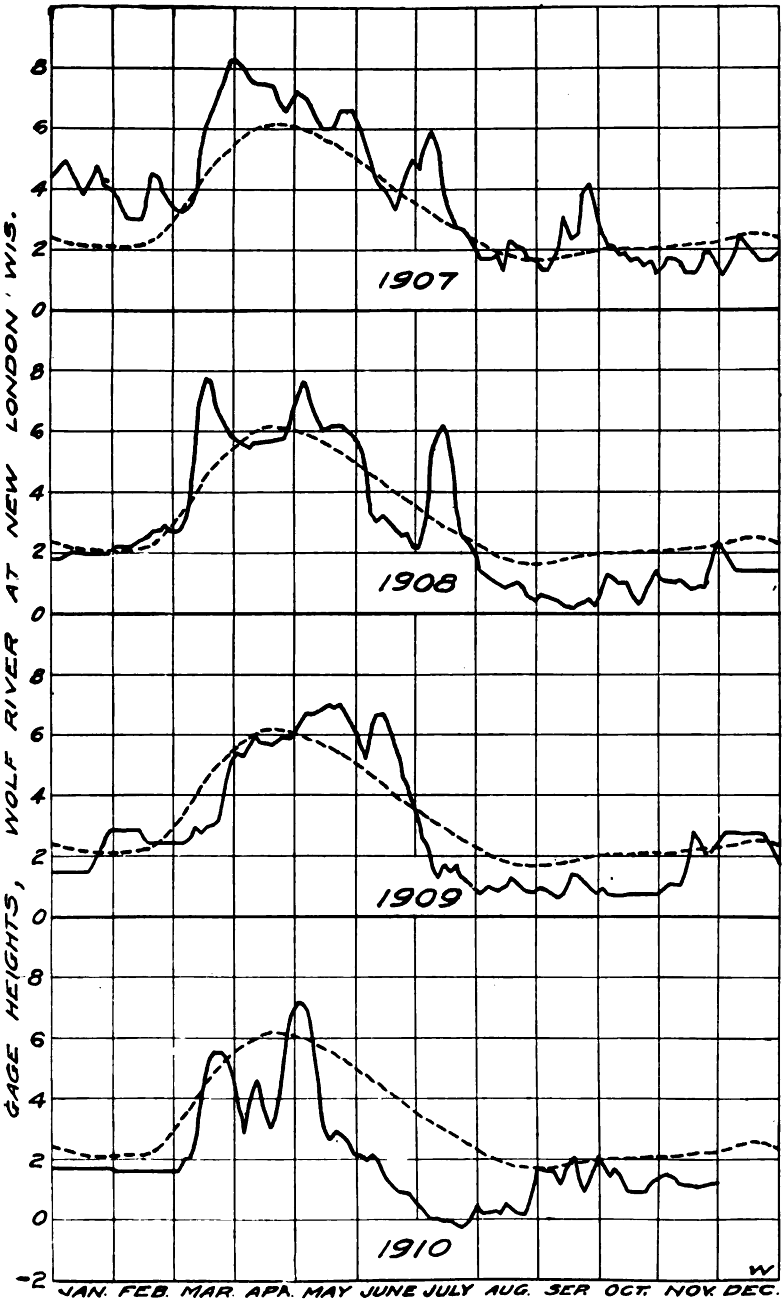


Fig. 91.—Hydrographs of the Daily Flow of the Wolf River at New London.

TABLE 11.—MEAN MONTHLY AND ANNUAL PRECIPITATION IN WOLF RIVER VALLEY ABOVE NEW LONDON, WIS.

Year.	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total annual.
1888.....	3.55	2.10	3.55	3.25	6.30	3.25	3.30	4.35	6.15	2.20	2.63	3.20	43.85
1889.....	3.50	3.45	0.90	1.20	4.20	3.50	4.70	3.00	2.75	0.40	2.35	3.80	33.75
1890.....	3.75	3.10	2.00	2.70	4.40	6.60	5.80	7.05	3.40	2.35	1.85	1.05	44.05
1891.....	2.05	2.26	2.23	1.79	0.28	2.38	2.27	3.58	1.37	1.72	1.20	3.34	24.47
1892.....	1.32	1.31	1.27	2.18	6.23	6.79	3.04	2.72	1.87	2.55	1.86	0.76	31.90
1893.....	1.66	2.39	1.99	3.94	2.78	1.73	2.93	1.46	2.15	2.72	1.65	3.09	28.49
1894.....	1.54	0.53	1.73	3.02	4.64	2.63	1.44	0.98	2.95	3.03	2.68	0.98	26.15
1895.....	1.53	0.52	1.06	1.57	3.02	1.69	2.51	2.43	3.38	0.78	1.36	1.46	21.31
1896.....	1.08	0.45	1.20	5.07	5.47	3.49	3.08	3.89	3.27	2.93	2.65	0.79	33.37
1897.....	1.69	1.29	1.84	1.86	2.17	5.37	3.67	2.06	2.09	1.71	1.43	0.88	26.06
1898.....	0.70	1.31	2.24	2.24	3.13	3.56	3.44	3.06	3.10	3.51	1.53	0.38	28.20
1899.....	1.35	1.11	2.57	3.21	2.90	5.62	4.68	2.41	2.00	2.65	0.78	1.41	30.69
1900.....	1.09	2.02	1.31	3.17	1.48	2.51	6.73	4.20	7.53	8.12	1.48	0.85	90.49
1901.....	0.88	0.47	3.26	0.76	2.71	3.45	6.18	2.20	4.41	3.52	1.07	0.93	29.84
1902.....	0.52	1.47	1.05	1.94	5.67	4.25	7.27	1.12	3.00	1.65	2.65	1.97	32.56
1903.....	0.76	1.09	3.21	3.27	4.83	1.46	4.69	5.45	4.75	2.13	1.27	0.62	33.53
1904.....	0.46	1.51	2.33	2.49	6.75	2.68	3.69	2.08	6.06	4.24	0.38	2.22	34.89
1905.....	1.45	1.11	2.22	1.47	4.44	5.72	4.68	5.73	3.47	2.50	1.93	1.29	36.01
1906.....	2.70	0.79	3.26	1.91	3.39	5.09	3.66	5.65	2.72	3.58	4.17	1.70	38.62
1907.....	2.22	0.35	1.75	3.26	2.34	3.81	2.94	3.71	4.98	0.06	1.34	1.55	28.88
1908.....	1.22	1.61	2.13	3.78	4.49	3.73	4.33	1.53	2.22	1.10	2.14	1.54	29.82
1909.....	0.63	1.67	1.72	4.14	3.34	3.43	2.44	2.95	2.81	0.83	3.44	1.88	29.28
1910.....	1.13	0.97	0.22	5.01	1.81	1.00	1.69	4.05	3.47	......	......	......	......
Means..	1.60	1.43	1.95	2.76	3.76	3.64	3.88	3.29	3.47	2.49	1.90	1.62	32.10

TABLE 12.—GAGE HEIGHTS AT NEW LONDON, WIS., ON THE WOLF RIVER, SHOWING COMPARATIVE AVERAGE AND EXTREME CONDITIONS OF FLOW FOR VARIOUS YEARS.

Year.	Extreme high water.	Extreme low water.	Average for year.	No. days above 8 ft.	No. days above 6 ft.	No. days above 4 ft.	No. days below 2 ft.	No. days below 1 ft.	No. days below 0.5 ft.	Remarks.
1888..	11.5	0.3	4.11	55	65	101	62	15	3	Missing Jan., Feb., Mar., Dec.
1889..	9.4	0.8	3.32	13	23	71	80	6	0	Missing Jan., Feb., ½ Mar., ½ Nov., Dec.
1890..	10.0	2.4	5.0	16	54	128	0	0	0	Missing Jan., Feb., Mar., Nov., Dec.
1891..	9.9	1.0	4.16	19	40	49	38	0	0	Missing Jan., Feb., Mar., Sept., Oct., Nov., Dec.
1892..	9.7	1.2	4.29	22	79	102	54	0	0	Missing Jan., Feb., Mar., Dec.
1893..	9.6	0.4	3.33	29	60	80	125	67	1	Missing Jan., Feb., Mar., Dec.
1894..	8.0	0.3	3.24	0	57	89	121	71	15	Missing Jan., Feb., Mar., Dec.
1895..	4.3	0.1	1.70	0	0	6	117	76	10	Missing Jan., Feb., Mar., Nov., Dec.
1896..	7.1	0.0	2.21	0	16	41	188	51	10	Missing Jan., Feb.
1897..	8.0	0.3	2.71	0	31	104	201	49	6	
1898..	6.2	0.0	2.18	0	9	50	185	48	9	
1899..	8.8	0.4	2.74	15	52	77	219	29	1	
1900..	8.9	0.3	3.39	7	36	111	125	24	1	
1901..	9.1	0.4	3.42	11	36	99	71	2	1	
1902..	6.5	0.5	2.82	0	21	66	162	31	0	
1903..	8.5	1.5	3.60	4	45	138	63	0	0	
1904..	8.6	1.3	3.50	10	68	111	116	0	0	
1905..	9.2	2.0	4.05	15	59	155	0	0	0	
1906..	9.5	1.6	4.52	31	69	188	8	0	0	
1907..	8.4	1.0	3.76	8	71	139	109	0	0	
1908..	8.0	0.0	2.85	0	43	101	128	66	23	
1909..	7.0	0.6	2.81	0	37	103	156	67	0	
1910..	7.2	—0.4	1.95	0	11	42	201	82	50	Missing Nov., Dec.

TABLE 13.—GAGE HEIGHTS AT NEW LONDON, WIS., ON THE WOLF RIVER SHOWING AVERAGE MONTHLY AND ANNUAL GAGE HEIGHTS FOR VARIOUS YEARS.

Year.	AVERAGES.												
	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year
1888.....	......	......	......	9.45	9.04	5.90	3.13	1.37	2.40	2.20	2.12	......	4.11
1889.....	......	......	8.44	5.50	3.82	3.65	3.07	1.69	1.67	0.99	1.06	......	3.32
1890.....	......	......	......	8.57	5.37	6.22	3.74	4.84	3.09	3.69	......	......	5.07
1891.....	......	......	......	9.24	5.26	2.85	2.11	1.32	......	......	......	......	4.16
1892.....	......	......	......	5.65	7.74	7.84	5.04	2.87	2.15	1.49	1.57	......	4.29
1893.....	......	......	......	8.66	7.20	4.50	2.39	1.03	0.78	1.22	0.83	......	3.33
1894.....	......	......	7.14	5.25	6.38	3.86	1.94	0.68	0.70	0.91	2.30	......	3.24
1895.....	......	......	......	2.50	4.60	2.50	1.10	0.90	0.60	0.70	......	......	1.70
1896.....	......	......	1.6	3.00	6.0	3.10	1.30	1.40	1.10	0.60	1.70	2.30	2.21
1897.....	2.6	1.8	4.1	6.00	4.0	4.70	3.60	1.40	0.96	0.80	1.05	1.50	2.71
1898.....	1.6	1.95	4.1	4.95	3.80	2.20	1.20	1.45	0.63	1.35	1.70	1.40	2.18
1899.....	1.26	1.47	2.42	6.89	6.31	5.74	2.61	1.25	1.07	0.99	1.26	1.48	2.74
1900	1.23	1.42	2.55	4.52	2.28	1.21	5.00	2.25	3.26	7.00	6.39	3.60	3.39
1901.....	2.96	2.54	4.01	7.44	4.90	3.79	3.53	2.47	1.56	3.89	2.24	1.69	3.42
1902.....	2.03	2.19	4.57	2.64	6.12	3.36	2.04	1.50	1.20	0.95	1.84	5.94	2.82
1903.....	1.80	1.70	5.60	4.90	5.7	3.70	4.50	3.1	4.0	3.50	2.10	1.90	3.60
1904.....	1.70	1.60	3.70	6.50	7.00	6.30	3.90	2.00	2.4	4.60	2.40	1.70	3.50
1905....	2.30	2.39	4.47	6.78	5.35	6.31	4.96	3.60	3.96	3.02	2.81	2.68	4.05
1906.....	2.54	3.66	5.88	8.92	5.80	5.04	3.83	2.65	2.83	2.74	4.55	5.78	4.52
1907.....	4.37	3.56	5.66	7.41	6.54	4.49	3.79	1.80	2.56	1.73	1.51	1.73	3.76
1908.....	1.95	2.47	5.60	5.81	6.49	3.25	3.74	0.95	0.38	0.83	1.15	1.49	2.85
1909.....	1.80	2.62	3.24	5.71	6.67	5.41	1.26	0.94	0.96	0.78	1.77	2.59	2.81
1910.....	1.69	1.20	3.92	4.22	3.99	1.34	0.00	0.46	1.51	1.20	......	......	1.95

APPENDIX C.

THE UPPER MISSISSIPPI RIVER.

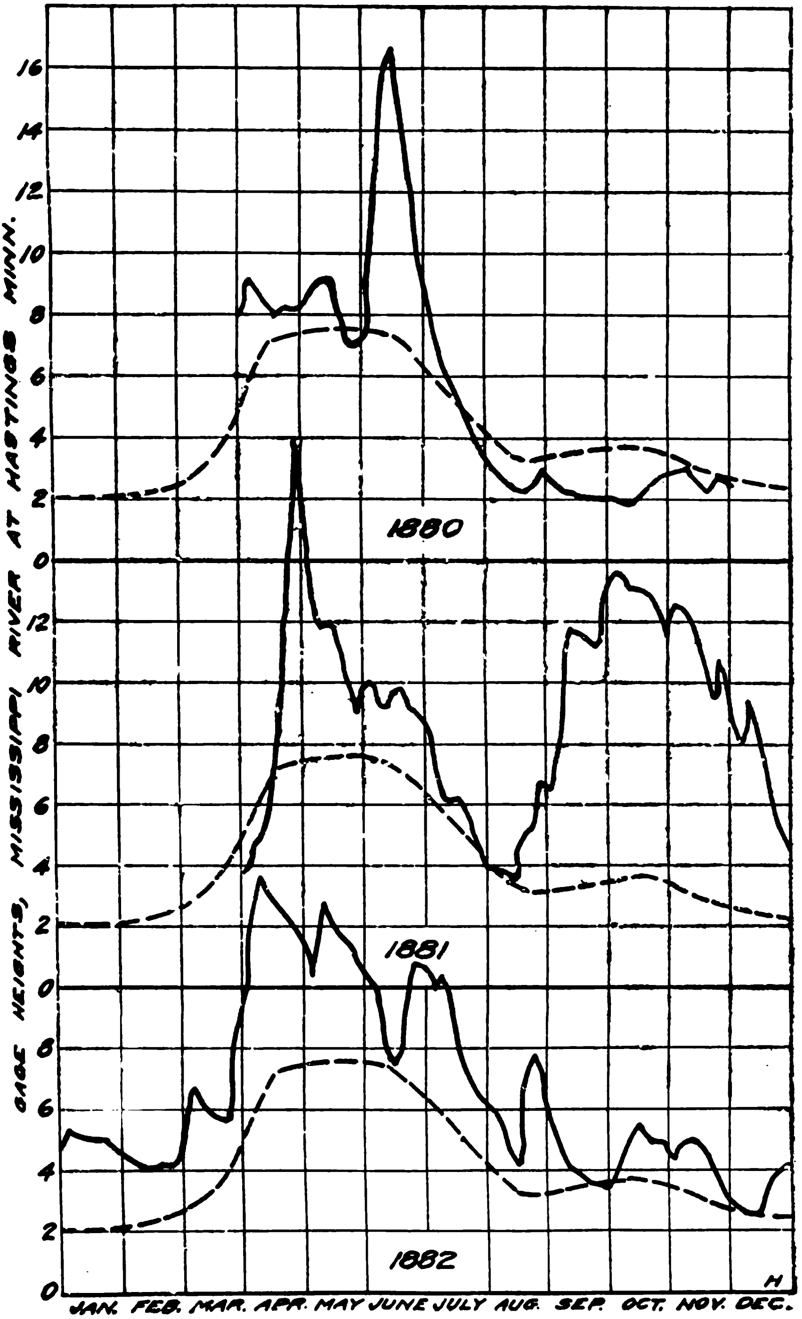


Fig. 92.—Hydrographs of the Daily Flow of the Upper Mississippi River at Hastings, Minnesota.

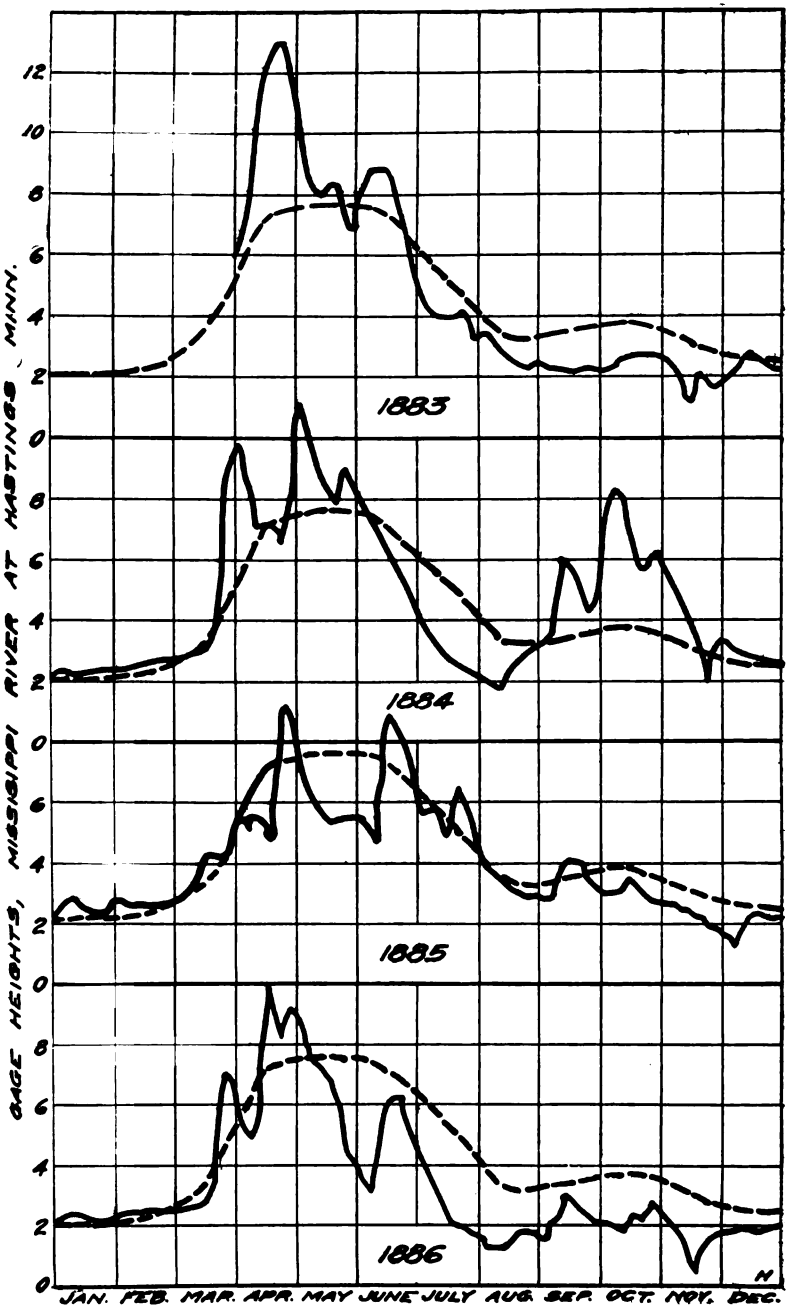


Fig 93.—Hydrographs of the Daily Flow of the Upper Mississippi River at Hastings, Minnesota.

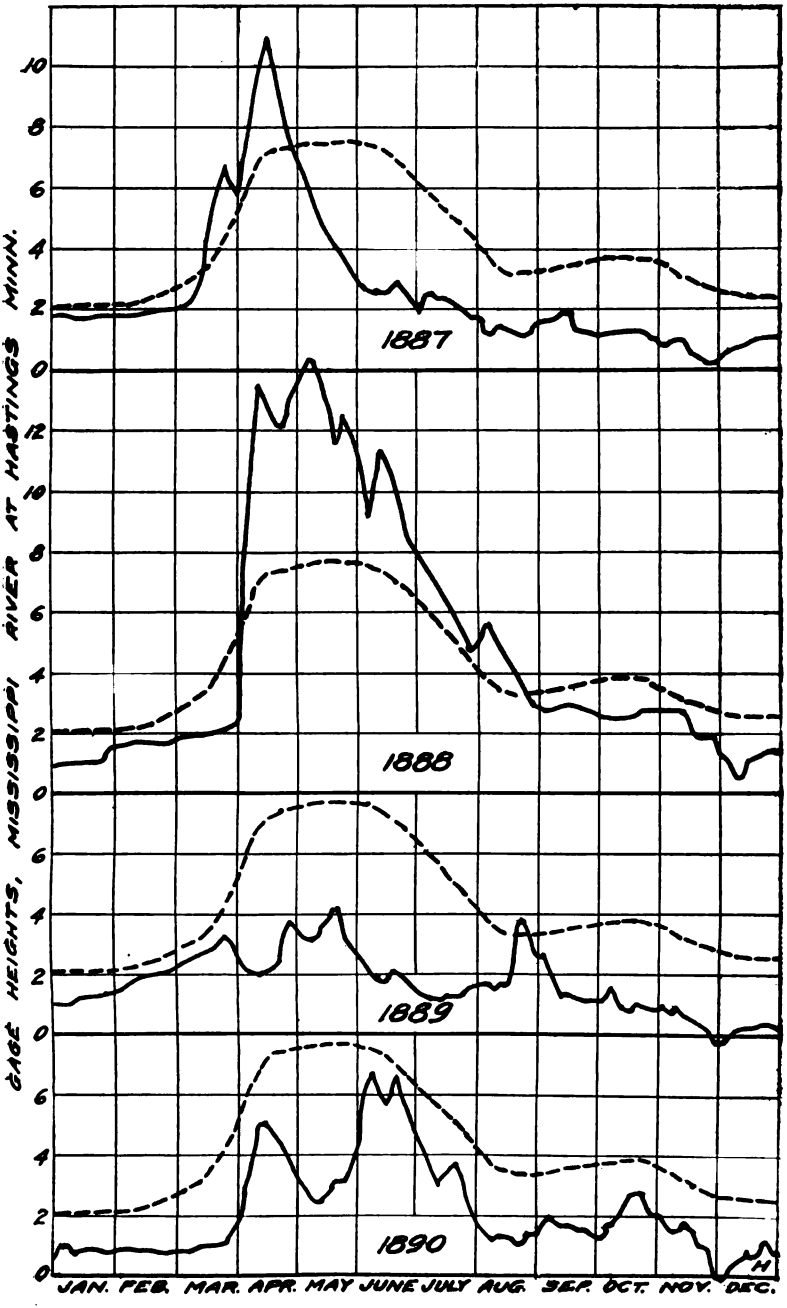


Fig. 94.—Hydrographs of the Daily Flow of the Upper Mississippi River at Hastings, Minnesota.

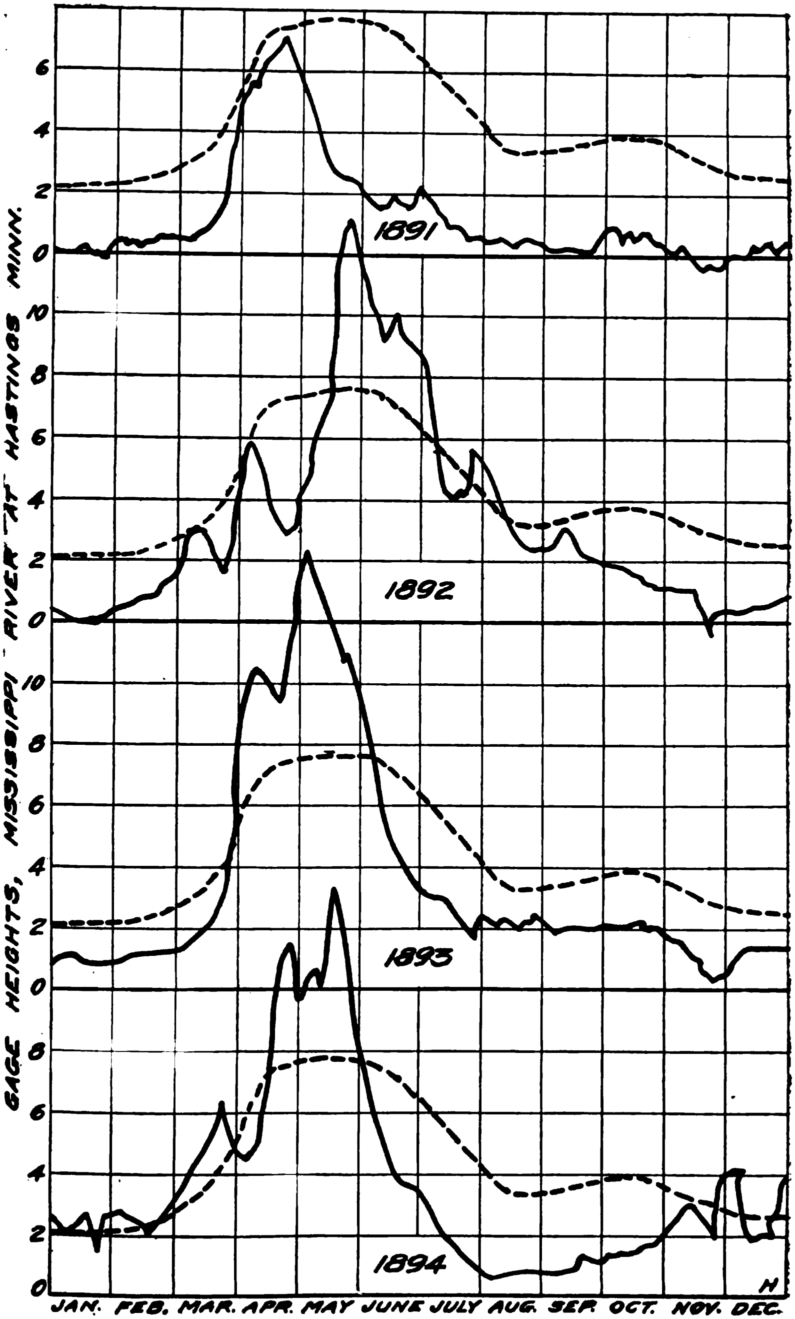


Fig. 95.—Hydrographs of the Daily Flow of the Upper Mississippi River at Hastings, Minnesota.

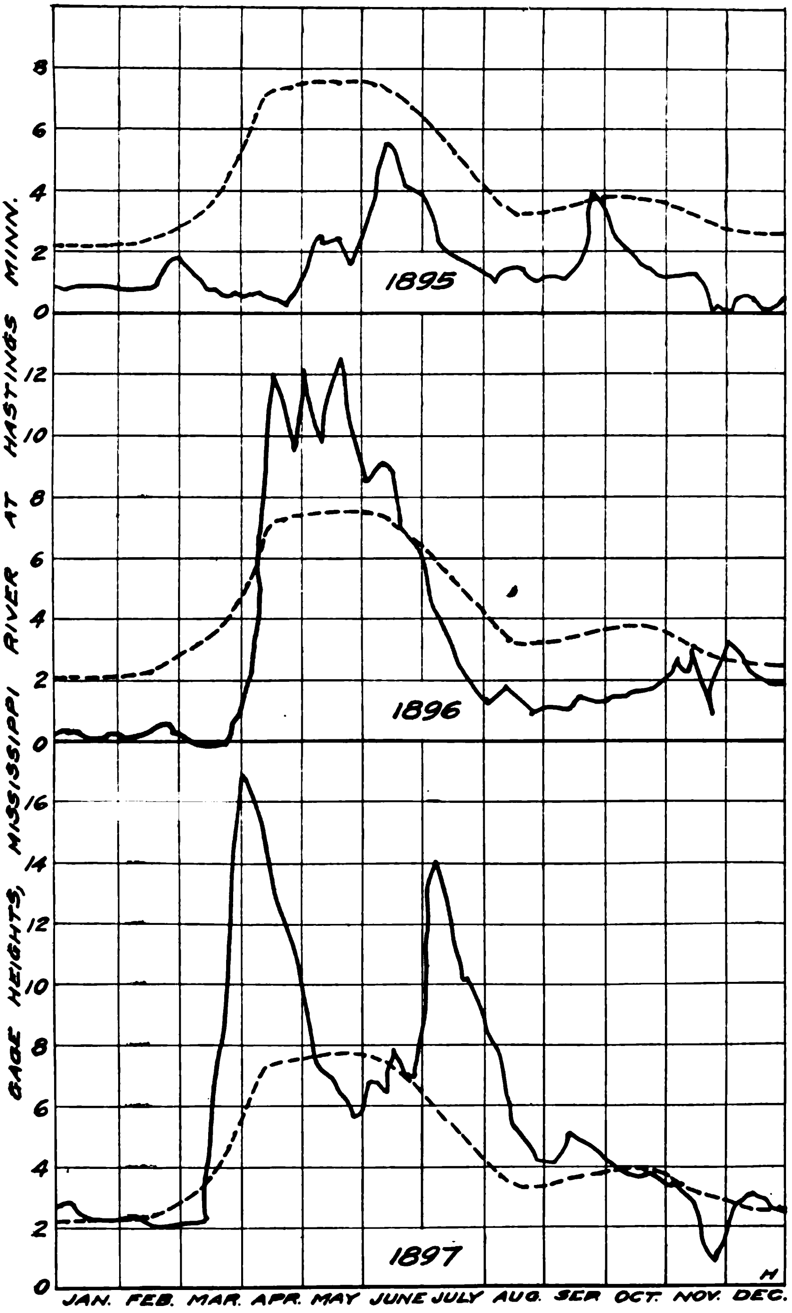


Fig. 96.—Hydrographs of the Daily Flow of the Upper Mississippi River at Hastings, Minnesota.

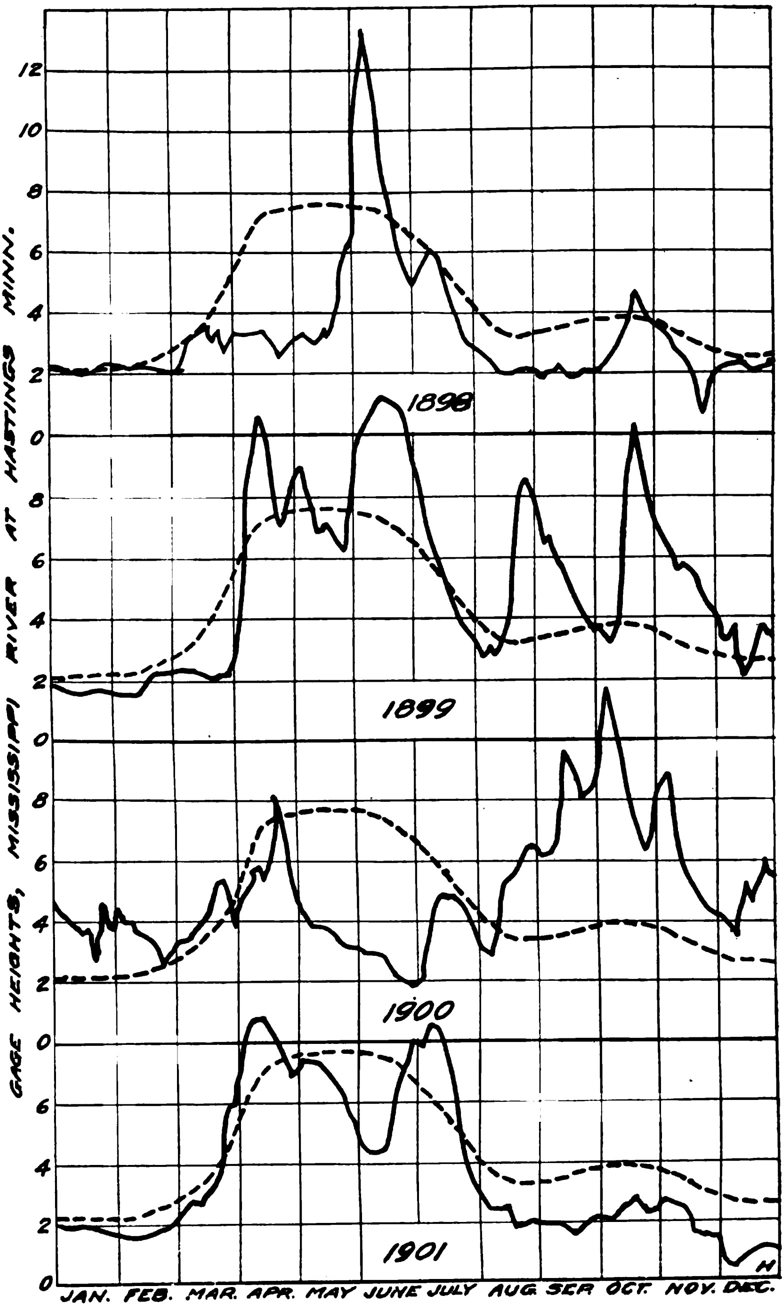


Fig. 97.—Hydrograhs of the Daily Flow of the Upper Mississippi River at Hastings, Minnesota.

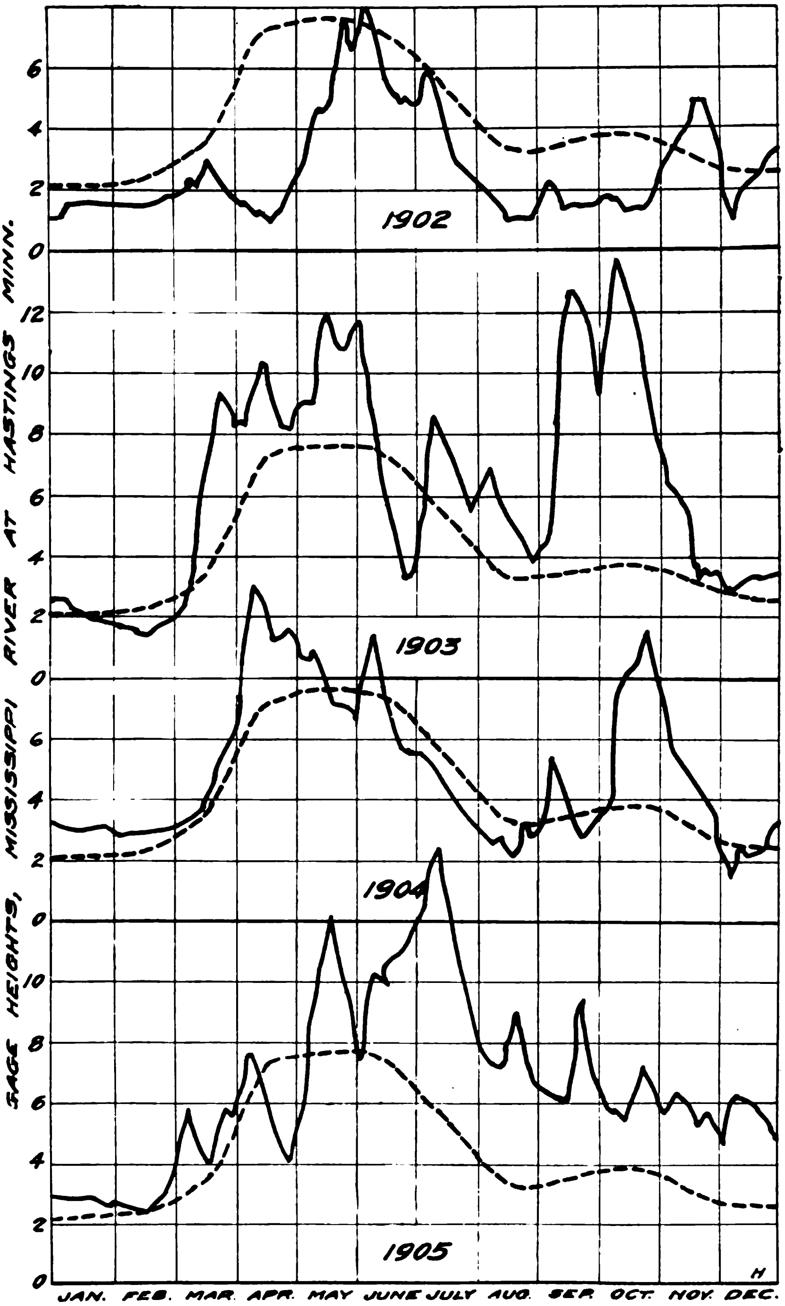


Fig. 98.—Hydrographs of the Daily Flow of the Upper Mississippi River at Hastings, Minnesota.

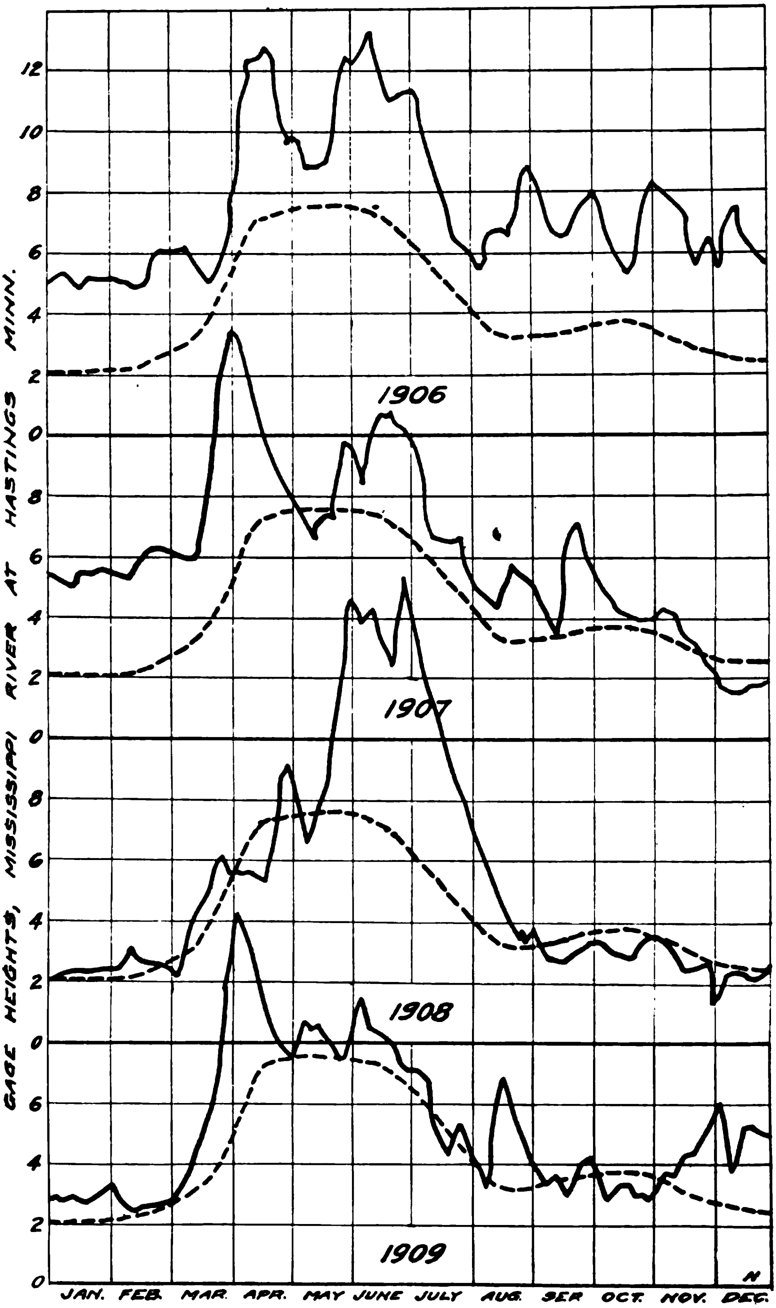


Fig. 99.—Hydrographs of the Daily Flow of the Upper Mississippi River at Hastings, Minnesota.

TABLE 14.—MEAN MONTHLY AND ANNUAL PRECIPITATION IN UPPER MISSISSIPPI VALLEY ABOVE HASTINGS, MINN.

Year.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total annual.
1880.....	0.88	0.98	2.03	1.62	2.87	5.19	2.62	3.20	1.22	1.38	2.30	1.70	25.99
1881.....	2.92	2.55	0.99	0.50	4.47	3.29	2.04	4.28	9.92	3 74	1.78	0.44	36.92
1882.....	0.58	1.88	2.18	2.15	3.23	2.68	2.65	2.34	0.21	1.95	1.70	1.49	23.04
1883.....	1.43	0.68	0.19	4.38	2.33	6.82	4.38	0.84	2.11	2.09	0.84	1.70	27.79
1884.....	0.61	1.74	1.14	1.86	2.54	3.88	2.29	3.40	4.88	1.80	0.66	2.14	26.94
1885.....	0.50	0.20	0.63	2.79	1.84	4.88	5.53	2.87	3.68	0.97	0.76	0.58	25.23
1886.....	1.49	0.55	0.71	4.22	1.14	4.15	3.16	2.26	3.57	1.15	1.74	0.94	25.08
1887.....	1.17	0.99	0.44	2.51	2.07	3.94	3.39	2.85	3.25	1.76	0.55	2.63	25.55
1888.....	0.70	0.62	1.30	4.05	4.68	2.02	5.88	2.32	1.70	1.00	0.28	0.47	25.07
1889.....	0.84	0.66	0.83	1.42	2.59	2.56	3.33	2.67	1.10	0.04	0.71	1.12	17.82
1890.....	0.70	0.60	1.30	1.38	3.22	5.21	2.59	2.70	2.38	2.05	0.45	0.34	22.92
1891.....	0.71	1.38	1.31	2.01	1.48	3.89	3.27	2.75	1.80	1.42	0.83	2.49	23.34
1892.....	0.09	1.38	0.98	2.04	6.70	5.32	8.14	4.06	1.15	0.67	0.55	0.55	31.63
1893.....	1.23	1.37	1.52	4.90	2.26	1.75	2.87	4.14	1.58	1.60	0.76	1.84	25.82
1894.....	0.88	0.14	2.34	4.49	4.71	2.43	0.70	0.85	1.68	3.73	0.72	1.08	23.75
1895.....	0.63	0.46	0.29	1.40	3.35	4.22	3.72	2.18	4.15	0.05	0.90	0.25	21.60
1896.....	0.74	0.28	2.37	6.12	5.05	3.73	1.66	2.35	2.48	3.68	3.34	0.60	32.40
1897.....	1.97	1.38	2.63	1.67	1.53	6.93	8.76	1.83	2.15	1.65	0.61	0.33	31.44
1898.....	0.11	1.19	1.48	1.19	3.47	3.86	2.89	3.17	1.49	4.22	1.23	0.13	24.33
1899.....	0.57	0.87	2.30	1.62	4.61	5.71	2.24	6.57	1.45	3.69	0.69	0.89	31.21
1900.....	0.49	0.65	1.40	1.27	0.49	1.50	5.49	7.16	5.59	3.40	0.60	0.54	28.68
1901.....	0.38	0.28	1.97	1.66	1.43	5.77	2.70	1.90	3.42	1.70	0.92	0.54	22.67
1902.....	0.50	0.46	0.66	2.00	4.22	3.12	4.78	4.14	2.42	1.75	1.84	1.87	27.76
1903.....	0.42	0.70	2.20	3.02	4.93	1.58	6.60	4.54	6.76	3.56	0.38	0.91	35.60
1904.....	0.49	0.91	1.36	1.64	2.57	4.49	4.68	3.71	4.09	4.15	0.13	0.75	28.97
1905.....	0.80	0.44	1.00	1.59	4.87	7.69	4.48	4.59	4.23	2.41	2.48	0.07	34.65
1906.....	1.52	0.35	0.96	2.07	6.49	5.89	3.18	4.85	3.81	2.02	1.71	0.92	33.77
1907.....	1.50	0.60	0.90	0.90	2.35	4.28	3.22	3.73	4.32	1.17	1.00	0.50	24.47
1908.....	0.35	1.00	1.43	3.00	7.12	7.17	3.33	1.70	3.07	1.73	1.15	0.78	31.83
1909.....	1.27	2.15	0.45	2.89	3.57	2.88	4.58	3.79	2.98	1.83	2.32	1.79	30.50
1910.....	1.22	1.62	0.18	1.38	1.47	1.67	1.85	2.61	......	......	......	......	......
Means..	0.89	0.91	1.27	2.38	3.34	4.15	3.77	3.23	3.08	2.08	1.13	1.01	27.56

TABLE 15.—GAGE HEIGHTS AT HASTINGS ON THE MISSISSIPPI RIVER SHOWING COMPARATIVE AVERAGE AND EXTREME CONDITIONS OF FLOW FOR VARIOUS YEARS.

Year.	Extreme high water.	Extreme low water.	Average for the year.	No. of days above 5 ft.	No. of days above 8 ft.	No. of days above 12 ft.	No. of days below 2 ft.	No. of days below 1 ft.	No. of days below 0 ft.	Remarks.
1880	16.80	1.70	5.60	112	73	18	11	0	0	Data for Jan., Feb., Mar., Dec., missing.
1881	18.00	3.60	8.94	232	170	50	0	0	0	Data for Jan.. Feb.. Mar., missing
1882	13.70	2.40	6.63	212	101	30	0	0	0	
1883	13.10	1.00	4.62	93	56	13	27	0	0	Data for Jan.. Feb., Mar., missing.
1884	11.20	1.70	4.88	142	48	0	8	0	0	
1885	9.20	1.10	3.97	111	17	0	18	0	0	
1886	10.00	0.30	3.29	74	22	0	86	6	0	
1887	11.00	0.10	2.61	55	17	0	217	39	0	
1888	14.30	0.30	4.75	119	82	40	120	10	0	
1889	4.20	−0.40	1.68	0	0	0	245	82	10	
1890	6.60	−0.40	2.09	28	0	0	227	101	8	
1891	7.00	−0.60	1.20	29	0	0	286	239	32	
1892	13.10	−0.70	3.33	84	46	10	153	103	4	
1893	14.30	0.10	3.47	80	68	19	203	53	0	
1894	13.00	−0.50	2.64	52	42	6	242	92	12	
1895	5.80	−0.20	1.44	8	0	0	283	154	4	
1896	12.60	−0.60	3.19	83	65	10	217	105	21	
1897	17.00	0.80	5.69	152	80	42	19	5	0	
1898	13.40	0.50	3.25	46	16	6	47	2	0	
1899	11.30	1.40	5.02	169	75	0	64	0	0	
1900	7.70	0.30	3.30	63	0	0	56	17	0	
1901	8.80	0.40	4.67	100	16	0	133	16	0	
1902	8.10	0.80	2.69	30	1	0	184	10	0	
1903	13.80	1.50	6.36	212	137	23	37	0	0	
1904	11.00	1.40	5.05	148	67	0	6	0	0	
1905	14.40	2.30	6.50	260	89	15	0	0	0	
1906	13.50	4.80	7.68	343	136	36	0	0	0	
1907	13.50	1.40	6.20	245	84	13	29	0	0	
1908	15.40	1.30	5.37	143	76	43	6	0	0	
1909	12.20	2.50	5.28	153	61	3	0	0	0	

TABLE 16.—GAGE HEIGHTS AT HASTINGS ON THE MISSISSIPPI RIVER SHOWING AVERAGE MONTHLY AND ANNUAL GAGE HEIGHTS FOR VARIOUS YEARS.

Year	AVERAGE.												
	Jan.	Feb.	ar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	ea:
1880	.. ..	......	......	8.33	8.17	12.35	6.08	2.64	2.32	2.24	2.63	......	5.6
1881	......	......	......	7.37	12.05	9.25	6.43	4.43	9.87	12.78	11.32	7.44	8.94
1882	5.11	426	6.05	12.05	11.60	8.95	9.28	5.69	4.47	4.55	4.57	2.99	6.63
1883	......	......	......	9.84	8.70	7.74	3.99	2.69	2.10	2.36	1.89	2.25	4.62
1884	2.26	4.96	4.03	7.93	9.08	6.67	3.28	2.31	4.48	6.57	4.18	2.86	4.88
1885	2.69	2.69	3.60	5.98	6.20	6.67	5.06	3.49	3.39	3.63	2.33	1.90	3.97
1886	2.25	2.44	3.50	2.17	6.87	4.75	3.08	1.50	2.26	2.27	1.69	1.71	3.29
1887	1.75	1.82	3.68	8.48	5.17	2.71	2.18	1.37	1.56	1.25	0.58	0.77	2.61
1888	1.03	1.62	1.95	9.17	12.90	10.10	6.40	4.37	2.81	2.59	2.33	1.09	4.75
1889	1.15	1.73	2.68	2.30	3.54	2.02	1.29	2.18	1.69	1.00	0.50	0.05	1.68
1890	0.81	0.80	0.90	3.79	2.88	5.65	3.53	1.24	1.52	2.00	1.50	0.44	2.09
1891	0.00	0.56	0.97	5.38	3.50	1.79	1.21	0.41	0.01	0.59	0.01	0.10	1.20
1892	0.15	0.79	2.42	4.28	8.12	9.96	5.60	3.50	2.43	1.53	0.63	0.43	3.33
1893	0.84	1.08	2.23	9.85	12.38	6.27	2.69	1.22	1.92	2.00	0.98	1.14	3.47
1894	1.10	1.16	3.03	6.28	10.73	4.08	1.01	0.46	0.78	1.21	1.08	0.73	2.64
1895	0.86	0.83	1.05	0.40	1.81	4.00	2.25	1.23	1.68	2.04	0.86	0.23	1.44
1896	0.13	0.28	0.14	6.66	11.12	8.12	3.31	1.35	1.20	1.47	2.03	2.41	3.19
1897	2.54	2.08	4.30	14.65	7.43	6.62	11.00	6.34	4.42	3.74	2.62	2.46	5.69
1898	2.08	2.15	2.79	3.13	3.57	8.97	4.50	2.29	1.98	3.19	2.38	2.06	3.25
1899	1.84	1.66	2.07	5.82	7.40	10.18	6.08	4.42	5.80	6.02	5.93	3.01	5.02
1900	2.74	2.46	2.64	4.18	2.63	1.60	2.19	3.30	6.00	6.23	3.90	1.85	3.30
1901	1.83	1.60	3.40	7.78	6.76	5.12	6.58	2.30	1.78	2.25	2.06	9.23	4.67
1902	1.32	1.44	2.22	1.34	4.55	6.43	4.26	1.60	1.57	1.52	3.76	2.24	2.69
1903	2.34	1.65	5.56	7.98	9.88	7.23	6.54	5.38	8.88	11.28	5.21	3.25	6.36
1904	3.12	3.07	4.08	9.30	8.03	7.30	4.43	3.05	3.87	6.95	5.12	2.23	5.05
1905	2.74	2.55	4.60	5.57	8.88	9.70	12.05	7.64	6.96	6.08	5.68	5.53	6.50
1906	5.13	5.21	5.80	11.16	9.68	12.10	8.74	6.65	7.53	6.68	7.19	6.44	7.68
1907	5.34	5.74	7.44	11.18	7.58	9.90	7.47	4.98	5.11	4.47	3.54	1.73	6.20
1908	2.36	2.69	3 98	5.90	8.68	13.80	10.95	5.08	3.03	3.09	2.83	2.06	5.37
1909	2.86	2.79	4.68	10.05	8.06	8.23	5.76	4.92	3.67	3.24	4.03	5.02	5.28

APPENDIX D.

After the preceding pages had gone to the printer, the writer's attention was called to an article in "Engineering News" of June 23rd, 1910, by C. W. Durham, principal assistant, U. S. engineer of Rock Island. The table accompanying Mr. Durham's article contains some important data not included in the previous pages, and his article is here reproduced in part.

It may be remarked that while it is sometimes possible, by a judicious selection of data, to sustain various conclusions in regard to the effect of forests on stream flow, yet, when practically all of the data concerning any one stream, is carefully considered, only one conclusion is possible. The data offered by Mr. Durham is not complete, but it will be found that the data already discussed in this bulletin contain considerable matter that will further illustrate his article, and emphasize his conclusions. (See especially the tables and hydrographs in Appendices A, B, and C.)

RECORDS OF DEFORESTATION AND GAGE HEIGHTS FOR THE ST. CROIX AND CHIPPEWA RIVERS.*

* * *

"The Chippewa, which has furnished enormous quantities of pine and other timber, has its source near the Michigan boundary of the state of Wisconsin and flowing in a southerly direction joins the Mississippi at Reeds Landing, 77 miles below St. Paul. It is 267 miles in length with a drainage area of 9,530 square miles, the greater part of which, before deforestation began, was covered with forests. Of late years the land from which the timber has been removed has been rapidly taken up by settlers so that there is practically no second growth.

"As regards the Chippewa, extraordinary overflows or floods occurred in 1838, '47, '55, '58, '66, '80 and '84, but since 1884 there has been no memorable flood. It will be noted that these floods took place before or during the comparative infancy of deforestation. The flood of 1884, the latest of those mentioned (26 ft. at Chippewa Falls, 27 feet. at Eau Claire) was productive of great damage and attributable to an unusually great rainfall over a comparatively small area, in combination with the breaking of two reservoirs containing together 900,000,000 cu. ft. of water, but even this great quantity of water was productive of only an ordinary high-water stage at the mouth of the river.

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*Engineering News, June 23, 1910. Charles W. Durham, Prin. Asst. U. S. Engineer.

"The St. Croix also rises near the Michigan boundary and flows in a direction parallel to that of the Chippewa. It empties into the Mississippi at Prescott, about 30 miles below St. Paul. It is 168 miles long and its drainage area of 7,146 sq. mi. was once well-nigh covered with forest growth. These lands were formerly thought to be barren and worthless, but since deforestation settlers are rapidly coming in and taking up for cultivation the former sites of pine forests.

"The highest floods recorded at Stillwater, 23 miles up the St. Croix, were 19.7 ft. in 1850, 16.9 ft. in 1880 and 17.8 ft. in 1881.

"Setting aside for the present the consideration of the precipitation of summer rain and winter snow, which governs in no small degree the extent and duration of floods and low waters, it is found necessary on account of lack of sufficient records of precipitation, except in late years, to use the river-gage records in forming our opinion as to both floods and droughts, and it was further thought best to use the records taken at or near the mouths of the rivers, as being less affected by logging dams and reservoirs than points at a distance up the streams. For this reason gage records for the St. Croix used in the following table are taken at Prescott, supplemented by those at Hastings, three miles above on the Mississippi, and for the Chippewa at Reeds Landing, and Winona, 38 miles below, which gages in both cases have always been in close accord especially at low stages.

"In Table 17 deforestation begins with 118 sq. mi. in 1875 and increases to a total of 6,306 sq. mi. at the close of 1909, which is 37.8% of the combined drainage areas (16,676 sq. mi.) of the two streams. While this mileage of deforestation is incomplete, as not including the ravages of former years by lumbermen and the smaller destruction by farmers, it affords a good idea of the enormous extent to which such deforestation has been carried, and it will be noted that the very highest floods took place before deforestation had made much progress, due attention being given to cases previously cited, especially in regard to the Chippewa.

"The quantity of logs cut on the St. Croix and the Chippewa has been ascertained from year to year by the "Northwestern Lumberman." In this article the cut on the drainage areas of the two rivers has also been combined. The cutting and rafting of logs had been going on for many years prior to 1875, the year of our first record, but not on so large a scale as subsequently.

"For the past few years the log cut has been chiefly on the upper Mississippi, above the Falls of St. Anthony, as the pineries of the St. Croix, Chippewa, Black and Wisconsin were practically exhausted some years ago. The average cut is estimated at 7,000 ft. to the acre, as ascertained from lumbermen most familiar with log cutting, and with this unit the mileage of deforestation as given in Table 17 was calculated.

"The low stages at Prescott in 1894 and 1895 were prevalent on the Mississippi river and its tributaries above the mouth of the Missouri and were due to the small precipitation in the summer months.

"High water (over 12 ft.) at the mouths of the St. Croix and Chippewa, occurred, as shown in Table I, but none of the high waters, except those of 1880, 1881 and 1897 in the St. Croix, and 1880 in the Chippewa, are to be considered as floods, the remainder being good navigation stages only and not detrimental in any way. It is to be noted, also, that when such good stages occur, the low water periods of the same year are generally higher than mean low water, which may be put at 1.5 ft.

"The big floods in the St. Croix and Chippewa, except that of 1897 in the St. Croix, occurred before deforestation was much advanced, and if we consider the floods of the St. Croix and Chippewa in early years, the influence of deforestation, if any, on floods cannot be considered as detrimental on these streams.

"As regards low-water (stages below 1.5 being so considered) the deductions from Table I are not so obvious, but it is noted that in the 16 years from 1878 to 1893 inclusive, the stage fell below mean low water 12 times, and from 1894 to 1909 inclusive, only 6 times. * * *

"My conclusions are that the destruction of the pine forests in Wisconsin has had no effect whatever in creating or increasing floods and droughts in tributary streams or in the upper Mississippi, inasmuch as these features are controlled by precipitation, which has not increased or decreased to any marked extent, at least not during the periods of accurate observations made by the Weather Bureau. The high waters are not higher nor the low waters lower than formerly, nor are they of greater frequency and duration, and the tendency appears to be of late years favorable to navigation in both respects."

TABLE 17.—SHOWING BY YEARS FOR THE ST. CROIX AND CHIPPEWA, THE LOG CUT, THE AREAS OF DEFORESTATION, THE HIGHEST AND LOWEST OPEN RIVER WATERS AND THE PERCENTAGE OF DEFORESTATION.

Year	Logs cut M. ft. B. M.	Area deforested per yr. sq. mi.	Total Area deforested sq. mi.	St. Croix at Prescott and Hastings.		Chippewa at Reeds and Winona.		Percentage of entire drainage area deforested.
				H.W.	L.W.	H.W.	L.W.	
1875..	530,032	118	118	....	....	....	....	.007
1876..	355,318	79	197	....	....	....	....	.011
1877..	204,600	45	242	....	....	....	....	.014
1878..	435,238	97	339	....	0.5	....	0.1	.020
1879..	650,036	145	484	10.7	1.1	9.0	0.7	.035
1880..	919,139	205	685	16.8	1.3	16.9	1.6	.041
1881..	1,000,000	223	908	17.6	3.1	14.4	3.0	.054
1882..	878,284	198	1,104	13.3	3.9	11.9	2.5	.066
1883..	825,640	184	1,288	12.7	1.3	12.2	1.8	.077
1884..	905,148	202	1,490	10.8	1.3	11.4	1.7	.089
1885..	920,249	205	1,695	8.8	2.3	8.8	2.5	.101
1886..	941.676	120	1,715	9.6	0.7	10.6	1.1	.103
1887..	632.550	143	1,858	10.6	0.4	10.9	1.4	.111
1888..	671.328	149	2.007	13.9	2.0	14.9	2.2	.123
1889..	678,495	151	2,158	3.8	0.4	5.3	1.1	.140
1890..	789,427	176	2,334	6.7	0.8	8.2	1,9	.148
1891..	651.372	145	2,479	7.5	0.0	8.9	0,2	.163
1892..	1,125,304	251	2,730	13.2	1.1	12.1	1.5	.171
1893..	1,014,372	226	2,956	14.0	0.5	12.6	1.2	.189
1894..	900,320	200	3,156	13.5	0.7	13.3	0.4	.202
1895..	996,166	222	3,378	6.5	0.1	5.5	0.2	.212
1896..	726,667	162	3,540	12.8	0.5	9.5	0.6	.230
1897..	937,187	209	3,749	16.2	3.1	12.3	2.2	.236
1898..	859,164	191	3,940	14.0	1.4	9.0	0.3	.247
1899..	802,153	179	4,119	11.3	2.6	8.9	1.2	.258
1900..	805,515	180	4.299	8.4	0.8	8.5	0.1	.272
1901..	1,090,651	243	4,542	9.5	2.0	7.1	0.9	.285
1902..	984,577	219	4,761	8.8	1.3	5.3	0.7	.293
1903..	590,424	131	4,892	12.5	3.3	11.0	2.8	.303
1904..	524,513	117	5,009	11.3	2.2	7.9	1.7	.366
1905..	499,082	111	6,120	14.7	3.8	10.1	2.6	.367
1906..	301,564	67	6,187	13.5	4.9	10.1	3.4	.371
1907..	265,748	59	6,246	13.6	3.4	10.0	2.1	.374
1908..	143,839	32	6,278	14.8	3.4	10.3	0.9	.376
1909..	123,283	28	6,306	11.6	3.1	7.7	1.7	.378

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SOME ALLOYS OF CALCIUM

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A THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF PHILOSOPHY
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SOME ALLOYS OF CALCIUM

INTRODUCTION

Until recently calcium has been a metal of high cost and great rarity on account of the difficulty of separating it from its compounds. Through improvements in electrolytic reduction methods it has become of moderate price and may now be classed amoung the technically available metals. It is as yet a matter of speculation as to what uses it may serve, and it is natural, therefore, that there should be some general interest in a study of alloys containing calcium as an ingredient.

A review of the literature shows that some alloys of calcium have been made. Three general methods have been employed. First lime was heated with metal to be alloyed in the presence of metallic sodium. Second, calcium carbide and an oxide of the metal were heated together, and third, fused calcium choloride was electrolyzed, using the metal to be alloyed as cathode.

METHODS USED

In this investigation which was carried out in the chemical engineering laboratories of the University of Wisconsin, the alloys were made by adding metallic calcium to the molten metal, the calcium being added in small pieces. This slow addition tends to moderate the violence of the reaction which occurs where the two metals have reached the necessary temperature. In some cases, even when a small quantity of calcium (one or two grams) was added, the violence of the reaction forced some of the molten metal from the crucible.

Crucibles made from Acheson graphite were employed in the preparation of these alloys, an electric furnace of granular resistor type being used to furnish the necessary heat.

Preliminary experiments were made to determine some of the characteristic changes produced by the addition of calcium to pure metals. In each case there was found to be a decrease in the tensile strength accompanied by an increase in the brittleness of the alloys produced. A further increase in the calcium content in some cases produced an alloy which could be pulverized easily.

ALLOYS OF CALCIUM WITH ALUMINUM, MAGNESIUM AND SILICON

Since calcium is a strong reducing agent is was thought that a considerable number of binary and ternary alloys could be made which would be of value as reducing agents. This suggested a study of the alloys of calcium with aluminum, magnesium and silicon.

Several alloys of calcium-aluminum and calcium-aluminum-silicon were made. The metals were melted in a graphite crucible containing sufficient calcium chloride to cover the molten alloy. The metals were added in this order; aluminum, magnesium, calcium. In the case of alloys containing silicon it was found advantageous to add the silicon to the binary alloys of aluminum-calcium or of aluminum-magnesium as the solubility of this element was greater in these combinations than in the aluminum alone. In each case there was an evolution of heat when the alloy was formed, and in some cases the reaction was sufficiently violent to force a portion of the molten metal from the crucible.

The violence of the reaction was diminished to a considerable degree by inoculating the molten metal with a small quantity of metallic calicum after which larger quantities of calcium could be added without losing any portion of the charge. The resulting alloy was stirred thoroughly by means of a graphite rod and when sufficiently fluid was poured into an iron mould. This method of operation tended to keep the alloy uniform in composition.

A series of calcium-aluminum alloys was made to ascertain how much calcium at one end of the series would make aluminum brittle and how much aluminum at the other end of the series would make calcium brittle. It was noticed that the point of brittleness sufficient to enable the alloys to be pulverised in a disc grinder lay between 20 and 25 per cent calcium. The alloy containing 20 per cent calcium was somewhat malleable and could not be pulverized in a disc grinder. An alloy containing 80 per cent calcium could be pulverised without difficulty.

Several alloys of calcium-aluminum-silicon and calcium-magnesium-silicon were made. All of these alloys containing 25 per cent calcium, or its equivalent of calcium and magnesium, or of magnesium alone could be pulverized in a disc grinder.

The binary alloys of calcium and aluminum containing the lower percentages of calcium were made without any special precaution being taken to prevent oxidation. As the percentage of calcium increased the tendency to oxidize increased to such an extent, that even with a protective layer of calcium chloride, it seemed impossible to prepare in an open crucible an alloy containing more than 85 per cent calcium. The alloys containing the higher percentages of calcium and magnesium were difficult to prepare on account of the low specific gravity of the resulting product. This rendered calcium-chloride too heavy to be used as a protective covering. No suitable protective material was secured for these lighter alloys.

CHEMICAL PROPERTIES

All of these alloys containing more than 5 per cent calcium decomposed water. The quantity of hydrogen increased with the increase in the calcium content of the alloy. In dilute hydrochloric acid the evolution of hydrogen was more rapid, and with alloys containing silicon, a spontaneously inflammable silicon hydride, gas was liberated.

The stability of some of these alloys in air was noteworthy. Pieces of various alloys weighing about 5 grams were placed on an exposure board and subjected to the atmosphere of the laboratory. At the end of four weeks it was observed that the alloys which contained the highest percentages of calcium and silicon, were disintegrating, while those alloys of the lower percentages of calcium had become coated with a layer of oxide which prevented further oxidation.

The time required to melt and pour a charge of about one pound of the alloys varied from five to twenty minutes, depending upon whether the furnace was hot or cold when the charge was put in, and also upon the calcium added as the heat generated was such as to assist materially in securing the necessary temperature. From two to five kilowatt hours were used for heating purposes.

Use in Goldschmidt Process

Since so many of the alloys of calcium, aluminum, magnesium and silicon could be pulverized to any degree of fineness, and since several of these alloys could be stored for considerable time without serious deterioration, it was thought that these might be used as a substitute for the aluminum used in the Goldschmidt process. This consists in the reduction of a metallic oxide by pulverized aluminum. An excess of this oxide to be reduced is mixed with powdered aluminum to prevent the formation of an alloy composed of the reduced metal and the reducing agent. The charge is placed in a suitable container and ignited by a magnesium ribbon fuse.

The success of the Goldschmidt reaction depends upon the heat liberated during the formation of alumina. The total heat liberated depends upon the differences of the heats of formation of the metallic oxides which are to be reduced and of the alumina formed in the reaction. From this it is evident that it is impossible to reduce by the Goldschmidt process those oxides which have a heat of formation greater than alumina, unless heat is supplied from an exterior source.

The values given by the various experimenters for the heat of formation of alumina, lime and magnesia differ considerably, as shown by the date gathered from various sources.

I. $2\,Al + 3\,O = Al_2O_3 + 386988$ calories. Strauss: Minet. Production of Aluminum, page 209.
$2\,Al + 3\,O = Al_2O_3 + 380200$ calories. Thomsen (L. & B.).
$2\,Al + 3\,O = Al_2O_3 + 392610$ calories. Richards (E. & M. I. 6–8).

II. $Mg + O = MgO + 148000$ calories. Strauss; Minet. Production of Aluminum, page 209
$Mg + O = MgO + 143300$ calories. Beketoff.
$Mg + O = MgO + 143400$ calories. E. & M. I. 6-8.

III. $Ca + O = CaO + 151900$ calories. Guntz and Besset. C. R. 1905, Vol. 140, pages 863, 864.
$Ca + O = CaO + 131500$ calories. Thomsen (L. & B.).
$Ca + O = CaO + 145000$ calories. Moissan.
$Ca + O = CaO + 131648$ calories. Strauss; Production of Aluminum, page 209.
$Ca + O = CaO + 130900$ calories. Thomsen (L. & B.).

Guntz used pure calcium and formed pure compounds from which he determined the heats of solution. This necessitated several corrections. Forcrand made pure CaO from $Ca(OH)_2$ and used this product in his determination. His value agrees with that of Guntz. (C. R. 1908, Vol. 146, pages 217 and 220). Moissan's value was determined from the heat of combustion of metallic calcium and is therefore less liable to error than the values determined by indirect methods.

From the above table it will be seen that the average value for the heat of combustion of magnesium is 145000 calories, of calcium 137000 calories and of aluminum 129000 calories; hence from the theory of the greatest heat evolution, magnesium should reduce lime and alumina, and calcium should reduce alumina, but not magnesia. Aluminum should reduce neither lime nor magnesia. However, Goldschmidt has succeeded in reducing lime by aluminum (Minet, Production of Aluminum, page 209), and Perkin has reduced alumina by calcium. Weston and Ellis (Trans. Faraday Society, 1908) have shown that aluminum powder will reduce lime at high temperatures and that the reaction

$2\,Al + 3\,CaO \rightleftharpoons 3\,Ca + Al_2O_3$[1].

is reversible. This leaves the question of the relative reducing powers of calcium and aluminum in doubt. It has been shown conclusively, however, that magnesium[2] will reduce alumina and lime vigorously at ordinary temperatures.

In view of the relative reducing powers of these three metals the preparation of binary and ternary alloys of calcium, magnesium and aluminum was undertaken with the hope of obtaining reducing agents which would have a higher heat of oxidation per unit of oxygen than has aluminum, and whose mixed oxides would form a slag more fusible than alumina. It was also thought that there would be a decided advantage, so far as the fluidity of the slag was concerned, in having the particles of the different oxides produced in intimate contact. This would be the case to a much greater degree when the metals are alloyed, than when the oxides are formed from isolated particles of each metal in a mechanical mixture.

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[1] Trans. Faraday Society, 1908.
[2] Trans. Faraday Society, 1908.

Twenty-three alloys of calcium with one, two or all three elements, aluminum, magnesium and silicon, were prepared. Their composition is shown in the following table.

No. of Alloy.	Formula.	Aluminum.	Calcium.	Magnesium.	Silicon.
1		75.	25.		
3		74.5	25.5		
4		45.	55.		
5			30.	70.	
6		60.		40.	
7	$CaMg_2$		45.5	54.5	
8		50.	30.	20.	
9	$AlCaSi$	28.4	42.		29.5
11	$AlCaMg$	29.7	43.9	26.3	
12	$AlCaMg_2$	23.5	34.8	41.7	
14	$Al_4CaMgSi_3$	41.4	15.8	9.4	33.8
15	$AlCa_2Mg_2$	17.4	51.6	31.	
16		93.8	6.4		
17		90.	10.		
18		84.	10.	6.	
19		67.1	12.9		
20		80.	20.		
21		80.	12.5	7.5	
22		27.2	72.8		
25		60.		40.	
31		65.			35.
32		45.		20.	35.
33		60.	40.		
34	40% Al + 60 $CaSi_2$	40.	25.		35.
35	$Al_2Ca_3Mg_2$	24.3	54.1	21.6	

The percentage composition given in the above table was calculated from the amounts of the constituents originally weighed out, and is subject to errors of from 2 to 3 per cent, as the balance used was for rough weighing only, and as there was a variable amount of calcium chloride adhering to the sticks of calcium. Determinations of calcium in two alloys containing a high percentage of this metal showed the errors to be within the limits mentioned.

Attempts were made to reduce the compounds MoO_3, MoS_2, WO_3, SiO_2, Fe_3O_4, Mn_3O_4, TiO_2 and mixtures of SiO_2 with MnO_2, of TiO_2 with MnO_2 and of TiO_2 with Fe_3O_4. The alloys employed in these reductions were Nos. 8, 11, 14 and 15.

The first experiment was an attempt to reduce MoO_3 by alloy No. 8. The oxide was the usual fine powder of commerce and the alloy a mixture of all sizes that would pass a twenty-mesh sieve. The reaction was very rapid, and was accompanied by the projection of melted material, flame and much smoke. At the end of the reaction the crucible was nearly empty, and contained no metal. Thinking that the sudden expansion of air contained between the particles of the charge might be responsible for much of the loss of material, the experiment was repeated in a vacuum cylinder constructed from a piece of eight-inch steam pipe. Less material was projected than before, but the loss was still great. Forty grams of well-fused molybdenum were obtained.

Several other experiments were tried in vacuo. SiO_2 was only partially reduced, yielding a brown product resembling silicon monoxide in appearance. A mixture of MnO_2 and SiO_2, reduced by alloy No. 11 (AlCaMg) gave an excellent ingot of silicon-manganese, well separated from the slag. The yield was only 22 per cent. of the theoretical, probably due to the large amount of material projected from the crucible. In order to reduce the velocity of reaction, pulverized fluor spar and lime were added to several charges, with a slight improvement in the yield of metal. After eight trials in the vacuum cylinder, experiments under atmospheric pressure were resumed.

Reduction of a mixture of MnO_2 and TiO_2 by No. 15 ($AlCa_2Mg_2$), in pieces one-fourth to one-eighth of an inch in length, was violent, with much flame and smoke, and the projection of some slag. Only a few globules of metal were obtained. Other reactions were tried with quite similar results. The only alloy that gave quiet reactions was No. 14 ($Al_4CaMgSi_3$). This acted much like aluminum itself.

To control the violence of the reaction the charges were fired in a closed steel cylinder designed to withstand a pressure of 10.000 pounds per square inch. A lining of magnesia was placed within the cylinder and the remaining space was completely filled with the charge which was ignited by means of an aluminum fuse wire. With this apparatus several difficult reductions were carried out. Mixtures of the alloys with molybdenite in theoretical proportions failed to react, but on adding a large excess of the alloy and enough calcium peroxide to oxidize it, an ingot of well fused metal was obtained.

The most difficult reduction carried out in the closed steel cylinder was that of rutile. By means of the calcium peroxide method[3] mentioned above, a single button weighing fifteen grams and several smaller globules of metal were secured. The metal obtained was hard but not sufficiently so to scratch glass. It was not in the least malleable and was broken by severe pounding with a hammer. In appearance it resembled freshly broken Goldschmidt manganese.

Several attempts to produce fused tungsten by the reduction of WO_3 with different alloys failed. Since the heat of reaction was not sufficient to produce this result, an attempt was made to pre-heat the charge in order to secure the desired temperature. For this purpose the charge containing WO_3 and the alloy in the necessary proportions was placed in a magnesia crucible four inches in diameter and six inches in depth. This crucible containing the charge was imbedded in a layer of thermit ($2\ Al + Fe_2O_3$) three inches in thickness which in turn was surrounded by a layer of magnesia one inch in thickness. The magnesia and the Fe_2O_3 were dried at 400°C in order to remove moisture. The thermit was ignited by a magnesium ribbon, and when the heat produced by this reaction was sufficiently great it ignited the charge in the magnesia crucible. The resulting temperature within the magnesia crucible was much higher, but was not sufficiently high to produce fused metal. An attempt to increase the temperature produced in this reaction by pre-heating the thermit failed to produce the desired temperature.

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[3] Bull. of Univ. of Wisconsin, No. 145, page 313.

The following table gives a list of the more difficult reductions carried out by means of these alloys:

Compound.	Reducer.	Result.	Yield.
MnO_3	15	58 grams fused metal............	46.7
MnO_3	15	58 grams fused metal............	47.2
MoS_2	15	31 grams fused metal............	32.0
WO_3	11	Finely divided metal	....
SiO_2	11	Amorphous silicon	....
Mn_2O_3	14	59 grams fused metal............	47.2
Mn_2O_3	36	31 grams fused metal............	21.
Fe_3O_4......	36	143 grams fused metal...........	78.1
Fe_3O_4......	36	143 grams fused metal...........	77.
TiO_2........	32	50 grams fused metal............	....

The cause of the violent nature of the reactions with these alloys, whether carried out in vacuo or in air, whether the reducing agents were coarsely or finely pulverized, was puzzling. The following explanation is offered: When all the products of a reaction are non-volatile, and when no gases are present, it would seem that mere rapidity of reaction would not account for the ejection of one-half the charge from the crucible. If the cause does not lie in the volatile nature of the products formed it must be sought in the original charge. This excessive violence is due to the presence of metallic calcium. This metal boils between 1290°–1300° [4] and the expulsion of the material is no doubt due to this particular physical characteristic of the metal in question.

An attempt was made to avoid the vaporization of calcium. Since calcium silicide is a product formed at high temperatures it should be stable at the temperatures reached in these reductions. Therefore, it should follow that if the calcium and silicon are added in the proportion to form calcium silicide, none of the calcium should be volatilized until the silicide is decomposed through the agency of oxidation. When the silicide is thus decomposed it is evident that the calcium would be oxidized to lime without vaporization.

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[4] Trans. A. E. S., Vol. XIII, p. 149.

Two alloys containing calcium and silicon in the proportions to form calcium silicide were made to test this theory. When Fe_3O_4 is reduced with either of these alloys the reaction is gentle. No material is projected from the crucible. The presence of the silicon in these alloys, whether combined or free, would greatly reduce the temperature obtained. It is possible, however, that the decrease in violence is due to a reduction of temperature, yet the fact that the temperature is high enough to make the iron sufficiently fluid shows that there is little, if any, reduction of temperature.

Conclusions.

(1) The alloys of this series containing up to seventy-five per cent calcium can be prepared without much difficulty.

(2) Alloys containing twenty-five per cent or more of calcium or magnesium can be pulverized.

(3) Violence of reaction is due to the calcium content of these alloys.

CALCIUM-ZINC ALLOYS

A study of the alloys of calcium and zinc was next undertaken. A review of the literature shows that several members of this series have been made. The general method employed was that used by Caron[5] and Tarugi.[6] This consisted in heating calcium chloride with metallic sodium in the presence of metallic zinc. Donski[7] has succeeded in making several of the alloys of this series by heating the two metals together in evacuated tubes or in neutral gases.

The alloys of this investigation were made by adding the metallic calcium to a bath of molten zinc, which has been heated in a graphite crucible. Several attempts to add metallic calcium to the molten zinc resulted in the loss of a considerable portion of the charge, owing to the violence of the resulting reaction. This loss of material was avoided by adding the calcium to the zinc as soon as the latter was melted. With this method of work the two metals were heated in contact so that the alloy could be formed at a lower temperature and thus the violence of the reaction could be greatly reduced.

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[5] Liebig's Ann. Vol. 115, 355.

[6] Bull. Soc. Chimie. Vol. 24, 250.

[7] Zeit. f. Anorg. Chem. Vol. 57, 186.

PROPERTIES

The hardness of this series of alloys increased up to 6 per cent calcium. Beyond this percentage the alloys increased in brittleness to such an extent that several fell to pieces before they were removed from the mould. Several attempts to form solid bars of alloys containing 12 per cent to 14 per cent failed. Beyond this point, although the alloys were brittle, they could be formed into bars without much trouble.

Several of this series of alloys were stable in air. Those alloys which contained less than 10 per cent calcium did not oxidize to any appreciable extent upon being exposed to the atmosphere of the laboratory for two weeks. Beyond this point the rate of oxidation increased to such an extent that the alloy containing 22 per cent calcium disintegrated in a few days. Between 22 and 35 per cent calcium the stability of the alloys increased so that those containing from 28 to 35 per cent were equally stable with those containing from 12 to 14. Beyond 40 per cent the disintegration was quite rapid.

The color and fracture of the alloys belonging to this series changed gradually from those characteristic of zinc to those characteristic of calcium. The combinations containing the smaller percentages of calcium lost the characteristic crystalline structure of zinc and assumed a very smooth even structure instead. No trace of the typical zinc crystallization was in evidence. A change of color accompanied this change in crystalline formation. The alloys containing from 3 to 8 per cent calcium were silver-white metals which did not tarnish even after six months' exposure to the atmosphere of the laboratory. Beyond 8 per cent the alloys slowly assumed a yellowish tinge which changed into black as the higher percentages of calcium were reached.

USE IN GALVANIZING PROCESS

Since the alloys containing the lower percentages of calcium possessed a silver-white color which is not modified on exposure to air it was thought that these might be used in the galvanizing process.

A series of tests was made using alloys containing 0.5, 1.0, 1.25, 1.5, 2.0, 3.4, 5.0 percentages of calcium. The method employed was as follows: Pieces of sheet iron were cleaned and dipped in a bath of molten zinc, protected from the air by a mixture of zinc chloride and ammonium chloride. These pieces were immersed for different periods and the physical characteristic of the coating noted. These were used as standards.

Each of the above series of alloys was used in place of the zinc. A difficulty arose from the fact that the calcium of the alloy gradually replaced the zinc of the zinc chloride forming calcium chloride which was pasty at the temperature of the bath. This interfered with the mechanical part of the process to such an extent that the covering had to be dispensed with. No suitable covering was found.

RESULTS

The coatings made by the alloys may be characterized as follows:—

(1) No traces of zinc crystallization appeared.

(2) The alloys produced a white silvery surface which remained unchanged upon exposure to air for several months.

(3) The coatings produced by the alloys were not so smooth and uniform as those produced by zinc alone.

(4) The coatings were equally adhesive with those of zinc when the deposit is in a very thin layer. When the coating is heavy the alloy becomes brittle and is easily removed by bending the plate.

(5) The brittleness of the coating is dependent upon the calcium content of the alloy.

CONSTITUTION OF ALLOYS

Since both the calcium-aluminum and the calcium-zinc series of alloys presented marked variations in the physical and chemical behavior of its individual members, it was deemed advisable to investigate the constitution of the alloys composing these series.

Modern research has shown that combinations of metals may take the form of simple mixtures, solid solutions or chemical compounds. In case of the two former it must be expected that the physical properties of the alloy in question must result from a blending of the properties of the original metals. With the compounds, on the other hand, the union of these metals in atomic proportions may result in a substance with properties entirely at variance with those of the original constituents.

The marked variations, above mentioned, in the calcium-aluminum and the calcium-zinc series must be due, therefore, to the formation of combinations of definite proportions.

Several methods are adapted to the study of the nature and constitution of alloys. Roberts-Austen[8] has classified them under the two following heads:—

(1) The chemical grouping of the metals in a solid alloy.
(2 The separation of the constituents during solidification.

The first group includes the following special methods:—

(a) The specific gravity of alloys.
(b) The electrical resistance of alloys.
(c) The diffusion of metals in alloys.
(d) Electrolytic conduction.
(e) The heat of combination of metals to form alloys.
(f) The electromotive force of alloys.
(g) Microscopic examination of alloys.

The second group deals with those methods involving a study of the separation of the constituents of an alloy on solidification and includes:—

(a) Measurement of the fall of temperature during solidification by means of a pyrometer.
(b) Mechanical separation of constituents of an alloy by heating to definite temperatures and pressing out liquid portion.

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[8] Alloys. Law. Page 39.

METHODS OF INVESTIGATION

In this particular investigation an attempt was made to locate the compounds formed by the following methods:—

(1) Measurements of single potentials in non-aqueous solutions.

(2) Measurement of electrical conductivity.

(3) Microscopic examination.

The measurement of single potential was introduced by Laurie[9] and has given valuable results. Puschin[10] has employed this method with a number of alloys and finds that the compounds located in this way agree exactly with the compounds located by means of cooling curves. Laurie and Puschin have found that a sharp depression in a single potential curve indicates the presence of a compound at that point.

These conclusions are in agreement with Toutourine[11] whose work goes to prove that:—

(a) The single potential is a function of the composition of an alloy.

(b) Depressions in curve indicate compounds.

(c) The initial and terminal point of each line is determined by the single potential of the phases forming the series.

(d) Solid solutions do not affect the series.

The study of the electrical conductivity of alloys began with the work of Matthiessen in 1860. He showed that the conductivity curve was similar to the cooling curve of the same alloy. Le Chatelier[12] and Rayleigh[13] have shown independently that the temperature factor for the conductivity of alloys is entirely independent of the temperature factor of pure metals. Kurnakow and Źemczuznyj[14] have shown that solid solutions lower the conductivity and that the curve formed by such solutions is continuous and has a minimum value. Guertler[15] has shown that it is possible to establish the exact relationship between the conductivity curve and the compounds formed in a series of alloys. He shows that Liebenow's[16] conclusion concerning the locations of compounds by means of sharp upward projections in the conductivity curve is not entirely true, for in many cases a compound occurs when there is but a slight elevation or perhaps a slight depression. Guertler concludes that the absence of an elevation does not necessarily indicate the absence of a compound but the presence of such an elevation is absolute evidence of the presence of a compound.

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[9] Chem. Soc. Journal. Vol. 53 (1888) 105.

[10] Zeit. f. Anorg. Chem. Vol. 56, 1.

[11] Soc. Chimique France. 1908.

[12] Compt. Rendu. Vol. III, 454. Vol. 119-126, 1709.

[13] Nature, Vol. 54-154.

[14] Ber. St. Petersb. Polyt. Inst. Vol. 6-559.

[15] Zeit. f. Anorg. Chem. Vol. 52-413.

The evidence afforded by the microscope in the study of the composition of metals and alloys has been of much value. The systematic study of metals as revealed by the microscope was first undertaken by Dr. Sorby[17] of Sheffield, who was studying the structure of meteoric irons. In 1865 his paper entitled *On the Microscopic Structure of Meteorites and Meteoric Iron* was published by the Royal Society. Nothing further appears to have been done until Professor Martens[18] published in 1878 the results of the researches which he carried on independently. The importance of the study of the micro-structure of metals and alloys was soon recognized, and the work of Dr. Sorby and Professor Martens has been taken up and greatly extended in recent years.

The preparation of the alloys for single potential and conductivity measurement in this investigation was as follows: The calcium was added to the fused metal with which it was to be alloyed as soon as the latter was liquid. This allowed the two metals to be heated in contact so that the resulting alloy formed at the lowest possible temperature. After the required amount of calcium had been added the molten metal was stirred vigorously so as to render the product as uniform as possible. The molten metal was poured as quickly as possible into an iron mould which had been heated to a temperature just below the freezing point of the alloy. This produced a chilling effect sufficiently great to prevent the segregation of the metals forming the alloy.

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[16] Zeit. f. Anorg. Chemie. Vol. 60-209.
[17] Br. Assoc. Reports. Part II, 1864-189.
[18] Engineering, Vol. 28-88.

The amount of metal poured in each case was sufficient to fill the entire mould as well as the entrance chamber above the mould. This enabled the alloy to pass from the temperature of the molten metal to a point considerably below its solidification temperature in the absence of air.

During the progress of these experiments it was found advantageous to hold the temperature of the alloy constant for some time after pouring. The mould was then cooled slowly.

Fused calcium chloride was used as a covering for these alloys during the process of melting.

SINGLE POTENTIAL MEASUREMENT

Several difficulties were encountered in the attempt to measure the single potential of these combinations. It was necessary to use a non-aqueous electrolyte on account of the calcium content of these alloys. After considerable experimental work with various non-aqueous electrolytes a solution of calcium chloride in ethyl alcohol was selected for this work.

The ethyl alcohol was allowed to stand over freshly ignited lime for five days. This mixture was shaken from time to time. The alcohol was distilled from the lime in a moisture free apparatus, after which it was treated with a fresh supply of lime and allowed to stand for some time. This process was repeated twice. The alcohol was then treated with anhydrous copper sulphate and allowed to stand for five days, during which time the mixture was shaken from time to time. The alcohol was distilled off, using the same precautions as before. After two treatments with anhydrous copper sulphate the material showed no evidence of the presence of water when placed upon the anhydrous sulphate, even after an interval of ten days.

In each distillation the first twenty-five cubic centimeters of the distillate were rejected.

A twelfth-normal solution of calcium chloride was prepared. Kahlbaum's anhydrous calcium chloride was used. This had been heated for eighteen hours at 120°C. A fine white precipitate remained in suspension in this solution. This stock solution was kept in a glass-stoppered bottle.

Considerable difficulty was encountered in an attempt to prevent the diffusion of the aqueous solution of the normal calomel electrode and the non-aqueous solution in which the single potential of the alloy was to be measured. The ordinary connection of the normal calomel electrode was worthless. A modification of this worked very well.

Pieces of glass tubing were heated and pulled so as to form a narrow constriction in the tube. These were packed on either side with finely macerated filter paper moistened with water. The constriction served to hold the plugs in place. These tubes were dried in an oven and the macerated filter paper was pressed firmly together. The filter paper plugs were three-quarters of an inch in length. These glass tubes containing the plugs were sealed to the tubes leading to the normal calomel electrode and to the half cells in which the single potentials of the alloys were to be measured. These tubes served as the connections with the intermediate vessel. That the diffusion was reduced to a minimum is shown by the fact that when the electrolyte was placed in the half cell and the alloy placed in contact with the solution, it was necessary to add more electrolyte in order to make a reading only after the lapse of several hours.

The half cells were dried for several hours at 115°C and allowed to cool in a desiccator containing sticks of sodium hydroxide. The electrolyte was poured in and the alloy placed in contact with the solution. The half cell was stoppered with a rubber stopper. Rubber stoppers were used in all cases. The half cells were kept in desiccators when not being actually measured.

The alloy whose single potential was to be measured, was thoroughly cleaned and its surface polished so as to remove the scratches visible to the naked eye. Electrical contact with the alloy was made by means of a fine copper wire. The junction of the wire and the alloy and that portion of the wire within the half cell was covered with a heavy layer of wax to prevent the formation of a couple in case the electrolyte should come in contact with the wire and the alloy.

The intermediate vessel was closed with a two-hole rubber stopper so that the arms from the normal calomel electrode and the half cell could be inserted without loss of time, when a single potential measurement was to be made. When not in use, this vessel was kept closely stoppered so as to prevent the entrance of moisture. An alcoholic solution of calcium chloride was used as the intermediate solution between the calomel electrode and the half cell. After each measurement the contents of the intermediate vessel were thrown away and a new supply used for the next measurement. This arrangement reduced the amount of moisture present to a minimum.

The single potentials were measured by means of a direct reading Leeds and Northrup potentiometer. This enabled individual readings to be made in less than one minute. A D'Arsonval galvanometer was used as a zero instrument.

The electromotive force varied considerably at first. Measurements were repeated every thirty minutes at the beginning of the work. As the amount of variation decreased the interval between the readings was increased. In some cases forty-eight hours were required to obtain constant readings.

Measurements were made at room temperatures.

A series of zinc-calcium and aluminum-calcium alloys were made. These were analyzed and the single potential curve plotted on the basis of analyzed percentages. Table *I* contains data for the calcium-zinc series and Table *II* for the calcium-aluminum series.

TABLE I.

E. M. F.	Time in Hours.	% Calcium.	Single Potential.
.9502	10	.7	.3972
.9421	10	1.3	.3821
.8930	10	1.7	.3330
.9000	9	2.0	.3400
.9060	9	2.8	.3460
.8580	8	5.6	.2980
.9650	6	6.0	.4050
.9730	3	8.2	.4150
.9675	9	9.6	.4075
.9641	7	11.9	.4041
1.0350	4	14.1	.4750
1.0430	9	15.3	.4830
1.0530	8	16.1	.4930
1.0580	10	18.2	.4980
1.0640	10	24.2	.5040
1.0510	10	26.2	.4910
1.0580	10	27.3	.4980
1.1608	19	28.9	.6008
1.1840	4	31.5	.6240
1.0580	14	32.0	.4980
1.2820	40	32.3	.7220
1.2720	29	33.2	.7120
1.2940	24	40.3	.7340
1.3480	14	56.3	.7880
1.3700	24	66.5	.8100
1.3790	12	70.6	.8190
1.5050	15	73.2	.9450

TABLE II.

E. M. F.	Time in Hours.	% Calcium	Single Potential.
.6550	24	Al.	.0950
.7203	36	3.07	.1603
.7300	48	6.0	.1700
.9108	24	11.5	.3508
.9300	28	12.7	.3700
.9480	22	19.4	.3880
.0	4	23.9	.4690
1.050	8	25.0	.4650
1 320	4	26.9	.4720
1.0450	9	27.0	.4850
1.0270	8	28.5	.4670
1.0400	7	31.2	.4800
1.0410	4	32.8	.4810
1.4720	10	34.3	.9120
1.4700	12	37.7	.9100
1.4810	16	50.3	.9210
1.5280	20	60.0	.9280
1.5220	21	70.5	.9620
1.5410	12	75.0	.9810
5400	21	78.8	.9800
.6240	15	80.2	1.0640

CONDUCTIVITY MEASUREMENTS

The method employed in making the conductivity measurements was as follows: Each bar of the individual members of the series was supported upon two knife edges which were connected to a direct reading Leeds and Northrup potentiometer. In this way the same length of bar was measured in each case. Several measurements of the diameter of each bar were made and the average was used in the computations. As there was some variation in the final conductivity values of the bars of each of the different percentages the average value was taken. A direct current of from ten to fifty amperes was passed through the bars. The ammeter and potentiometer readings were taken simultaneously. Contact with the bars was made by means of two heavy brass springs which held the bars firmly upon the knife edges.

Measurements were made at room temperatures.

Table *III* gives the data obtained in the measurement of the conductivity of the alloys of the calcium-aluminum series.

TABLE III.

% Calcium.	Diameter in Cm.	Amperes.	Milli-volts.	Microhms per Cm³.	Specific conductivity.
3.07	.819	37.50	.8	15.8	63291
6.00	.822	32.75	.9	20.2	49505
....	.812	34.75	1.25	27.1	36900
12.5	.822	28.75	1.50	38.4	26042
19.4	.829	29.25	2.0	49.5	20202
23.9	.835	32.25	4.5	99.5	10050
25.0	.836	28.25	4.5	113.4	8848
28.5	.842	10.75	1.8	117.0	8547
31.3	.855	11.50	2.0	118.0	8474
32.8	.887	26.00	4.6	112.0	8928
37.7	.860	19.00	3.1	110.0	9091
50.0	.862	20.0	4.2	141.0	7093
60.0	.847	28.50	7.0	171.0	5848
75.0	.850	6.25	1.7	189.0	5298
78.8	.837	9.5	1.5	112.0	8928

The length of bar measured in each case was 2.56 centimeters.

Five breaks occur in the single potential curve for the calcium-zinc series. These appear at the following percentages of calcium: 5.6 per cent, between 11.9 and 14 per cent, 29 per cent, between 32.5 and 33.5 per cent, and at 71 per cent. These breaks would correspond to combinations of the following formulae: $CaZn_{10}$, $CaZn_4$, Ca_2Zn_3, Ca_2Znx and Ca_4Zn.

A peculiarity of this series was observed during the measurements of single potentials of the alloys containing between 29 and 34 per cent calcium. Several combinations corresponding in composition to these alloys were made. Their single potentials varied, some having a value of 0.49 volts, while others had a value of 0.61 volts. The calicum content of these alloys differed in some cases by less than 0.5 per cent. This would tend to show that there were two combinations present. Tamman and Donski[10] have located two polymorphic forms between these two limits.

In the single potential curve for the calcium-aluminum series a marked increase in value occurs between 33 and 34 per cent

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[10] Zeit. f. Anorg. Chem . Vol. 57-191.

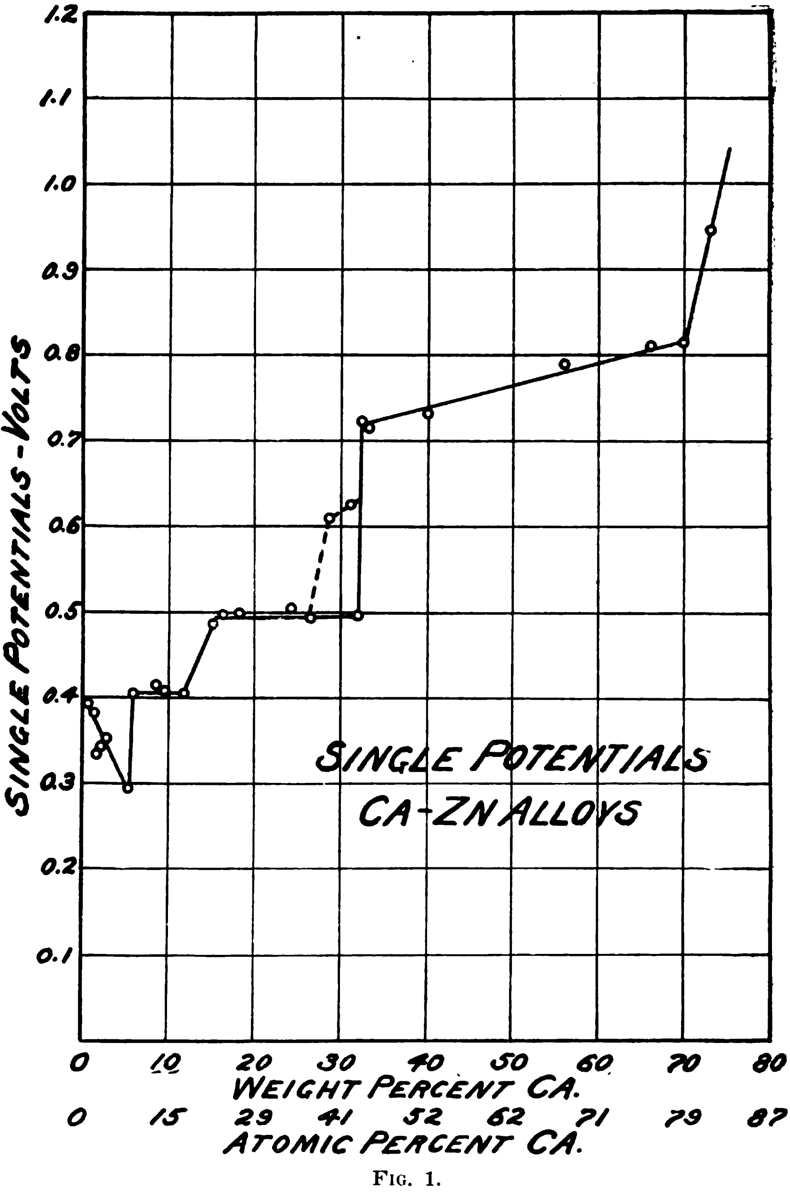
SINGLE POTENTIALS - VOLTS
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
SINGLE POTENTIALS
CA-ZN ALLOYS
0
10
20
30
40
50
60
70
80
WEIGHT PERCENT CA.
0
15
29
41
52
62
71
79
87
ATOMIC PERCENT CA.

FIG. 1.

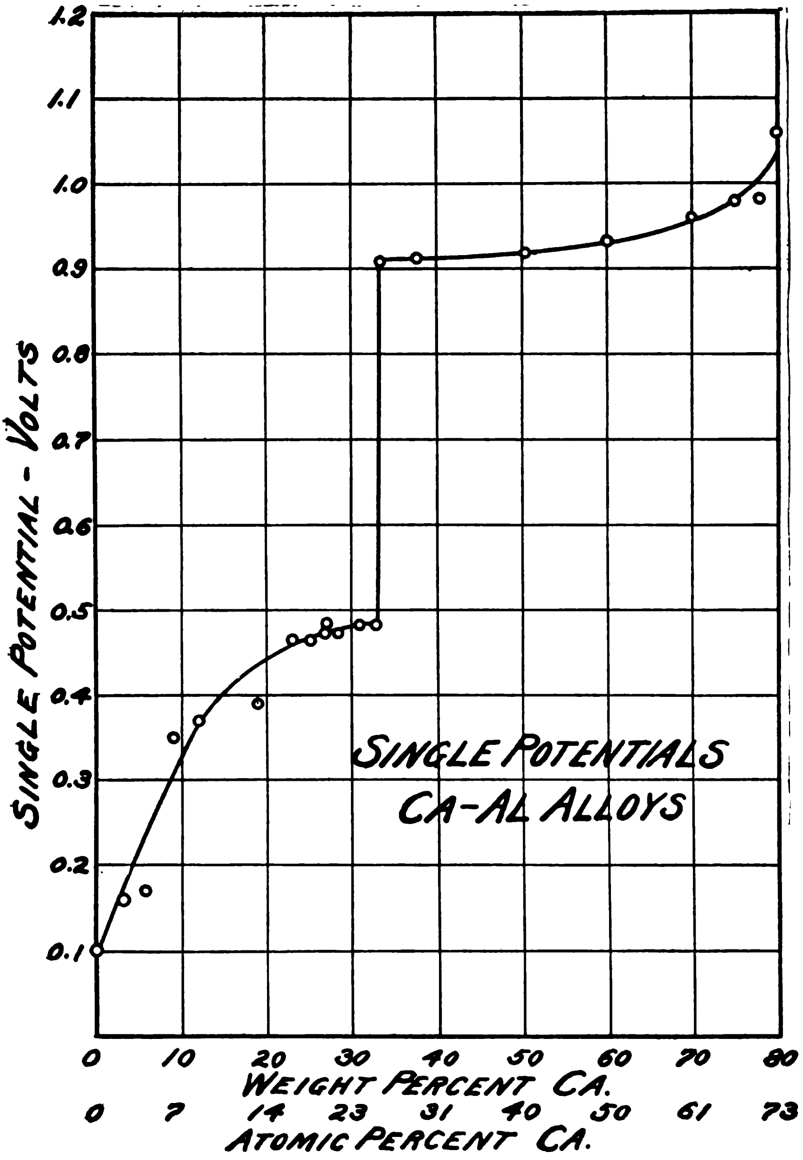
SINGLE POTENTIAL - VOLTS
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
SINGLE POTENTIALS
CA-AL ALLOYS
0 10 20 30 40 50 60 70 80
WEIGHT PERCENT CA.
0 7 14 23 31 40 50 61 73
ATOMIC PERCENT CA.

Fig. 2.

calcium. Several unsuccessful attempts were made to get an alloy whose calcium content would lie between these limits. This would define the composition of the alloy more exactly. This particular percentage of calcium corresponds to the compound Al_3Ca, which is in agreement with the work of Donski.[20]

The conductivity curve for this series shows a maximum value between 32.8 and 37.7 per cent calcium, which indicates the presence of the compound Al_3Ca.

MICROSCOPIC STUDY

The preparation of these alloys for micrographic study gave some trouble. In the calcium-zinc series the alloys over a considerable range are soft and brittle. After some experimental work it was observed that if a small quantity of rouge moistened with paraffin was rubbed on the surface of canvas, this would serve very well for the rougher polishing. For the final preparation of the specimen, a piece of very soft cloth was used. In many cases this produced a surface which revealed the structure without etching. Whenever an etching agent was necessary very dilute hydrochloric or nitric acid was used. In some cases oxidation tinting served very well.

The microscopic study of the structure of the calcium-aluminum series furnished evidence in agreement with the facts observed in the study of the conductivity and the single potentials. Alloys containing 28.5 and 32.8 per cent calcium presented a mass of polygonal crystals interspersed with a decreasing mass of eutectic. Beyond 34.7 per cent calcium the microscope revealed the same ground mass in which a new form of eutectic appeared. All these alloys were brittle and as a result it was impossible to secure suitable micrographs.

In the study of the zinc-calcium series the microscope revealed the presence of definite compounds. Figure 4 shows the homogeneous structure of the compound $CaZn_{10}$. Figure 5 shows the ground mass of Figure 4 in which an increased amount of eutectic has crystallized. Figure 6 shows a new

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[20] Zeit. f. Anorg. Chem. Vol. 57-201.

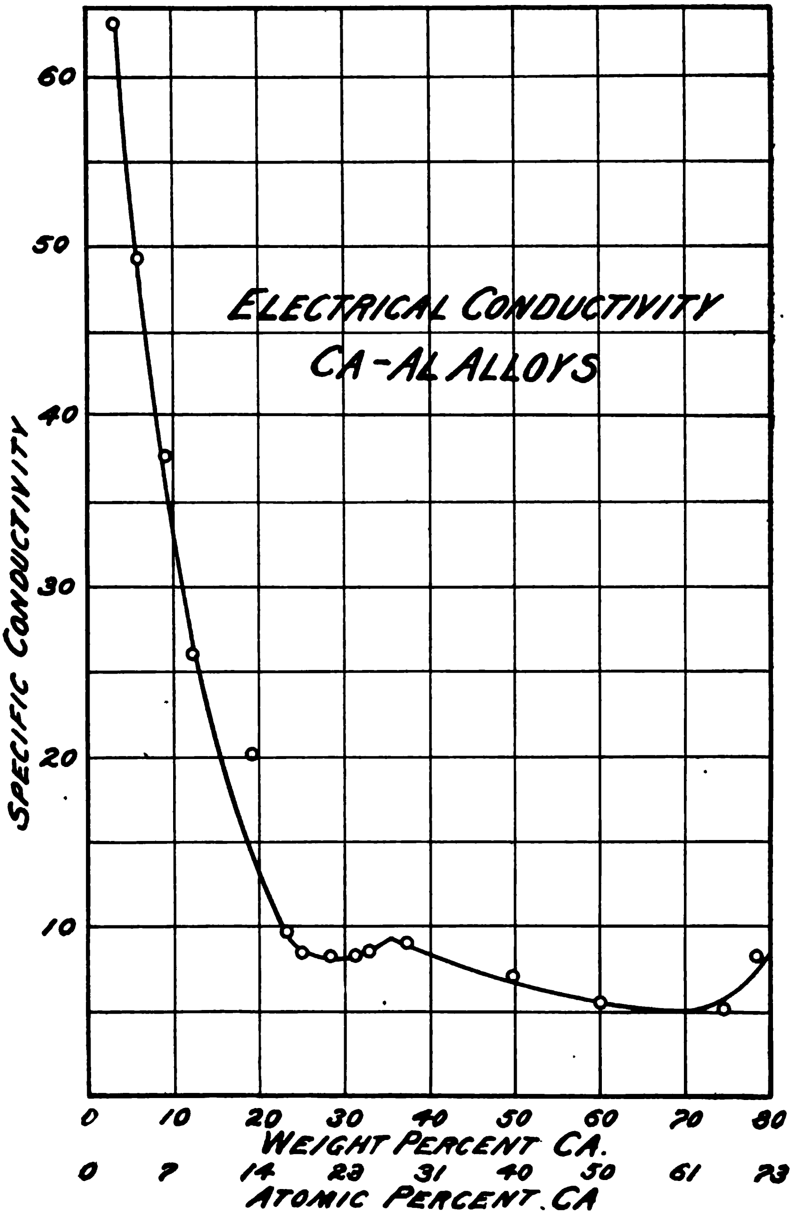
ELECTRICAL CONDUCTIVITY
CA-AL ALLOYS
SPECIFIC CONDUCTIVITY
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80
WEIGHT PERCENT CA.
0
7
14
23
31
40
50
61
73
ATOMIC PERCENT. CA

FIG. 3.

crystalline structure which is distinctly different from Figure 7. Figure 8 and Figure 9 present the homogeneous structure of the alloys represented by the formulae $CaZn_3$ and Ca_2Znx.

Micrographs were made at a uniform magnification.

CONCLUSIONS

(1) The breaks in the single potential and conductivity curves for the calcium-aluminum series correspond to the composition of the compound Al_3Ca.

(2) The breaks in single potential curve for the calcium-zinc series correspond to the compounds $CaZn_{10}$, $CaZn_4$, Ca_2Zn_3, Ca_4Zn.

(3) The study of the single potentials, conductivity and microscopic structure of alloys are valuable aids in determining the nature and constitution of the series.

In conclusion, the author wishes to express his gratitude for the timely suggestions and advice received from Professor C. F. Burgess and Professor O. P. Watts during the progress of this investigation.

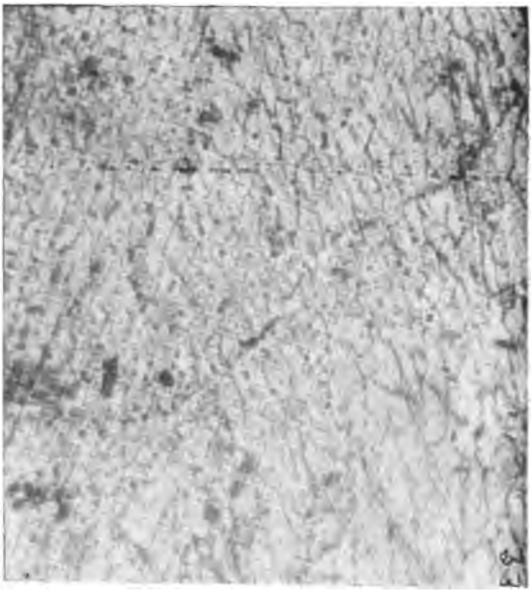

FIG. 5.

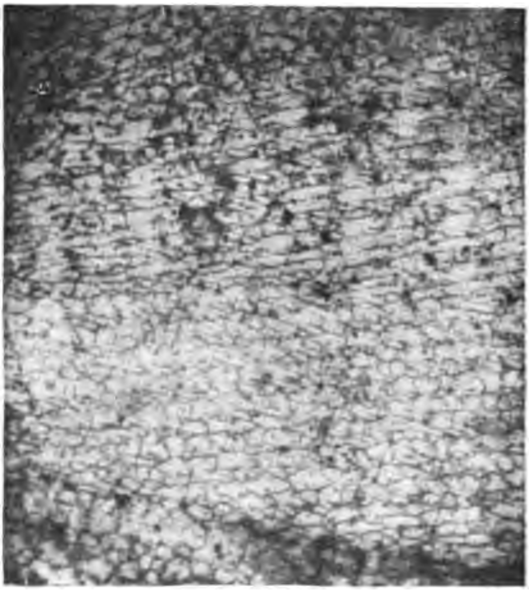

FIG. 4.

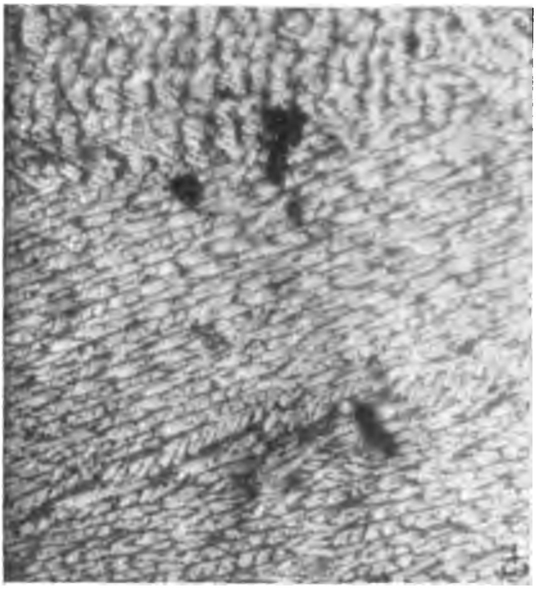

Fig. 7.

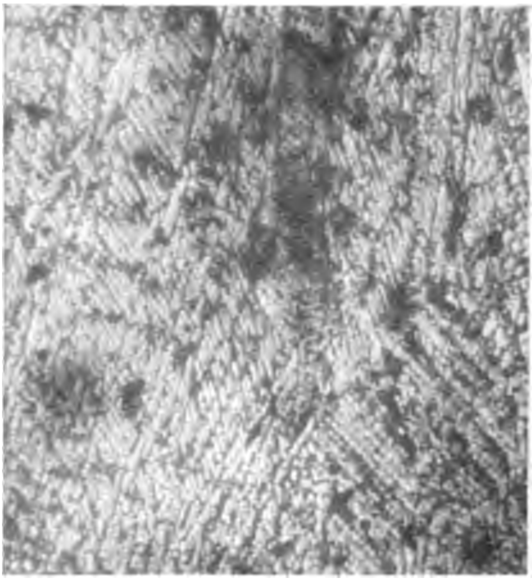

Fig. 6.

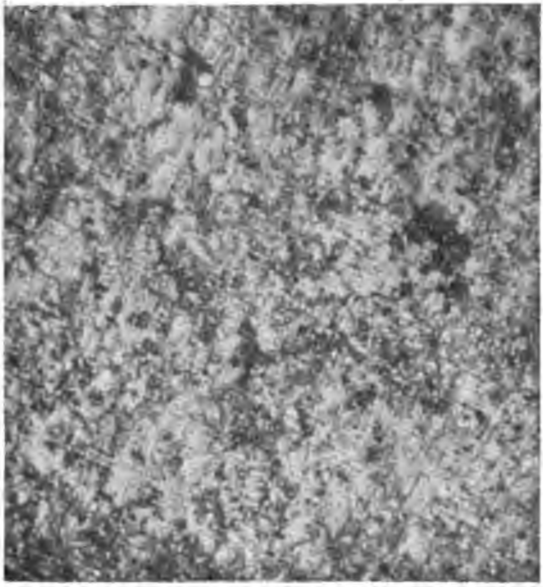

FIG. 9.

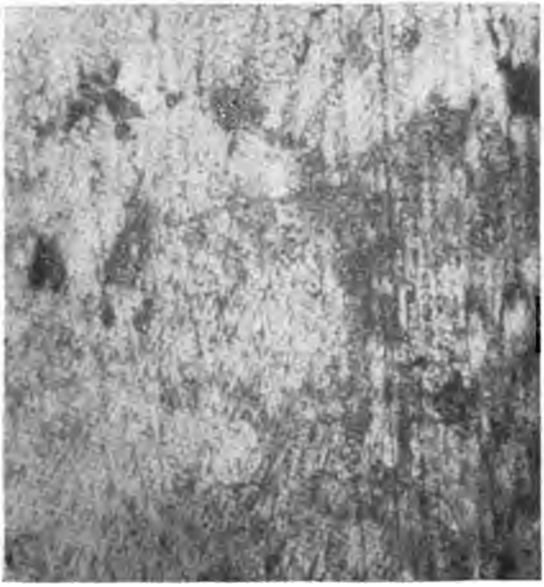

FIG. 8.

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AN INVESTIGATION OF THE AIR LIFT PUMP

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LIST OF SYMBOLS

AREAS

a_o = area of orifice in square feet.

a_p = area of eduction pipe in square feet.

COEFFICIENTS

c = coefficient of pipe friction and slip (variable).

c_e = coefficient of entrance.

c_o = coefficient of discharge of orifice.

c_p = coefficient of pipe friction and slip (average).

EFFICIENCY

e = efficiency.

DISTANCES

h = head.

h_a = head produced by the air used.

h_b = elevation of outlet above datum.

h_c = elevation of point C above datum.

h_e = head lost at entrance.

h_i = elevation of inlet above datum.

h_l = lift, in feet.

h_o = head lost due to elbow.

h_p = head lost in eduction pipe due to pipe friction and slip.

h_s = depth of submergence in feet.

WORK

L = total work, per pound of air, in foot pounds.

l = work output, in foot gallons per second.

l_i = work input, in foot pounds per second.

l_m = maximum work output, in foot gallons per second.

l_o = work output, in foot pounds per second.

PRESSURES

p = absolute pressure at any point, with variable specific weight of air.

p_a = loss of pressure due to air friction, when discharging into the atmosphere.

p_b = barometric pressure, acting on the surface of the water in the well and also on the discharge end of the pipe A (Fig. 3).

p_c = absolute pressure at the point C (Fig. 3).

p_d = absolute pressure of air at down stream side of orifice, in pounds per square inch.

p_g = absolute pressure at gage.

p_i = absolute pressure at inlet in the foot-piece.

p_s = standard atmospheric pressure (=14.7 pounds per square inch).

p_u = absolute pressure of air at upstream side of orifice, in pounds per square inch.

p_x = loss of pressure due to air friction when discharging against the pressure p_i in the foot-piece.

p_1 = absolute initial pressure of air, in pounds per square foot.

p_2 = absolute final pressure of air, in pounds per square foot.

VOLUMES

Q_i = volume of air at pressure p_i, in cubic feet.

Q_s = volume of air at standard atmospheric pressure (14.7 pounds per square inch).

q_a = discharge of free air, in cubic feet per second.

q_b = discharge of air at pressure p_b.

q_g = discharge of water in gallons per minute.

q_w = discharge of water, in cubic feet per second.

q_x = volume of air at pressure p_i.

q_1 = volume of air at pressure p_1.

q_2 = volume of air at pressure p_2.

SUBMERGENCE

s = percentage of submergence.

s_m = percentage of submergence corresponding to maximum discharge.

TEMPERATURE

T_u = absolute temperature at upstream side of orifice, in degrees Fahrenheit.

DENSITIES

u = density of mixture of air and water, variable.

u_a = density of air at 14.7 pounds per square inch pressure.

u_b = density of air at the discharge end of the eduction pipe.

u_i = density of air at inlet in foot-piece.

u_v = density of air, variable.

u_w = density of fluid pumped.

r = the ratio of compression in atmospheres.

VELOCITIES

v = variable velocity.

v_a = velocity of air in pipe, when discharging into the atmosphere.

v_b = velocity of the mixture of air and liquid, at the outlet of the eduction pipe, in feet per second.

v_c = velocity of water in the well outside of the eduction pipe at point c (Fig. 3).

v_i = velocity of the liquid in the eduction pipe below the air inlet.

v_t = velocity of the liquid in the tail-piece.

v_x = velocity of air in pipe, when discharging against pressure p_1.

WEIGHTS

w_a = weight of air used in pounds per second.

w_w = weight of water pumped in pounds per second.

INTRODUCTION

The air lift method of pumping, though not highly efficient as compared with some other methods, is nevertheless an important one, owing to the many advantages it possesses over other methods in the pumping of corrosive liquids, in pumping large quantities from wells of small bore, and on account of other features which will be discussed on a succeeding page.

Notwithstanding the fact that this method of raising liquids has been known for over a century and is now quite extensively used in both small and large pumping installations, the amount of reliable data, that are available to the practicing engineer, concerning the performance of this type of pump, is very meagre. Numerous tests of air lift pumps have been made by the manufacturers of air compressors and patented devices to be used in connection with air lift pumping plants, but the information gained from such tests has not been made public. The engineer in private practice has available, for use in designing, only the data from some tests on very small scale apparatus and those from a number of tests on actual installations, where it was not practicable to vary the conditions of operation much, nor to make accurate measurements of quantities.

With the purpose of supplying the demand for reliable data, from tests on pumps of commercial size and of various types, the investigations described and discussed in this bulletin were undertaken. The experiments, which comprise more than 600 runs, were carried on in the Hydraulic Laboratory of the University of Wisconsin. In taking the data on the first 318 runs the writers were assisted by B. R. McBride, then Instructor in Hydraulic Engineering, who supervised the work of C. J. Miller and E. J. Springer, then senior students in the College of Engineering, who used the data taken by them as the basis of a thesis for the baccalaureate degree. Assistance in the way of computing, drafting, and changing apparatus, has also been given from time to time by E. P. Abbott, G. P. Stocker, M. C. Koenig, E. B. Nelson, P. C. Dodge, Andrew Ludberg, and R. W. Hart, students in the College of Engineering. The observations on runs 319 to 500 inclusive were made by the writers, those from 501 to 608 inclusive were made by Messrs. Bingham and Hallauer, who included these data in a thesis submitted for a degree.

The experiments were not made on actual wells, but the apparatus was designed to reproduce as nearly as possible the practical working conditions of the air lift pump.

HISTORICAL NOTES ON THE DEVELOPMENT OF THE AIR LIFT PUMP

The application of compressed air as a means of pumping liquids was first used by Carl Emanuel Löscher, a German Mining Engineer, who in 1797 made some laboratory experiments, and described his invention in a pamphlet entitled "Aerostatisches Kunstgezeug." It was not until half a century later that the idea was put to a practical application, and then in a completely independent manner, by an American named Cockford, who in 1846 succeeded in pumping petroleum from some wells in Pennsylvania.

On May 23, 1865, a United States patent (No. 47,793) was issued to A. Brear on an "oil ejector," which the description and illustration accompanying the patent show to have been an air lift pump of the annular tube type (see page 34).

The idea was again revived by Mr. J. P. Frizell who obtained a patent on an air lift pump, dated Oct. 19, 1880. His invention was apparently made independently and without knowledge of the work done in this field by others, and grew out of his invention of a method of compressing air on which he was granted a patent on January 29th, 1878 (No. 199,819).

The Frizell method of compressing air consists in introducing the air within a column of water descending through a vertical shaft or pipe, from whence the mixture flows through a horizontal tunnel at the top of which the air is collected by means of a suitable receiver and from which it may be conducted to any desired point by a pipe. The water divested of the air passes through the tunnel and rises to the surface through an ascending shaft at the other end. The invention was based on the knowledge that air drawn into a current of water, descending through a vertical shaft or pipe with a velocity greater than that with which air bubbles would rise in still water, will be carried down with the descending column, and will be subjected to a pressure corresponding to the depth attained. The amount of air which may be carried down and compressed by a current of water depends upon the quantity of water and the difference of head available between inlet and outlet of the tunnel. Although the bubbles of air are actually moving downward their velocity relative to that of the water is upward. They, therefore, have a retarding effect on the velocity of the water due to fluid friction between the air and the water. The larger the quantity of air introduced the smaller will be the size of the water passages between the bubbles, which, with constant head, will result in a slower velocity of the water, until the limit is reached when the downward velocity of the water is just equal to the relative upward velocity of the air. From observation of these facts it was apparent that if compressed air be introduced at or near the submerged bottom of a vertical pipe in sufficient quantity, it will, in rising through the water in the pipe, cause the water to acquire an upward velocity. Mr. Frizell in the specifications of his air lift patent (No. 233,499) expressly states that his method of pumping is a reversal of the principle of his method of compressing air, and since in the latter device the air was admitted in the form of small bubbles, so in the pump also it was admitted through a great number of small orifices with the object of producing small bubbles, so as to aerate the water, as illustrated in Fig. 4.

The air lift pump was used to increase the discharge of flowing wells as early as 1884, a patent (309,214) being issued Dec. 16th of that year to S. S. Fertig on an annular tube type of pump.

Apparently without any knowledge of the previous inventions Werner Siemens in 1885 made use of the air lift pump for draining a mining shaft near Berlin. In France, Laurent in 1885 and Goudry in 1886 used a similar contrivance, which they called an "emulseur," for pumping sulphuric acid.

The term air lift, as applied to the above described method of pumping, was first used by Dr. Julius G. Pohle in the specifications for his patent (No. 487,639) which was issued Dec. 6,

1892. The chief distinction between this type of pump and that of Mr. Frizell lies in the method of introducing the air. To use his own words, "The invention * * * consists in improved processes and apparatus whereby the compressed air is delivered in bulk into the lower end of the water eduction pipe, and the water and air are caused to ascend through said pipe in distinct alternate layers of definite dimensions." In the specifications of his patent he explains his understanding of the working of the pump as follows:

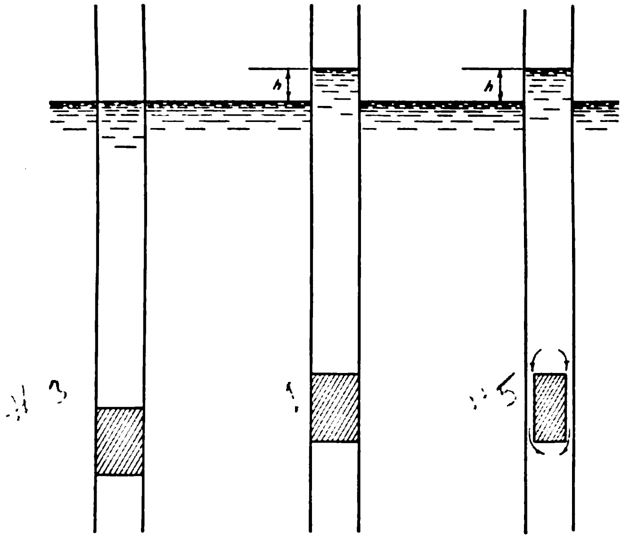
"I have discovered that when air of suitable pressure is allowed to enter in a constant stream and in suitable quantity into an eduction pipe at or near its lower end when it is submerged in water while its upper end rises above the water about the same distance that its lower end is submerged, the compressed air thus introduced will at first expel the standing water from the pipe in an unbroken column free from air, and subsequently, by the continued inflowing of the compressed air under a pressure just sufficient to overcome the resistance of the water outside of the eduction pipe, it will arrange itself in alternate layers with the water, while the latter flows into the lower end of the eduction pipe by force of gravity until it is discharged at the upper or exit end of the pipe. This alternate interposition of determinate quantities of air between the also determinate quantities of water elongates the entire column of air and water, thus facilitating, without materially adding to the weight of the column, the discharge of the water at a higher level than would be the case were these air sections or layers absent. I have also discovered that under the above mentioned conditions the compressed air will not escape through the water overlying it, and also that the water overlying the compressed air will not fall back through the underlying air while both are in upward motion, but find that the elasticity stored in the compressed air layer, pressing alike in all directions, forms a temporary water-tight air piston, which lifts the water above it to its final discharge without appreciable loss by leakage or so called "slip," while this compressed air piston after having expended its elastic energy in work of lifting water is dispelled with only a practically unimportant loss of power.'' In Fig. 3(b) a pump is shown in which the air is in large bubbles or pistons.

In addition to the patents mentioned, many others have been granted covering various supposed or real improvements. Some of these will be mentioned on following pages in the discussion of the features of the pump to which they relate.

THE PRINCIPLE OF THE AIR LIFT PUMP

The precise action going on in an air lift pump is not thoroughly known and it doubtless differs under the various conditions of operation, but the basic principle on which the pump works is simple and may be illustrated in the following manner.

First, consider a vertical pipe, open at both ends and partly immersed in a liquid, as shown in Fig. 1 (a). The liquid will stand at the same height inside and outside of the pipe. Assume that a block of material, like cork or wood, lighter than



(a) (b) (c)

Fig. 1.

the liquid, made to fit the pipe snugly but able to move without friction, is made to replace part of the liquid near the bottom of the pipe. The hydrostatic pressure on the underside of the block is now greater than the combined weight of the block and the liquid above it. The block and the liquid in the pipe will therefore be pushed up in the pipe, as shown in Fig. 1 (b), until the head h balances the difference between the weight of the block and the weight of an equal volume of the liquid. If more blocks of the light solid material be introduced into the pipe, the liquid will be raised a distance h for each block until the top of the pipe is reached, when an overflow of liquid and blocks will occur leaving an unbalanced head in the pipe, which would keep up the discharge as long as the supply of liquid and blocks was kept up at the bottom of the pipe. In the Pohle air lift system the claim is made that the pump works as described above, with the exception that compressed air is used instead of a light solid, and that work is done by the expansion of the air as it is relieved of the weight of the liquid when approaching the top of the pipe.

A closer approximation is made to usual working conditions in an air lift pump by the illustration (c) in Fig. 1. In this case the block of light material, cork, wood, or air, does not entirely fill the cross-section of the pipe. By virtue of its buoyancy it will tend to rise in the pipe and the liquid in the pipe will tend to flow down past it. The height h, to which the water rises in the pipe in this case, represents the head necessary to force the liquid down through the restricted passage-way past the block. The same conditions would obtain if the single block nearly filling the pipe were replaced by a large number of small blocks. It would require some head h to overcome the resistance offered to the liquid in its flow between the small blocks or between the blocks and the pipe walls. If a sufficient quantity of the small blocks of air or other light material are inserted, the head h will reach the top of the pipe and will cause a discharge of the liquid. The flow of liquid down past the buoyant material is called the slip of the pump. It is the cause of a serious loss of energy.

A commonly accepted conception of the principle of operation of the air lift pump may be had by considering that the air bubbles, in rising through the water in the discharge pipe, reduce the specific gravity of the mixture and therefore the weight of the column, causing an unbalanced condition between the column inside and outside of the tube. The excess pressure at the base of the column, due to the external water pressure, therefore, forces the mixture above the supply level and out of the top of the pipe. This excess pressure increases with the depth of submergence of the pipe, and the latter must be regulated to suit the height of delivery.

THEORY OF THE AIR LIFT

Echol's Theory.—An attempt was made by Professor W. H. Echols, to develop a theory of the air lift pump based upon a mathematical analysis of the problem. The results of his studies were presented before the Philosophical Society of the University of Virginia in 1891, but they were not available to the writers.

Harris's Theory.—A further effort in this direction was made by Professor Elmo G. Harris of the University of Missouri, with the purpose of obtaining a rational formula by which a pump could be designed intelligently, and on which experiment could be based. His discussion of the subject was published in the Journal of the Franklin Institute, Vol. 140, p. 32, July, 1895.

In deriving his formulas for the design of a pump the work done by the air is divided into four parts by Professor Harris, as follows:

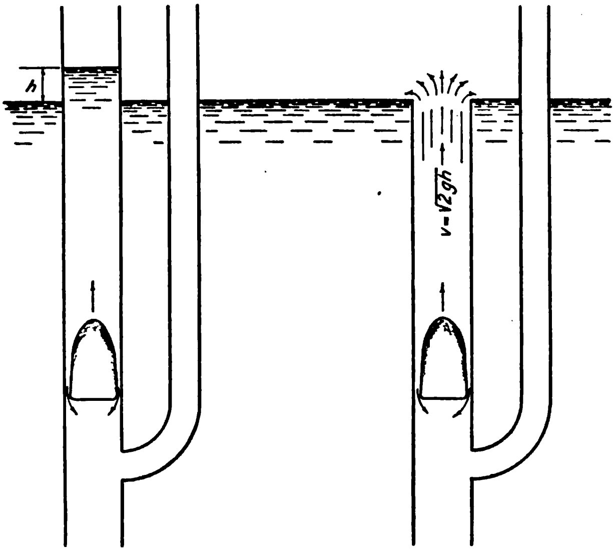
(1) The kinetic energy in the liquid discharged at the top of the pipe.

(2) The energy necessary to raise the liquid to the top of the discharge pipe.

(3) The energy lost by the liquid slipping down by the bubbles.

(4) The energy consumed by friction in passing through the pipes.

Theoretical expressions may be found for each of the above quantities. That for the slip is too complicated for use in practice, so an approximate formula based on a number of assumptions is derived. For the fourth term the value of the friction factor is assumed, as is the relation of the loss to the velocity. In-as-much as it is not possible to verify the correctness of the individual terms for the losses by experimental means, since the loss due to slip and that due to friction could not be differentiated under working conditions, and in-as-much as the formulas are complicated and difficult to use, they will not be given in detail, but a brief review of the general principles on which they are based will be presented as a further aid to an understanding of the action of the air lift pump.



(a) (b)

Fig. 2.

In his consideration of the subject Professor Harris first proposes the following problem:

A vertical pipe, open at both ends, is partly immersed in a liquid, as shown in Fig. 2 (a). A quantity of gas is released within the pipe and below the surface of the liquid. What effect will the gas have on the column of liquid and what will be the action of the bubble of gas?

The pipe is assumed to be so large that capillary forces cannot control the action. Then the bubble will ascend in the pipe. Assuming for the present that no liquid is pumped out of the top of the pipe, then during the ascent the liquid above the bubble must pass by it in order to get below. Hence, the bubble cannot occupy the whole cross-section of the pipe. In order to ascend the bubble must become elongated until the liquid can pass down. In order to pass down through the contraction formed by the bubble, the liquid must have a certain absolute velocity. The presence of this velocity is evidence of the existence of an unbalanced head somewhere above.

Expressions are found for the upward velocity of the air bubbles and for the downward velocity of the water, and from the relations of these quantities the area of cross-section of the bubble and the rate of loss of liquid from above to below one bubble is computed, thereby giving a basis for finding the loss of energy due to slip.

Under the conditions of the above problem all of the useful energy supplied by the air is wasted in fluid friction caused by the water slipping past the bubble.

A closer approximation to working conditions is illustrated in Fig. 2 (b), in which the arrangement is the same as in the preceding problem with the exception that the top of the pipe is flush with the surface of the supply reservoir. In the first problem the bubble produced the standing head h. In the absence of the standing head in this problem, the liquid will flow out of the top of the pipe with a velocity theoretically equal to $\sqrt{2gh}$. Under the conditions of this problem the entire column of liquid in the pipe will be moving upward. The downward velocity of the water past the bubble will not be actual but only relative to the velocity of the bubble, but the loss due to slip is assumed to be the same as under the conditions of the first problem.

In air lift pumps as actually operated the bubbles are not always of the proper size to fill the pipe in the manner assumed above. In the pump specified in the Frizell patent, for example, the air is admitted to the water, as already described, in the form of very minute bubbles. When the size of the bubble is small the surface tension in the liquid tends to compress the bubble into a sphere. When the bubbles are small their motion is irregular and the formulas deduced by Professor Harris for loss due to slip are not supposed to hold.

In his book on *Compressed Air*, published in 1910, Professor Harris has modified his original theory, and shows that the slip varies as the square root of the volume of the bubble, and that the head produced is independent of the size of the bubbles, hence it would seem desirable to have the air in the form of small bubbles.

Anderson's Theory.—A simple theory of the air lift pump, proposed by Mr. Robert M. Anderson, was published in Bulletin No. 53 of the Hudson Engineering Company in 1905. In developing this theory static conditions were assumed to exist in the pump. Under such conditions the pressure at the air inlet due to the depth of submergence is equal to that produced by the mixture of air and water in the eduction pipe. The lift is found by computing the length of eduction pipe required to give a pressure equal to that due to the submergence. This length is inversely proportional to the average density of the mixture. To find the latter quantity an expression is developed for giving the mean volume of the air, while expanding isothermally from its volume at the inlet to its volume at atmospheric pressure. The various terms when combined give the volume of air at barometric pressure required to pump one volume of water, as

$$q_b = \frac{h_l}{h_s \dfrac{p_b}{p_1 - p_b} \log_e \dfrac{p_1}{p_b}} \qquad (1)$$

in which

q_b = discharge of free air at the pressure p_b.

h_l = the lift, in feet.

h_s = the depth of submergence, in feet.

p_b = barometric pressure.

p_1 = pressure at the air inlet in the foot-piece.

For convenience of reference these and all other symbols used in this bulletin have been tabulated and defined on page 9.

Under the static conditions assumed for developing this theory, there would be no losses of head, such as those occasioned by entrance to the eduction pipe, pipe friction, slip, elbow loss, etc., and accordingly no terms for these quantities are found in the formula. To adapt the formula for practical use a constant, found by comparing with experimental results, has been introduced giving

$$q_b = 1.9 \frac{h_l}{h_s \dfrac{p_b}{p_l - p_b} \log_e \dfrac{p_l}{p_b}} \qquad (2)$$

It is not claimed that this formula gives accurate results, but only approximations.

Gibson's Theory.—This theory, published in 1908 by A. H. Gibson in his "Hydraulics and Its Application," is based on the same fundamental ideas as the preceding one except that it is not confined to static conditions. Under operating conditions the column of mixed air and water in the eduction pipe is not long enough to create a pressure at the air inlet sufficient to balance that due to submergence, the difference being made up in the losses of pressure due to friction, etc. Mr. Gibson takes account of the losses due to friction and velocity at exit, introducing terms for the loss of head due to these causes so that formula (1) becomes

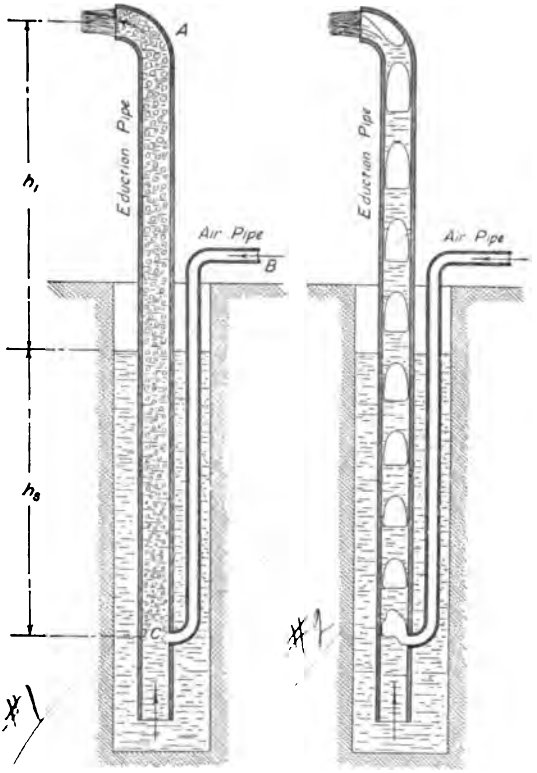
$$q_b = \frac{h_l + h_p + \dfrac{v_b^2}{2g}}{h_s \dfrac{p_b}{p_l - p_b} \log_e \dfrac{p_l}{p_b}} \qquad (3)$$

Lorenz's Theory.—A very simple mathematical theory, explaining the action of the air lift pump, was published by Dr. H. Lorenz, in Zeitschrift des Vereines Deutcher Ingenieure, Vol. 53, page 545, April, 1909. The formulas he deduces take account of the losses of energy occasioned by slip, pipe friction, etc., and are therefore of practical use in designing air lift pumps, provided the necessary experimental coefficients are known. The theoretical discussion in Dr. Lorenz's article is therefore given in full below. Let

p_l = the pressure in the foot-piece.

p_b = the barometric pressure acting on the surface of the water in the well and also on the discharge end of the pipe A, Fig. 3.

u_w = the density of the fluid pumped.
w_w = the weight per second of the water pumped.
w_a = the weight per second of the air discharged through the pipe B.
u_i = the density of the air at the air inlet in the foot-piece.



(a) (b)

Fig. 3.—Comparison of the Frizell and Pohle Systems of Operation.

u_b = the density of the air at the discharge end of pipe A.
v_i = the velocity of the liquid in the pipe A below the air inlet.
c_e = the coefficient of entrance.
h_s = the depth of submergence.

Referring to Fig. 3 (a), it may be seen that during the operation of the pump the following equation of heads holds between the point C in the pump and a point at the same elevation outside the pump:

$$h_s = \frac{p_1 - p_b}{u_w} - \frac{v_1^2}{2g}\left(1 + c_e\right) \qquad (4)$$

For flow in the discharge pipe A the following differential equation holds on account of the variable value of the density u of the mixture of gas and liquid:

$$dh - \frac{dp}{u} = \frac{v\,dv}{g} - cv^2\,dh \qquad (5)$$

in which v equals the variable velocity, and p the pressure at any point; and with the variable specific weight of air as u_v, equation

$$\frac{w_a + w_w}{u} = \frac{w_a}{u_v} + \frac{w_w}{u_w} \qquad (6)$$

designates the momentary volume of the mixture $w_a + w_w$.

If the mixture of air and fluid is very intimately commingled; that is, if the air penetrates the fluid in the form of small bubbles, it can be assumed that the air expands isothermally, so that

$$u_v = \frac{p}{p_b}\,u_b \qquad (7)$$

By means of equations (6) and (7) the fundamental formula (5) becomes

$$dh - \frac{w_a}{w_a + w_w}\,\frac{p_b\,dp}{u_b\,p} - \frac{w_w}{w_a + w_w}\,\frac{dp}{u_w} = \frac{v\,dv}{g} - c_f v^2\,dh \qquad (8)$$

Integrating this equation between the limits $h_s + h_1$ and 0, p_1 and p_b, and v_1 and v_b (the velocity of the mixture at the discharge end of the eduction pipe) there results

$$-\left(h_1 + h_s\right) + \frac{w_a}{w_a + w_w}\,\frac{p_b}{u_b}\log_e\frac{p_1}{p_b} + \frac{w_w}{w_a + w_w}\,\frac{\left[p_1 - p_b\right]}{u_w} = \frac{v_b^2 - v_1^2}{2g} + \int_0^{h_1 + h_s} c_f v^2\,dh$$

Replacing the last term in this equation, for the sake of simplicity, by assuming a mean coefficient c_p so that

$$\int_0^{h_1 + h_s} c\, v^2\, d\, h = c_p \frac{l}{d} \frac{v_b^2}{2g} \qquad (9)$$

what is $\frac{l}{d} = \frac{(h_s + he)\,12}{\text{diam}}$

there results

$$-\left(h_1 + h_s\right) + \frac{1}{w_a + w_w}\left(w_a \frac{p_b}{u_b} \log_e \frac{p_1}{p_b} + \frac{w_w}{u_w}\left(p_1 - p_b\right)\right) -$$

$$\frac{v_b^2 - v_1^2}{2g} + c_p \frac{l}{d} \frac{v_b^2}{2g} \qquad (10)$$

Adding this equation to (4) gives

$$\frac{w_a}{w_a + w_w}\left(\frac{p_b}{u_b} \log_e \frac{p_1}{p_b} - \frac{p_1 - p_b}{u_w}\right) -$$

$$h_1 + \frac{v_b^2}{2g}\left(1 + c_p \frac{l}{d}\right) + \frac{v_1^2}{2g} c_e \qquad (11)$$

Neglecting the second term on the left hand side of the equation, which will be very small in comparison with the first term on account of the large difference between the values of u_w and u_b, w_a and w_w, and neglecting w_a in the denominator, this equation reduces to the simple form

$$\frac{w_a}{w_w} \frac{p_b}{u_b} \log_e \frac{p_1}{p_b} - h_1 + \frac{v_b^2}{2g}\left(1 + c_p \frac{l}{d}\right) + \frac{v_1^2}{2g} c_e \qquad (12)$$

In developing this energy equation Dr. Lorenz assumed the velocity of air entering the foot-piece as equal to that of the water; that is free from any losses due to impact which may be readily assumed on account of the small kinetic energy of the air.

Now let

$$l_1 - w_a \frac{p_b}{u_b} \log_e \frac{p_1}{p_b} \qquad (13)$$

the work of isothermal expansion of the weight of air w_a, and

$l_o = w_w h_l$ the work done in lifting the fluid weight w_w, from which the hydraulic efficiency

$$e = \frac{l_o}{l_i} = \frac{w_w h_l u_b}{w_a p_b \log_e \frac{p_l}{p_b}} \tag{14}$$

can be computed with the aid of equation (12)

$$\frac{1}{e} = 1 + \frac{v_b^2\left(1 + c_p \frac{l}{d}\right) + v_l^2 c_e}{2 g h_l} \tag{15}$$

For the practical use of these formulas it will be better to eliminate the velocities v_b and v_l, by introducing the volumes q_b and q_w of weights w_a and w_w and using the area of the discharge pipe a_p, by means of the following formulas:

$$\begin{array}{ll} w_a = q_b u_b & w_w = q_w u_w \\ q_b + q_w = a_p v_b & q_w = a_p v_l \end{array} \tag{16}$$

Writing now in place of equation (12)

$$\frac{q_b}{q_w}\frac{p_b}{u_w} \log_e \frac{p_l}{p_b} = h_l + \left(\frac{\left(1 + c_p \frac{l}{d}\right)\left(q_b + q_w\right)^2 + c_e q_w^2}{2 g a_p^2}\right) \tag{17}$$

and differentiating this equation with respect to q_b, putting

$$\frac{d q_w}{d q_b} = 0,$$

there results as a requirement for maximum discharge q_w:

$$\frac{1}{q_w}\frac{p_b}{u_w} \log_e \frac{p_l}{p_b} = -\frac{1 + c_p \frac{l}{d}}{g a_p^2}\left(q_b + q_w\right) \tag{18}$$

or in connection with equation (17), that is after eliminating the pressures p_l and p_b,

$$\left(1 + c_p \frac{l}{d}\right)\left(q_b^2 - q_w^2\right) = 2 g h_l a_p^2 + c_e q_w^2 \tag{19}$$

If now the maximum discharge determined from the capacity of the well, and the area a_p of the discharge pipe determined from the diameter of the well, and also the lift and the known coefficients c_e and c_p are given, the volume of free air required may be computed by means of the formulas (18) and (19) from which the submergence h_s can then be computed by means of equation (4). For these fixed conditions equation (17) then gives the relations between any desired values of q_b and q_w using the same pressure p_1.

There may be other losses of energy than those accounted for by Dr. Lorenz, such as the loss due to the elbow or bend which generally forms the upper end of the pump. In comparing the experimental results of the Wisconsin experiments with Dr. Lorenz's theory (see page 77), his formulas have been modified to take account of the elbow loss, and in computing experimental values of the coefficient of pipe friction and slip, a term has been introduced to correct for the loss of energy due to friction in the air pipe (see page 73).

Green's Theory.—In the Engineering and Mining Journal of Aug. 7, 1909 (Vol. 88, p. 251), Leonard M. Green has published an article entitled "Efficiency of the Air Lift as a Solution Pump," in which he discusses mathematically the theory of the air lift, amount of air required, minimum air pressure, efficiency of the lift under given conditions, etc.

He makes the assumption that the water and air rising in the eduction pipe are in layers; the layers of water being equal in volume and the layers of air being of equal weight. Formulas are deduced for computing the ratio of the volumes of water and free air in each layer for given conditions of lift and submergence and for giving the number of these layers, or the volumes of water and air discharged.

No provision is made in the formulas for correcting for entrance loss or for the loss caused by the elbow, at the top of the eduction pipe, which may amount to as much as 15 per cent. of the total work done. Nor has any account been taken of the slip. An attempt has been made to correct for pipe friction by assuming that the velocity in the tail-piece of the eduction pipe is equal to the velocity of flow of water in a clean iron pipe of length equal to that of the eduction pipe and under a difference of head equal to that caused by the compressed air. The experiments made by the writers indicate that this last assumption may be approximately true for some ratios of volume of air to volume of water, but that it is very far from the truth when this ratio is small.

By neglecting the entrance and elbow losses and making the above assumption as to pipe friction the author computes theoretical efficiencies of more than 90 per cent. for small amounts of air, and draws the conclusion that under proper working conditions the total combined efficiency of the compressor and lift should not be less than 70 per cent. Experiments do not justify this opinion.

The Green formulas are not so convenient for use as the Lorenz formulas, so no attempt has been made by the writers to supply the missing terms or to work out experimental coefficients for use with them.

METHOD OF OPERATION

To start the operation of an air lift pump requires a greater air pressure than is necessary for normal operating conditions. When the air supply is first turned on, the air pressure must be greater than that due to the submergence of the air inlet, while after the discharge of the liquid from the pump has commenced the pressure at the air inlet will be reduced by the amount of the entrance and velocity heads of the liquid entering the eduction pipe. The conditions existing while starting an air lift pump are accurately illustrated and described above in connection with Fig. 2 (a) with the exception that instead of only one bubble of air there would be many. When a sufficient number of bubbles have been introduced to raise the head through the entire lift, some of the liquid will begin to spill out over the top of the pipe. The loss of this liquid causes a reduction in the pressure in the eduction pipe, which under some conditions allows a sudden influx of the high pressure air, resulting in a violent discharge of liquid and air which may exhaust the store of compressed air. Following this the liquid would regain its full static head, requiring the operation to be started over again.

To prevent such intermittent action the escape of air into the eduction pipe must be controlled. It should be throttled the instant the discharge of liquid commences. That intermittent action does not always occur is probably due to the effect of friction in the air pipe. As this friction increases with the square of the velocity, it is evident that in long pipes of small cross-section it will serve to some extent as a governor, tending to control the discharge of air.

DESCRIPTION OF AN AIR LIFT PUMPING PLANT

THE PUMP

The essential structural features of the air lift pump are exceptionally simple and few in number, and this fact constitutes one of its principal advantages. In its simplest form, as illustrated in Fig. 3, it consists of a pipe for the discharge of the water and a smaller pipe for conveying compressed air to it at a point near its lower end.

The Eduction Pipe.—The discharge pipe is designated by various writers as the eduction pipe, lift tube, lift pipe, and rising main. It should not touch the bottom of the well or reservoir from which it is to pump but should be elevated above it so as to freely admit the water or other liquid through its lower open end. This end of the pipe should, however, be submerged below the liquid surface a distance which, our experiments indicate, should be greater than the height above the water surface to which the liquid is to be lifted. This latter distance is technically called the lift of the pump. The distance measured from the water surface down to the point of admission of the air into the eduction pipe is known as the submergence of the pump. Submergence is generally expressed as a percentage of the total length of the pump, measured from the point of air inlet to the point of discharge. The discharge should be free into a reservoir at atmospheric pressure. Submergence and lift should be measured from the elevation of the water as it stands under working conditions rather than under static conditions.

The Foot-Piece.—In most air lift pumps the compressed air from the air pipe enters the eduction pipe through a casting designed to cause the air to enter in a special manner. The casting used for such a purpose is technically called a foot-piece.

The type of foot-piece devised by Mr. Frizell is shown in Fig. 4. In the specifications for his patent it is described as follows:

"Into the bottom of the rising-pipe is fitted the hour-glass-shaped pipe 5, inclosing between the two pipes the annular space E E.

"The upper end of the pipe 5 is perforated with a great number of minute orifices, F, as indicated by the black dots.

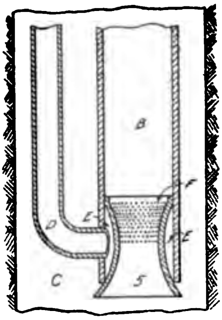


Fig. 4.—Frizell's Foot-Piece.

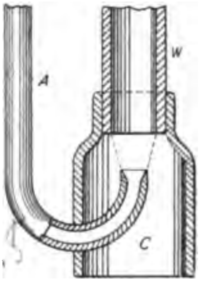


Fig. 5.—Pohle's Foot-Piece.

The lower end expands to a greater width than that of the rising-pipe in order to diminish the resistance of the water in entering.

"The pipe D, leading from the source of compressed air opens into the annular space E E.

"The pipe D, which conveys the compressed air may pass down in the pit C, as shown, or inside the rising-pipe B, or outside the pit C in the ground, if preferred."

Dr. Pohle's foot-piece is illustrated in Fig. 5. The description of it, taken from the specifications for his patent, is as follows:

"The exit end of the air pipe is enlarged by beveling off the inner edge thereof, in order to permit the free delivery of the air in mass or bulk, and thus to avoid the formation of air bubbles. The enlargement C of the pipe W is of sufficient area to compensate for the space occupied by the exit end of the air pipe A, and said end of said pipe A passes through the vertical side of the enlargement C, as shown, and derives support therefrom."

Quite a large number of different foot-pieces have been devised and patented and in most cases extravagant claims have been made for them as regards their effect in increasing the efficiency of the pump. A few of the types will be illustrated and discussed on a following page, in connection with the description of the experiments carried on at the Hydraulic Laboratory of the University of Wisconsin.

As will be shown later, experiments indicate that changes in the form of the foot-piece have little if any effect on the efficiency and capacity of the pump, other than the effect due to the amount of obstruction of the water passages; a factor which differs in the various types.

Tail-Piece.—Most forms of foot-piece are arranged for the attachment of a pipe onto their lower ends, thus extending the eduction pipe some distance below the point of air admission, for the purpose of preventing the escape of compressed air from the bottom of the eduction pipe. The pipe used for such an extension is termed a tail-piece. The tail-piece is often made a larger size of pipe than is the remainder of the eduction pipe. The length and size to be used for a tail-piece are problematical.

A number of distinct types of air lift pump have been produced through the various methods that have been devised for piping the wells. The methods in most frequent use are illustrated in Figs. 6 and 7 and are described below.

Side Inlet Pump.—The side inlet pump, in which the air pipe is on the outside of the discharge pipe, is shown at (a) in Fig. 6. The air pipe is connected to the bottom of the eduction pipe by means of standard fittings, special castings, or one of the various patented foot-pieces, examples of which are illustrated in Figs. 11 and 12. This method is used when the well is large enough to admit of the air and water pipes being placed side by side from top to bottom and is probably the most economical of the systems shown.

Annular Air Tube Pump.—Fig. 6 (b) shows the annular air tube pump in which the well top is sealed and the air passes down through the annular space between the discharge pipe and the air pipe or well casing.

In the illustration accompanying the 1865 patent of A. Brear's annular tube pump, a foot-piece is shown attached to the lower ends of the eduction and air pipes. This nozzle is so arranged that the air is directed upward into the center of the eduction pipe, the liquid entering the lower part of the foot-piece and surrounding the air nozzle. The air is introduced differently in the Bacon pump, which has been quite extensively used. In the original patent (No. 542,620) of James E. Bacon, issued July 16, 1895, the statement is made that it was found "advantageous to make an opening into the uptake-pipe near the bottom end thereof, so that the air may flow through that opening in a uniform or nearly uniform stream." Many annular tube pumps have been built with no foot-piece or opening in the side of the pipe. In such pumps the air forces the water level in the well down until the bottom of the discharge pipe is uncovered. Air then enters the discharge pipe and the pressure in the annular space is lowered. This causes the fluid to rise again in the air space and discharge pipe until the pressures balance and then the operation is repeated. This causes the water in the well approaching the bottom of the eduction pipe to surge more violently than it would if it were allowed to rise in the well around the eduction pipe to its normal height, as it does in the side inlet system. It is claimed for

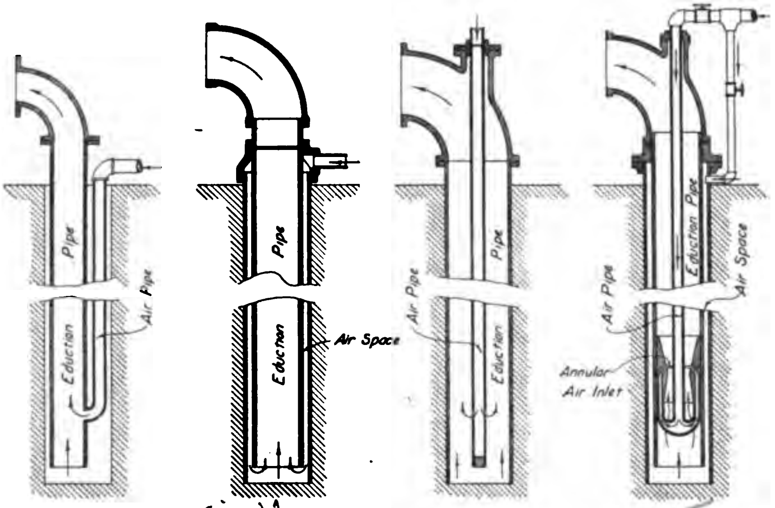


Fig. 12 (a) Fig. 13 (b) (c) Fig 15 (d)

Fig. 6.—Different Methods of Piping Wells.

pumps built in this way that this surging promotes the entrance of the water and air into the eduction pipe in a manner conducive of high efficiency.

Central Air Tube Pump.—Fig. 6 (c) shows the central air tube pump which uses the well casing as the discharge pipe, and introduces the air through a small pipe usually fitted with some special device or foot-piece attached to the bottom through which the air escapes. Usually a number of small holes are drilled, or a number of slits cut into the lower joint of pipe and the end plugged. An objection to this method is that when the well is cased for only a portion of the distance, the air and water may escape out of the well into fissures in the rock. The hydraulic radius of the water passage in the discharge pipe is reduced by the air pipe, which increases the frictional losses and so diminishes the efficiency, but in this method of piping the entire cross-section of the well, less the area of the air pipe, is available for use as a discharge pipe; so a well piped in this way will have a greater theoretical capacity than wells of the same size piped by the other methods; notwithstanding the obstruction caused by the central air pipe. This fact is shown by the following comparison of the three methods. Assuming in each case that the wells are of 6 inches diameter, the largest eduction pipe which can be got into the well with a 1-inch air pipe beside it, is 3½ inches in diameter, while in the annular tube method of piping a 4½-inch eduction pipe can be accommodated. In the central tube system it is assumed that a 1½-inch air pipe is used with the 6-inch casing serving as the eduction pipe. The tabulated values of the areas and perimeters of the pipes were taken from the table of standard dimensions of wrought iron and steel pipe published in the Crane Company's catalogue. It may be noted in the following tabulation that the hydraulic radius of the central air tube pump is less than that of the annular tube pump, but the net area of the former is nearly twice as large as that of the latter. The discharge through a pipe under a given head is proportional to the product of the area and the square root of the hydraulic radius of the pipe. This product has been calculated and tabulated in the last column and shows the relative capacities of the wells piped according to the three different methods, assuming the value of the coefficient of pipe friction and slip to be the same in all three cases.

	Nominal Size of Well, in Inches.	Nominal Size of Eduction Pipe, in Inches.	Nominal Size of Air Pipe, in Inches.	Approximate Net Area of Eduction Pipe, in Square Inches.	Approximate Wetted Perimeter, in Inches.	Hydraulic Radius, in Inches.	Discharge Function.
				a_p		r	$a_p\sqrt{r}$
Side Inlet Pump..........	6	3½	1	9.89	11.15	0.89	9.33
Annular Tube Pump..........	6	4½	6	15.96	14.62	1.13	16.9
Central Air Tube Pump..........	6	6	1¼	26.05	25.62	1.04	26.5

Combination Pump.—Fig. 6 (d) shows a combination of the annular air tube and central air tube methods of piping. It is evident that the hydrostatic head in the well cannot be greater than that due to the ground water to permit of continuous operation. Therefore, no special advantage is to be gained in introducing compressed air above the water surface in the well, unless the increased surging, due to the less depth of water on the outside of the eduction pipe, might effect the size of bubbles of air admitted. The results of the authors' experiments show, that with the well piped according to the side inlet method using a Harris Air Pump Company's foot-piece, there was no appreciable difference when compressed air was introduced above the water surface in the well and when the well was open to atmospheric pressure, the percentage of submergence and lift being the same in both cases. The cut shows this system with a patented foot-piece on the end of the air-pipe.

Multiple Air Lift Pump.—When the lift is very high and the proper submergence difficult to obtain, the arrangement shown in Fig. 7 may be used where the cross-section of the well permits of its use. This arrangement employs a series of successive lifts and it is claimed that it works more economically than when the water is raised in a single lift. The cross-sectional area of the ordinary deep well will not permit of such an arrangement, but it may be used to advantage in a mine shaft.

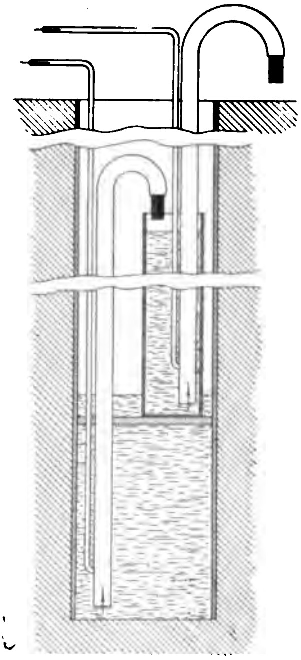


Fig. 7.—Multiple Air Lift.

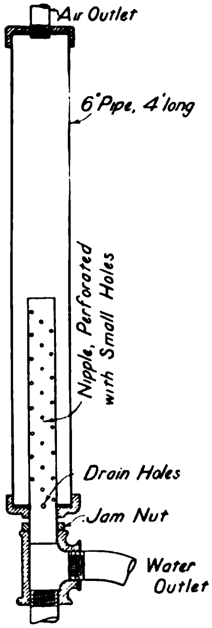


Fig. 8.—Air Separator.

Return Air Pump.—In rising through the eduction pipe there is a transfer of heat between the air and the water; the temperature of the two being practically equal at the point of discharge. Therefore, where the air lift is being used to pump from underground supplies, the temperature of the air issuing from the discharge pipe will be cooler than the atmosphere during the warm months of the year. For each five degrees fall in the temperature of the free air entering the compressor, a saving of about one per cent. in the expenditure of energy used in doing the work of compression may be effected. Hence, where the wells are situated in close proximity to the power house, considerable economy may be effected by connecting the air inlet of the air cylinder of the compressor with the top of the well casing head. An air separator used for this purpose consists of a cylindrical drum about 18 inches in diameter and 8 or 10 feet long, attached to the casing head from the side of which the discharge from the well passes out and from the top the air is piped to the compressor. An air separator is shown in Fig. 8. The return-air method of piping may be used with any of the types of pumps shown in Figs. 6 and 7.

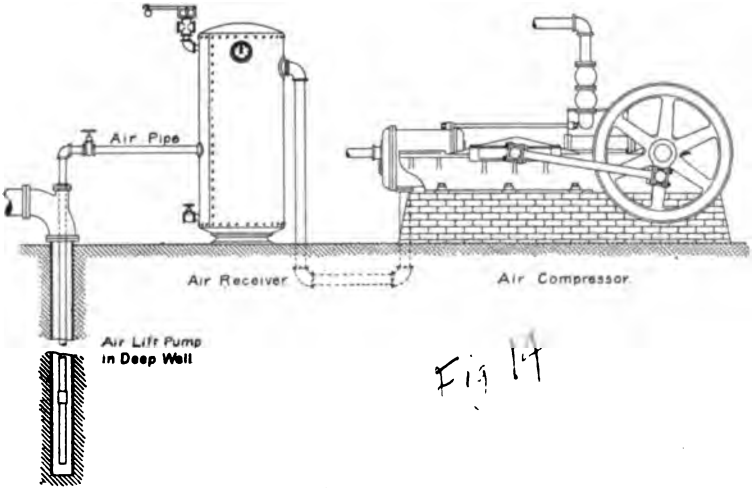


Fig 17

Fig. 9.—Air Lift Pumping Plant.

The air separator described above is useful also where it is desired to pump to points situated some distance from the well, as described on page 42.

Diverging Outlet Pump.—When the eduction pipe is of uniform diameter throughout, as shown in Figs. 3, 6, 7 and 9, the discharge occurs at quite high velocity, resulting in a considerable loss of energy. To conserve this kinetic energy of the water Mr. Jos. Price, an English engineer, fitted his pump with an eduction pipe which increased in diameter towards the top, so that as the compressed air expanded in rising the velocity would not be greatly increased. This device could be used with any type of foot-piece and any method of piping. A few experiments by the authors indicate that a considerable saving may be effected by the use of a diverging outlet of proper design.

THE PLANT

An air lift pumping plant comprises the motive power, an air compressor, an air receiver, and an air pipe leading to one or more pumps such as have been described in the preceding paragraphs. Fig. 9 serves to illustrate in a general way the machinery and other essential apparatus.

The compressor and receiver should be located where the expense of installing and operating the plant will be the least. They are often situated a considerable distance from the wells. The power used to drive the compressor may be either steam, electric, water, or internal combustion engines.

The Compressor.—The type of compressor installed will depend on the pressure required to pump the wells, the nature and amount of power available for compressing the air, and the number and size of the wells. Where the pressure required is small and the quantity to be compressed not very large, the ordinary straight line or duplex compressor may be used. Where high air pressures are necessary economy demands the cross compound type fitted with an intercooler.

The Receiver.—The receiver is used to store and equalize the air pressure. It acts in much the same way as the air chamber on a force main and reduces the effects of the pulsations of the compressor. It also acts as a separator to catch the water and oil which are carried by the air. It is quite necessary to provide some kind of a separator for this purpose to prevent the air pipes becoming clogged, and where a foot-piece with small air openings is used it is especially desirable to have the air free from any clogging material. The air receiver is usually built of boiler iron and designed so as to permit a steady flow of air to the well. The air from the compressor passes down from the top of the receiver through a pipe and discharges beneath the surface of some water which is usually kept in it. The outlet pipe for the air is located near the top of the receiver, and a drain is provided at the bottom to carry away the oil and water.

The Air Line.—From the receiver the main air line runs to the wells. The piping to the wells should be arranged to avoid unnecessary valves, elbows and bends, as these reduce the efficiency of the plant. The same precaution should also be taken in the design of the water pipes. A valve should be placed on the air main in the power house so that the wells may be controlled from a central point. It is desirable to place a regulating and a stop valve at each well so that after the proper amount of air has been adjusted to the conditions of lift, submergence, etc., at the well, it will not be necessary to disturb the adjustment in order to shut down the well.

In the following pages the discussion will be confined to the air lift pump alone, leaving out of consideration the efficiency of the air transmission pipes, compressors and other appurtenances, which have an important bearing on the desirability of installing air lift pumping plants, but the consideration of which is beyond the scope planned for this bulletin.

DISADVANTAGES OF THE AIR LIFT PUMP

The air lift pump, in common with every other device, has a number of disadvantages, the principal ones being its low efficiency, the great depth of submergence required and its poor adaptability to conditions requiring the discharge to be conveyed great horizontal distances.

Low Efficiency.—The most serious of the above mentioned disadvantages is the low hydraulic efficiency of the pump. The pump is generally credited with efficiencies of only 25 to 33 per cent, but notwithstanding the low efficiency of the pump itself, the entire air lift pumping plant in some cases develops a duty which compares favorably with other systems of pumping.

Great Depth of Submergence.—A single air lift pump cannot be used in a shallow well or reservoir except to raise the liquid a very small distance, owing to the high percentage of the total length of the pump which must be submerged to give good efficiencies. This fact limits the field of the air lift pump principally to deep well pumping. The multiple stage pump described on page 37 overcomes the difficulty, but probably at the cost of reduced efficiency.

Limited Horizontal Pumping.—Several plants have been installed where the air lift was used to pump the water a considerable horizontal distance after it had been raised to the surface of the ground, but such plants are not considered efficient. The air in passing through the horizontal or even an inclined pipe, is not likely to be evenly distributed throughout the cross-section of the pipe, but is likely to pass along under the upper side, allowing a large space in the lower portion of the pipe for the water to slip back past the bubble. In a horizontal pipe the air cannot exert any buoyant effort to aid in discharging the water, and its expansive force, which might be used in overcoming pipe friction is not likely to be effective on account of the serious amount of slip which occurs under these conditions. Where it is desired to convey the water to a point distant horizontally from the well, the eduction pipe should be carried vertically above the well a height equal to the friction head of the water in the horizontal conductor, and at its top it should be fitted with an air separator, such as the one described on page 39. From the separator the water would flow by gravity to the desired point.

Aeration.—The thorough aeration of the water pumped is generally regarded as an advantage, but under some circumstances it is a disadvantage. It doubtless promotes the rusting and consequent destruction of the eduction pipe and in some cases causes a deposit of salts which clogs the water passages, especially in the foot-piece. The opinion has also been expressed that compressed air causes an excessive growth of algae. The bacterial content of water is somewhat increased by the air lift unless the air supply is filtered.

ADVANTAGES OF THE AIR LIFT PUMP

The air lift pump possesses many features which give it decided advantages over other types of pumps, when applied to certain conditions.

Large Capacity.—Its principal claim to superiority lies in its large capacity. When the conditions are suitable for its installation, an air lift pump will discharge more liquid from a well of small bore than will any other type of pump. This is due to the fact that almost the entire cross-sectional area of the well is available for the flow of liquid and the action is nearly continuous. The quantity that can be discharged by an air lift pump is only limited by the capacity of the well and the expense of pumping at unreasonably high rates; while deep well pumps, the majority of which are single acting, limit the discharge by the allowable piston speed, which usually does not exceed 100 feet per minute. The air lift affords a ready means for testing the capacity of a well even if it is not to be permanently installed.

Low Maintenance Cost.—Owing to the simplicity of the pump the expense of maintenance of the plant is very low, and is due principally to the expenses in connection with the compressor. The operation of the air lift is exceedingly simple and the life of the pump is almost indefinite. Sometimes the air pipes and foot-piece become clogged with the oil carried over from the compressor cylinders and in that case have to be removed and cleaned, but this rarely occurs, and when it does the cost is small compared with the cost of replacing a mechanical pump. The fact that there are absolutely no moving parts in the well makes the pump especially fitted for handling dirty or gritty water, sewage, mine water and acid or alkaline solutions in chemical or metallurgical works, or other corrosive liquids. Mechanical pumps suffer from fine sand in the water, which cuts the packing, plungers, and valves in a short time and makes frequent repairs necessary. To repair such pumps they have to be stopped for a period of several days, resulting in a consequent waste of time and increase in expenses. Liquids that attack metals such as brine, sulphuric acid, etc. may be pumped by the air lift pump, because the pump and appurtenances may be replaced at a small expense and loss of time. The application of the air lift as a dredge pump has been tried and found successful, but it has not been extensively used for this purpose.

Low Operating Cost.—In places where the wells are scattered over a considerable area, or are remote from the power house, the air lift pump has an advantage over the steam driven pump. In a deep well pump driven by steam each well must be equipped with a separate engine and working barrel, which entails heavy condensation losses through long steam supply pipes. The expense of attendance of a plant of this sort with its scattered pumping units is great. In the air lift pump the transmission loss is much smaller and as no attendance at the well is required, they may be put in operation or controlled by a valve in the power house.

Not Affected by High Temperatures.—Fluids of different densities and temperatures may be handled to advantage by the air lift in cases where the use of other types of pumps would be prohibited. In the case of a hot liquid the air absorbs part of the heat of the liquid and hence is increased in volume, so that the discharge of liquid for the same expenditure of free air is greater with hot than with cold liquids. This results in a considerable gain in efficiency for the pump.

Aeration.—Where the water is to be used for a domestic supply and there are impurities in it such as iron, it has been noticed that the iron is oxidized by the aeration of the water and the supply is thereby improved. Aeration is especially advantageous in the pumping of sewage on account of the aid it gives in the oxidation of the impurities.

Reliability.—Air lift pumps are not liable to sudden stoppages or breakdowns.

WISCONSIN EXPERIMENTS

The experiments made by the writers at the Hydraulic Laboratory of the University of Wisconsin, were begun during the summer of 1908 and were continued at various times thereafter with modifications in the apparatus.

NOTATION

The notation and definitions used in the following discussion are given below:

"Reading" is the term used to designate a single value obtained by the inspection of the scale of an instrument.

"Run" is used to designate such a combination of readings, made under practically constant conditions, as form a complete unit in the result.

"Series" is used to designate those runs made with varying rates of pumping but with constant conditions of apparatus.

DESCRIPTION OF APPARATUS—1908 EXPERIMENTS

The 1908 experiments, comprising runs 1 to 318, inclusive, were made with the apparatus arranged as shown diagrammatically in Fig. 10.

Eduction Pipe.—In this series of experiments the eduction pipe, which consisted largely of 1¼-inch glass tubing supported by a slotted 2-inch iron pipe, was exposed to view, thus affording opportunity to observe the air and water rising in the pipe. The upper end of the glass pipe was coupled to a 1¼-inch iron pipe, about a foot long, on the upper end of which was a cast iron elbow fitted with a nipple 2 inches long. A piece of canvas fastened to this nipple deflected the spray downward into a 4-inch galvanized spout. This arrangement provided a free discharge into the air at the elevation of the elbow.

Foot-Piece.—The foot-piece used in the 1908 experiments was a No. 2 Harris air pump* having a 1¼-inch discharge, a 1½-inch suction, and ½-inch air pipe. A sectional view of the foot-piece used is shown in Fig. 11 (a) and an exterior view in Fig. 12.

The Harris pump, as described in patent No. 814, 601, consists essentially of an ejector having a contracted passageway formed by a sleeve snugly fitted into its upper end, and having immediately below the sleeve a nozzle screwed into an air pipe. The walls of the ejector are enlarged about the air tube so that the dimensions of the passageway through the ejector are substantially uniform. The air nozzle in the ejector differs from the form described in the above patent, in having the air discharge into the body of the ejector through a circular slot in the end of the nozzle, instead of from an open diverging tube. The sleeve and nozzle are shown in their relative positions in Fig. 12.

Tail-Piece.—As used in the 1908 experiments the foot-piece was connected by a short nipple to a reducing flange on the top of a 6-inch flanged cast iron tee. This arrangement made

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* **Made** by the Harris Air Pump Co., Indianapolis, Ind.

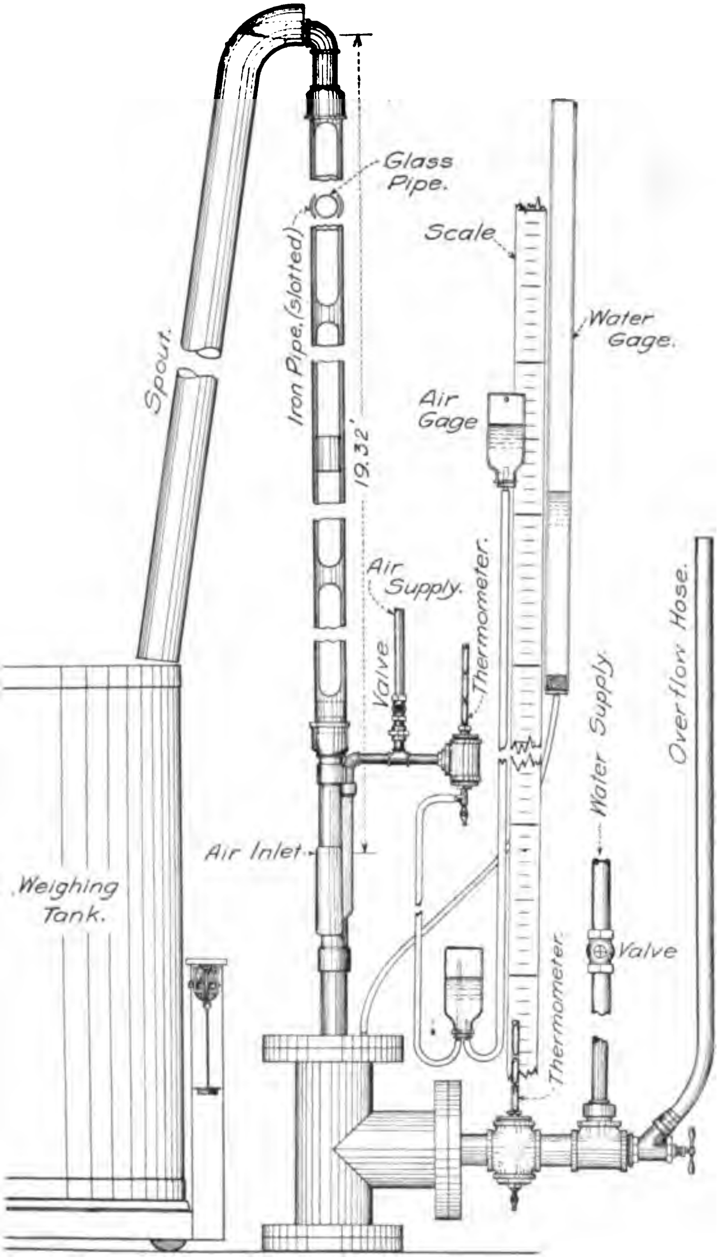
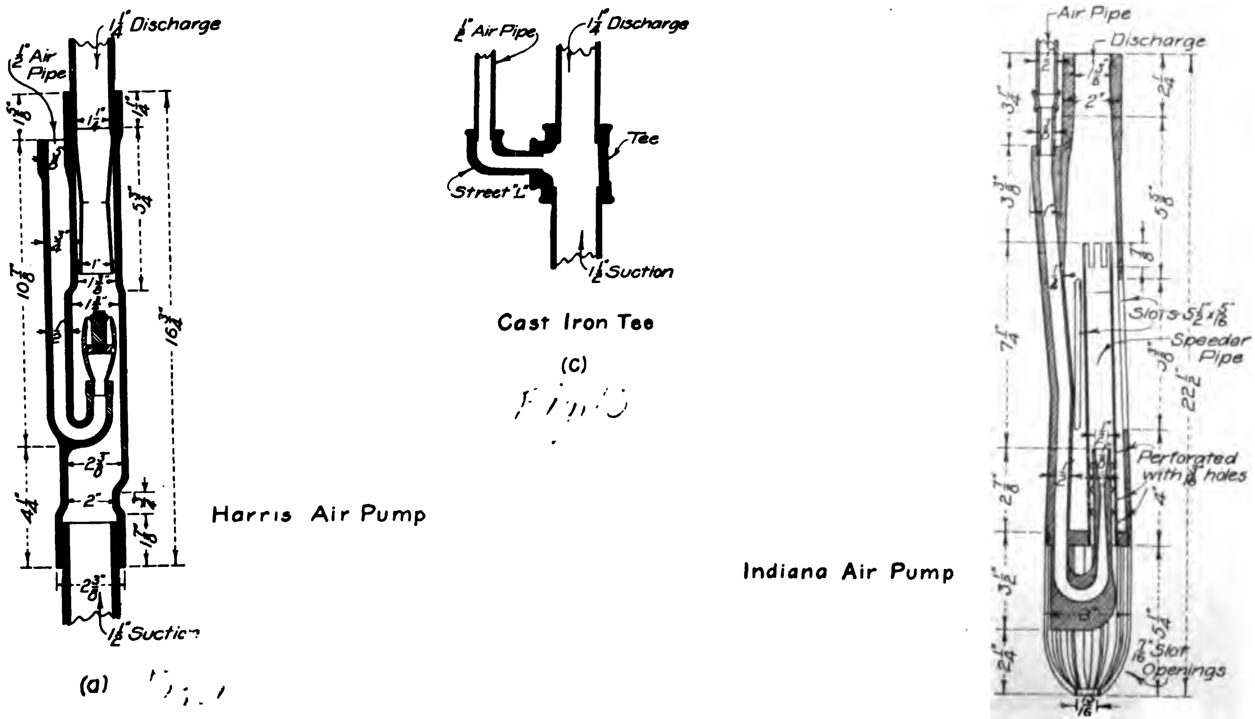


Fig. 10.—Experimental Apparatus—1908 Experiments.



Harris Air Pump

(a)

Cast Iron Tee

(c)

Indiana Air Pump

(b)

Fig. 11.—Foot-Pieces Used in Wisconsin Experiments.

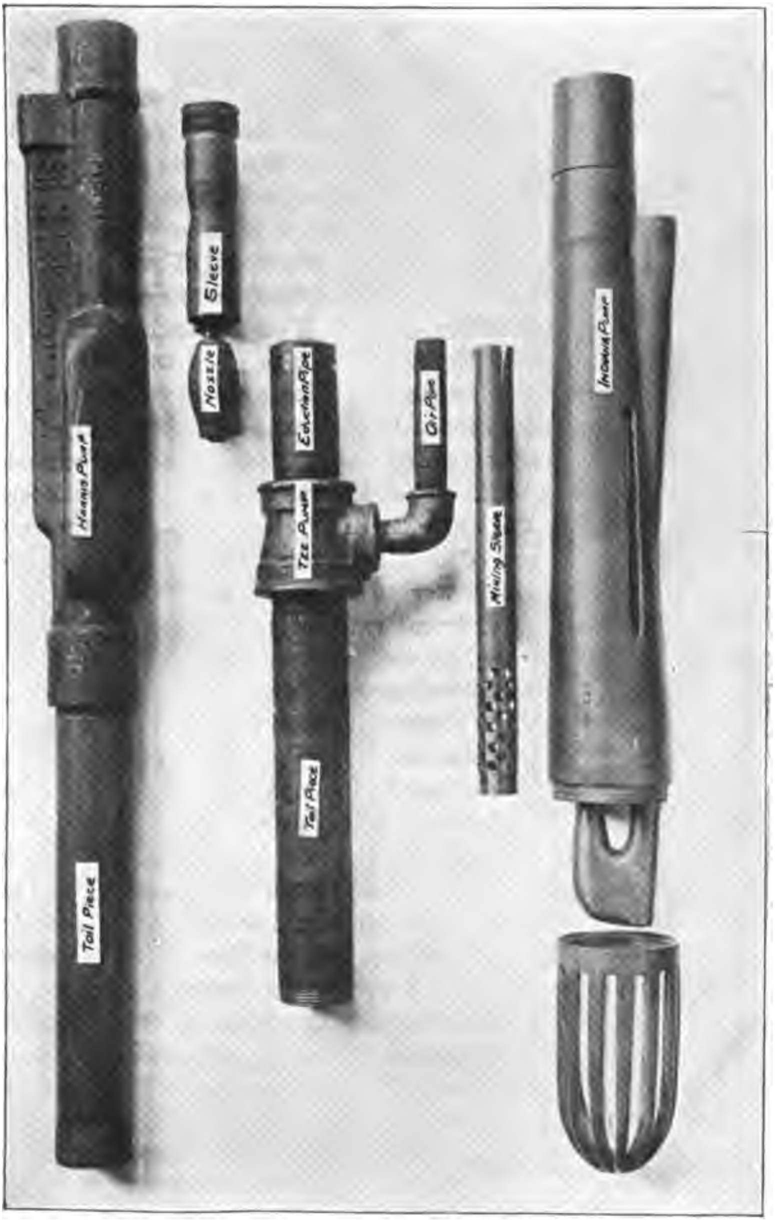


Fig. 12.—Foot-Pieces Used in Wisconsin Experiments.

the entrance to the pump approximately one foot below the air inlet.

Water Supply.—The water supply, controlled by a 1-inch valve, entered the tee through the branch, the water passing through a 2-inch cross in which was inserted an iron thermometer well. The bulb of the thermometer in the well was surrounded by mercury in order to make the thermometer respond quickly to changes in the temperature of the water.

For the purpose of determining the head of water at the air inlet, a ¼-inch nipple was tapped into the top of the 6-inch tee and was connected by means of a 1¼-inch glass tube, marked water gage in Fig. 10. A large glass tube was used in order to reduce the amplitude of the oscillations due to the variations of pressure caused by the surges of air in the eduction pipe. The head of water was varied during the experiments by adjusting the height of the outlet of the overflow hose. This overflow hose was quite necessary for the proper working of the pump as it served as a reservoir and standpipe, which regulated the flow of water to the well and kept the submergence constant at different rates of pumping. The rate of pumping was determined by weighing, on a platform scale, the water discharged in a given interval of time.

Air Supply.—The air used in the experiments was drawn from one of the storage tanks of the University pneumatic pressure water works plant. This tank had a capacity of approximately 3,000 cubic feet and was normally maintained at about 140 pounds pressure, but during the air lift experiments the pressure was often reduced, owing to the use of the air, to the minimum under which the air lift would work. An automatic reducing regulating valve controlled the flow from the storage tank so as to maintain a practically uniform pressure in the receiving tank at the air orifice described below.

The quantity of air supplied to the air lift was controlled by a valve in the ½-inch supply pipe near the foot-piece. The temperature of the air was measured by a mercury thermometer placed in a well similar to the one used to measure the temperature of the water. It was placed in a 2-inch tee on a branch leading from the main air pipe. A rubber tube, one end of which connected to the tee and the other end of which terminated in a glass tube, served to transmit the air pressure at the tee to the air space in the upper part of the lower bottle on a water gage. The pressure head in the air at the 2-inch tee was indicated in feet of water by the difference of elevation between the water surfaces in the two bottles. A ⅜-inch hole in the side of the upper bottle insured the pressure in this bottle being equal to that of the atmosphere. The two bottles were connected by rubber hose, the ends of which terminated in short glass tubes under the surface of the water in both bottles. As the pressure of the air used in the different experiments varied, the height of one or both of the bottles was changed so as to prevent the surface of the water disappearing in the tube or overflowing through the vent hole. Bottles were used instead of glass tubes in this gage, in order to provide larger water surfaces, thus reducing the oscillations of the surfaces to one or two hundredths of a foot, instead of two feet or more which occurred when a tube was tried.

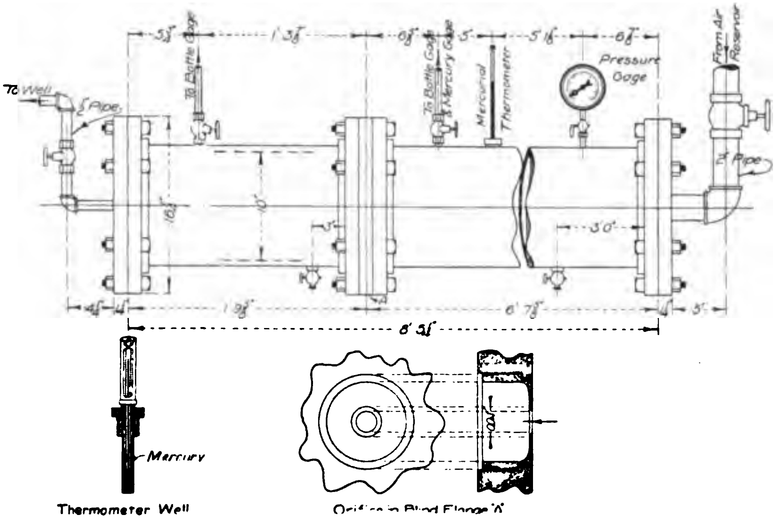


Fig. 13.—Drum and Orifice for Air Measurement.

Air Measurement.—To determine the quantity of air used in the experiments a standard sharp edged orifice, shown in Fig. 13, was used. Its diameter was measured by a Starrett micrometer caliper reading to 1/1000 of an inch and was found to be 0.480 inches. It was made of brass and was arranged to screw into an iron plate flush with its surface on the high pressure side. The iron orifice plate was bolted between two sections of 10-inch iron pipe. The air supply entered the longer section of 10-inch pipe through a 2-inch pipe. A Bourdon pressure gage connected to the drum was used to indicate the pressure in the drum while adjusting the pressure regulating valve. During the experiments the pressure in the high pressure side of the drum was more precisely determined by means of a mercury U tube gage, shown at A in Fig. 15, connected to the drum by a ¼-inch iron pipe at a point 6 inches from the face of the orifice plate. During the 1908 experiments about 100 feet of ½-inch pipe connected the orifice drum with the air lift pump. In Fig. 13, one of the thermometer wells, used for measuring the temperature of the air and of the water at the pump, is shown in detail. A mercurial thermometer placed in a similar well was used to determine the temperature of the air in the orifice drum.

To measure the difference of head on the two sides of the orifice a bottle gage of special design, was connected to the orifice drum by ¼-inch pipe at the points shown in Fig. 13. The bottle gage is shown in Figs. 14 and 15. It consisted of two inverted clear glass bottles about half filled with water and connected by a glass U tube, which passed through the stoppers but ended below the water surface. The air pressures on the two sides of the orifice were transmitted to the air spaces in the bottles by tubes which passed through the stoppers and ended above the water surface. When the pressures were equal the water stood at the same level in both bottles, but when unequal the difference of pressure was balanced by the difference in the heights of the water surfaces. This difference in height was measured by means of two pointed brass rods which were coated with grease and slid through holes in the rubber stoppers. In use the two rods were raised or lowered by a small worm gear until their points were in the level water surfaces in the bottles, the relative elevations of the points being then determined by means of the scales and verniers attached to the rods and reading to 1/1000 of a foot. To get the actual difference of elevation of the points,

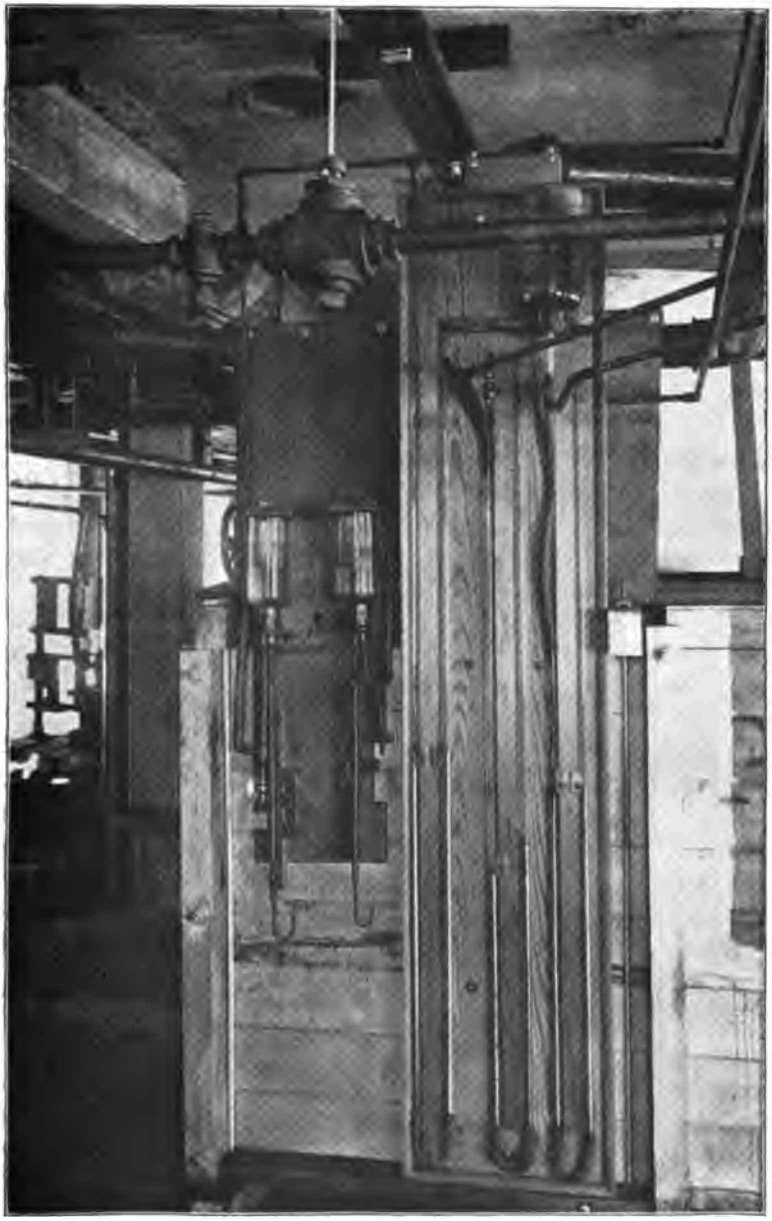


Fig. 14.—Gages Used in 1909 Experiments.

or of the water surfaces, it was necessary to deduct from the difference indicated by the vernier readings a similiar difference indicated when the air pressure in the bottles was equal, as when the tubes were open to the atmosphere. The scale and vernier on the high pressure bottle were arranged to give readings increasing downward, while those on the low pressure bottle gave readings increasing upward. This arrangement made the computations easy, for in order to calculate the true difference of elevation of the water surfaces it was only necessary to subtract the sum of the initial or equal pressure vernier readings from the sum of the average vernier readings of any run. This arrangement of the verniers and scales also nullified the effect of any leakage of water during an experiment, since a lowering of the water surfaces by leakage would increase the high pressure readings the same amount that it would decrease the low pressure readings, the sum of the two remaining the same. The leakage through the stoppers was very slight; not requiring replenishing in a year and a half.

DESCRIPTION OF APPARATUS—1909 EXPERIMENTS

The 1909 experiments, comprising runs 319 to 608, inclusive, were made with the purpose of determining the effect of variations in the length of the eduction pipe, the form of the foot-piece, the presence of compressed air in the casing, and other features. These experiments required that the apparatus as originally set up be changed somewhat.

The general arrangement of the apparatus as used in the 1909 experiments is shown in Fig. 15.

Well.—There was some doubt as to whether the arrangement, used in the 1908 experiments, for introducing the water into the eduction pipe represented closely enough the working conditions of a pump in an actual well, and it was further desired to find the effect of compressed air outside of the eduction pipe. In the 1909 experiments the pump was, therefore, placed in a well which consisted of a 6-inch wrought-iron pipe 22 feet long, capped at the top and flanged at the bottom to a 6-inch tee.

Eduction Pipe.—The eduction pipe was the same as used in the 1908 experiments. The entire glass portion of the pipe with its supporting slotted 2-inch iron pipe was suspended in the well from the cap. During runs 319 to 379, inclusive, the length of the eduction pipe from the air inlet to the center of the elbow was 19.32 feet as in the previous experiments. During runs 381 to 440, inclusive, the length was 26.74 feet, the increase being made by adding a length of 1¼ inch iron pipe above the well top. During runs 441 to 460 the length was 41.50 feet, the addition being made as above. Runs 461 to 500 inclusive were made with different forms of foot-pieces which made slight variations in the length, 42.08 feet for runs 461–480 and 41.6 feet for runs 481–500.

Foot-Piece.—During runs 319 to 460, inclusive, the same Harris foot-piece was used as was used in the 1908 experiments. An Indiana foot-piece* was used during runs 461 to 480. It was a 1½-inch foot-piece reduced to 1¼-inch by a reducer at the top. During runs 481 to 500 the foot-piece used consisted of a common cast-iron reducing tee, the eduction pipe, tail-piece and air pipe entering the tee only so far as allowed by the standard threads on their ends, as shown in Fig. 11 (c). The Harris foot-piece has already been described on page 48. The Indiana foot-piece was developed from the Harris foot-piece, the object, as stated in the patent specifications, being to produce a simple, efficient, center-nozzle air lift. This foot-piece is shown in Figs. 11 (b) and 12, the various parts being indicated. In Fig. 12 the strainer has been removed to show the lower part of the nozzle, and the mixing tube or sleeve, into which the nozzle discharges the air, has also been removed and placed beside the foot-piece. The reducer can be seen at the upper end of the foot-piece.

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* Presented by the Indiana Air Pump Co., Indianapolis, Ind.

Tail-Piece.—In the 1908 experiments the tail-piece was only about 2 inches long. Under the conditions of the apparatus used in these experiments, any air which may have escaped from the bottom of the eduction pipe into the 6-inch tee would have been carried into it again by the water. Under the conditions of the apparatus used in the 1909 experiments such would not have been the case. A tail-piece consisting of 1 foot of 1½-inch pipe was, therefore, screwed into the lower end of the Harris foot-piece, the length suitable for the conditions of our experiments having been determined by correspondence with the pump makers. The same piece of pipe was used as a tail-piece in connection with the tee used as a foot-piece in runs 481 to 608. No tail-piece was used on the Indiana foot-piece.

Water Supply.—Water was introduced into the well by means of a 2-inch pipe connecting the branch of the 6-inch tee with the University main. The depth of the water in the well was regulated by means of a 1½-inch overflow pipe connected at the lower end to the supply pipe and discharging at its upper end into the air. By varying the length of the overflow pipe any desired depth of submergence could be obtained. Provision was made, as in the 1908 experiments, for taking the temperature of the water as it entered the well, by screwing a thermometer well into a 2-inch tee in line with the supply pipe. The water was discharged at the upper end of the eduction pipe

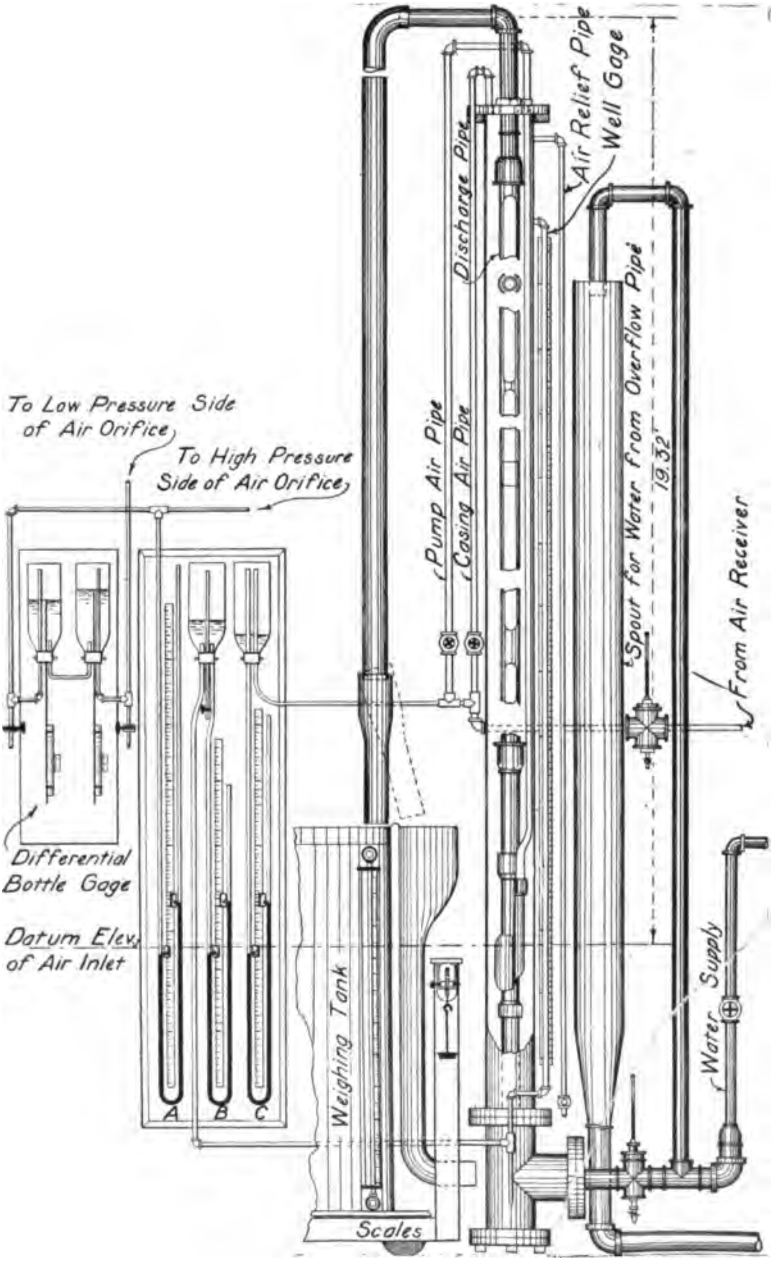


Fig. 15.—Experimental Apparatus—1909 Experiments.

at atmospheric pressure into a conductor pipe, by means of which it was led to the weighing tank or discharged as waste.

Air Supply.—The source of air and the apparatus for controlling it and for measuring it were the same as in the 1908 experiments, but in the 1909 experiments the pump was set up only a few feet distant from the air measuring orifice. The air line was, therefore, shorter in these experiments. At the well the air line divided into two branches; one branch, marked pump air pipe in Fig. 15, supplying the pump through a ½-inch pipe, which entered the well through a stuffing box in the cap of the well, the other branch, marked casing air pipe, introducing the air into the casing above the water in the well, through a ½-inch pipe screwed into the cap of the well.

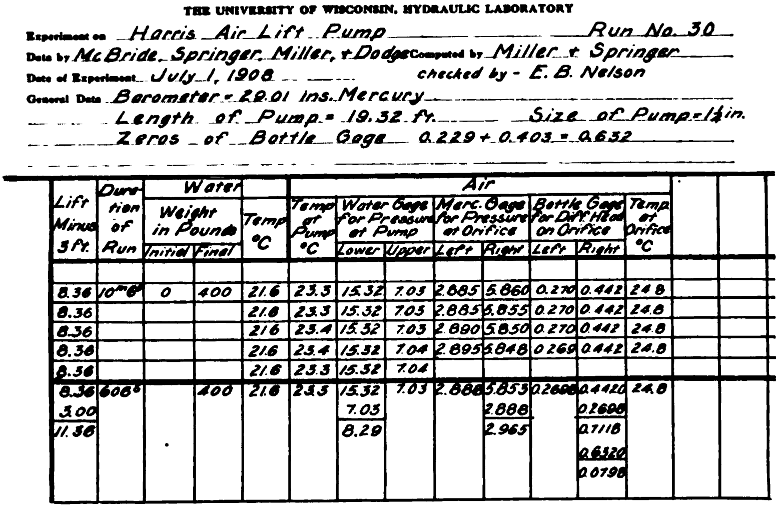
Gages.—There were four gages used in making these experiments. The differential bottle, described in connection with the 1908 experiments, was used to determine the difference of head on the air orifice. The gage, marked A in Fig. 15, was a mercury U tube used for determining the head on the high pressure side of the orifice. Gage B measured the pressure at the bottom of the well, by means of which the equivalent submergence could be calculated, when air pressure was exerted on top of the water in the well. The actual depth of water in the well was obtained by means of a glass gage attached to the outside of the well, from which the submergence could be directly calculated when the water in the well was open to atmospheric pressure. Mercury gage C measured the air pressure as it entered the pump, the valve in the pump air line being wide open. Air chambers were used on gages B and C to dampen the fluctuations of the gage readings, but this device was unnecessary as the readings were steady except when the water was so low in the well that air entered the pump through the tail-piece. The scales on the gages were made of steel, graduated to hundredths of a foot, and, by means of sliding verniers, readings could be obtained to 1/1000 of a foot.

Leakage Tests.—All pipe joints and other places where an escape of air was possible were tested by applying soap solution, which indicated small leaks by the formation of bubbles. The entire apparatus was made air-tight before any runs were made.

METHODS OF OBSERVING

Owing to the fact that different types of gages were used in the 1909 experiments than those used in the 1908 experiments for making some of the measurements, the arrangement of the data differed somewhat in the latter runs from the arrangement first used. The conditions of operation usually remained quite constant during a run, so that the gage and thermometer readings varied only slightly, if at all. Usually four or five readings of each instrument were made during a run, the length of a run varying from 3 to 15 minutes, and the discharge of water varying from 200 to 1,000 pounds. Time was measured by a calibrated stop-watch; temperatures were measured by mercurial thermometers graduated to the centigrade scale and reading to tenths of a degree, and lengths were measured in feet.

1908 Experiments.—Copies of the complete observed data of runs 30 and 132 are given below as samples to show the constancy of the conditions of operation and the method of making the observations. The scale by which data were obtained for



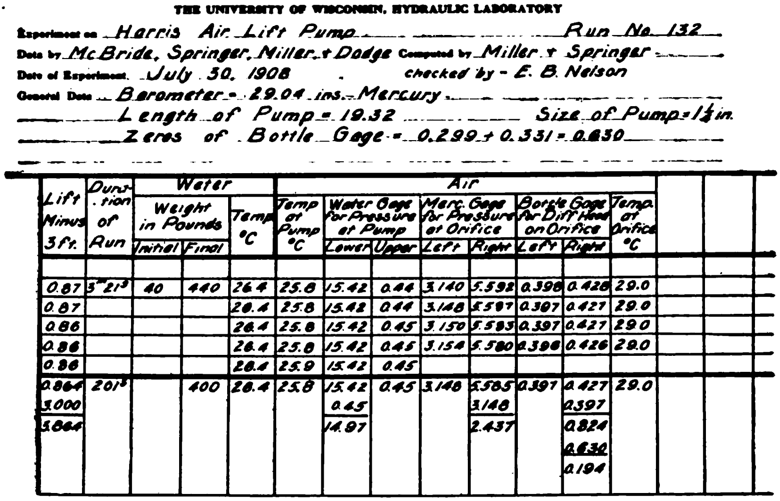
THE UNIVERSITY OF WISCONSIN, HYDRAULIC LABORATORY

Experiment on Harris Air Lift Pump Run No. 30
Data by McBride, Springer, Miller, + Dodge Computed by Miller + Springer
Date of Experiment July 1, 1908 checked by - E. B. Nelson
General Data Barometer = 29.01 ins. Mercury
Length of Pump = 19.32 ft. Size of Pump = 1¼ in.
Zeros of Bottle Gage 0.229 + 0.403 = 0.632

Lift Minus 3 ft.	Duration of Run	Water			Air									
		Weight in Pounds		Temp °C	Temp at Pump °C	Water Gage for Pressure at Pump		Merc. Gage for Pressure at Orifice		Bottle Gage for Diff. Head on Orifice		Temp at Orifice °C		
		Initial	Final			Lower	Upper	Left	Right	Left	Right			
8.36	$10^{m}6^{s}$	0	400	21.6	23.3	15.32	7.03	2.885	5.860	0.270	0.442	24.8		
8.36				21.6	23.3	15.32	7.05	2.885	5.855	0.270	0.442	24.8		
8.36				21.6	23.4	15.32	7.03	2.890	5.850	0.270	0.442	24.8		
8.36				21.6	23.4	15.32	7.04	2.895	5.848	0.269	0.442	24.8		
8.36				21.6	23.3	15.32	7.04							
8.36	606^{s}		400	21.6	23.3	15.32	7.03	2.888	5.853	0.2698	0.4420	24.8		
3.00						7.05			2.888		0.2698			
11.36						8.29			2.965		0.7118			
											0.6320			
											0.0798			

computing the lift and submergence was inverted and placed with its zero three feet below the center of the outlet of the pump. The readings of the scale, from its zero down to the water surface in the water gage (see Fig. 10), were, therefore, three feet less than the true lift.

1909 Experiments.—The apparatus for the 1909 experiments was so arranged that only two men were needed for conveniently making the observations; one man reading the gages while the other took the time of the run, read the thermometers, observed the depth of water in the well, and weighed the water. The gages were read as nearly simultaneously as possible, by first setting the verniers on all the different gages and afterwards taking the readings. About four accurate settings of the gages could be obtained in a ten minute run. The thermometers were read to a tenth of a degree centigrade and the temperature was obtained at least twice during a run; usually at the beginning and end. The method of weighing the water was as follows: The empty tank weighed about 150 pounds, so a 200-pound weight was placed on the scale beam and the water diverted into the tank. When enough water had run into the tank to cause the scale beam to rise, a stop watch reading to ⅕ of a second



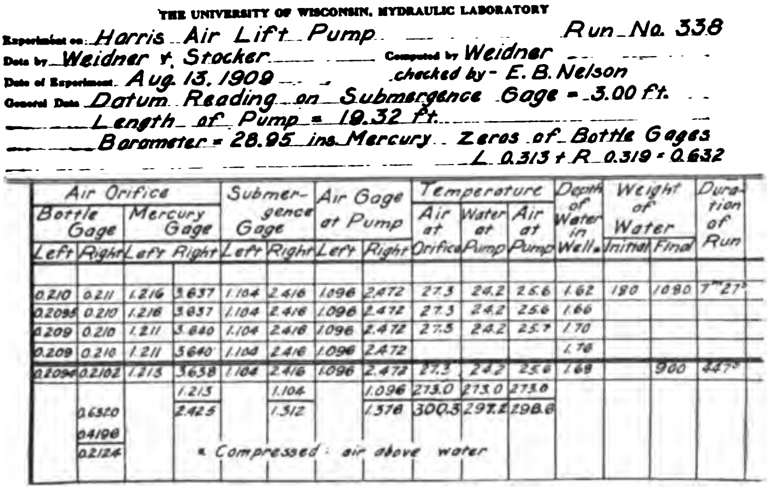
THE UNIVERSITY OF WISCONSIN, HYDRAULIC LABORATORY

Experiment on Harris Air Lift Pump Run No. 132
Data by McBride, Springer, Miller + Dodge Computed by Miller + Springer
Date of Experiment. July 30, 1908 checked by - E. B. Nelson
General Data Barometer = 29.04 ins. Mercury
Length of Pump = 19.32 Size of Pump = 1½ in.
Zeros of Bottle Gage = 0.299 + 0.331 = 0.630

Lift Minus 3 ft.	Dura-tion of Run	Water			Air										
		Weight in Pounds		Temp °C	Temp of Pump °C	Water Gage for Pressure at Pump		Merc. Gage for Pressure at Orifice		Bottle Gage for Diff Head on Orifice		Temp. of Orifice °C			
		Initial	Final			Lower	Upper	Left	Right	Left	Right				
0.87	$3^m 21^s$	40	440	26.4	25.8	15.42	0.44	3.140	5.592	0.398	0.428	29.0			
0.87				26.4	25.8	15.42	0.44	3.148	5.597	0.397	0.427	29.0			
0.86				26.4	25.8	15.42	0.45	3.150	5.593	0.397	0.427	29.0			
0.86				26.4	25.8	15.42	0.45	3.154	5.580	0.398	0.426	29.0			
0.86				26.4	25.9	15.42	0.45								
0.864 3.000 3.864	201^s		400	26.4	25.8	15.42 0.45 14.97	0.45	3.148	5.585 3.148 2.437	0.397	0.427 0.397 0.824 0.630 0.194	29.0			

was started. Additional 100-pound weights were then placed on the scale beam and the watch was stopped when the beam rose again. The time and number of weights added while the watch was running were then recorded. The length of a run varied from 3 to 15 minutes depending on the depth of submergence and the amount of air supplied to the pump.

The readings for some of the runs were taken by a single observer, and it was not possible for one man to get more than two sets of gage readings during a run, the length of which was limited by the capacity of the measuring tank. It was entirely unnecessary to take more readings, because as a rule the readings were identical, and two readings of each instrument served as a check to prevent blunders in observing and recording. The complete data of runs 338 and 482 are given below.



THE UNIVERSITY OF WISCONSIN. HYDRAULIC LABORATORY

Experiment on: Harris Air Lift Pump Run No. 338
Data by Weidner & Stocker Computed by Weidner
Date of Experiment Aug. 13, 1909 checked by- E. B. Nelson
General Data Datum Reading on Submergence Gage = 3.00 ft.
Length of Pump = 19.32 ft.
Barometer = 28.95 ins Mercury Zeros of Bottle Gages
L 0.313 + R 0.319 = 0.632

Air Orifice				Submergence Gage		Air Gage at Pump		Temperature			Depth of Water in Well.*	Weight of Water		Duration of Run
Bottle Gage		Mercury Gage						Air at Orifice	Water at Pump	Air at Pump				
Left	Right	Left	Right	Left	Right	Left	Right					Initial	Final	
0.210	0.211	1.216	3.637	1.104	2.416	1.096	2.472	27.3	24.2	25.6	1.62	190	1090	$7^{m}27^{s}$
0.2095	0.210	1.216	3.637	1.104	2.416	1.096	2.472	27.3	24.2	25.6	1.66			
0.209	0.210	1.211	3.640	1.104	2.416	1.096	2.472	27.5	24.2	25.7	1.70			
0.209	0.210	1.211	3.640	1.104	2.416	1.096	2.472				1.76			
0.2094	0.2102	1.213	3.638	1.104	2.416	1.096	2.472	27.3	24.2	25.6	1.69		900	447^{s}
			1.213		1.104		1.096	273.0	273.0	273.0				
	0.6320		2.425		1.312		1.376	300.3	297.2	298.6				
	0.4198													
	0.2124													

* Compressed: air above water

The observed temperatures of the air and water at the entrance to the pump were not used in the computations and hence are not tabulated in the final results of Table 1.

Most of the computing was done with a Thatcher calculating instrument and all computations have been carefully checked.

THE UNIVERSITY OF WISCONSIN, HYDRAULIC LABORATORY

Experiment on 1¼ Inch Tee Air Lift Pump Run No. 482

Data by Weidner Computed by Weidner

Date of Experiment Feb 23, 1910 checked by - Nelson

General Data Length of Pump = 41.6 ft.

Datum Reading on Submergence Gage = 6.67 ft.

Barometer = 29.42 ins. Mercury

Zeros of Bottle Gages 0.299 + 0.334 = 0.633

Air Orifice				Submergence Gage		Air Gage at Pump		Temperature			Depth of Water in Well	Weight of Water		Duration of Run
Bottle Gage		Mercury Gage						Air at Orifice	Water at Pump	Air at Pump				
Left	Right	Left	Right	Left	Right	Left	Right					Initial	Final	
0.241	0.355	0.790	4.145	0.853	2.873	0.910	3.480	18.0	7.7	18.3	5.6	200	1000	$5^{m}34^{s}$
0.241	0.355	0.790	4.145	0.855	2.873	0.910	3.480	17.9	7.6	16.2	5.6			
	0.633 0.596 0.037		4.145 0.790 3.355		2.873 0.853 2.020		3.480 0.910 2.570	17.9 273.0 290.9	7.6 273.0 280.6	16.2 273.0 289.2	5.6		800	3034

METHODS OF COMPUTING

Quantity of Air.—The amount of air supplied to the pump was measured, as previously stated, by means of a circular orifice in a thin brass plate. The following formula derived by Mr. S. A. Moss* was used in computing the discharge through the orifice.

$$q_a = 3860\, c_o\, a_o \frac{p_u}{\sqrt{T_u}} \sqrt{\left(\frac{p_d}{p_u}\right)^{1\cdot425} - \left(\frac{p_d}{p_u}\right)^{1\cdot713}} \qquad (20)$$

in which

q_a = cubic feet of free air discharged per second.

c_o = a coefficient of discharge.

a_o = area of orifice, in square feet.

p_u = absolute pressure of air at upstream side of orifice, in pounds per square inch.

p_d = absolute pressure of air at downstream side of orifice, in pounds per square inch.

T_u = absolute temperature of air, in degrees Fahrenheit, at upstream side of orifice.

The values of the coefficient of discharge were obtained from the curve shown in Fig. 16, in which the abscissas represent the difference in pressure in feet of water between the up and down stream sides of the orifice, and the ordinates represent the values of the coefficient of discharge for an orifice 0.48 inches in diameter. This curve was plotted from data published in an article "On the Measurement of Air Flowing into the Atmosphere through Circular Orifices in Thin Plates" by Mr. R. J. Durley.†

Mr. Durley regards these data as being reliable to within less than one per cent. In view of this fact and the accuracy with which the temperature, diameter of orifice, and difference of pressure were measured, the writers believe the probable error should be within one or two per cent, in measuring the quantity of air.

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* American Machinist, Volume 28, page 193, 1905.

† Transactions Am. Soc. M. E. Vol. 27, page 193—1906.

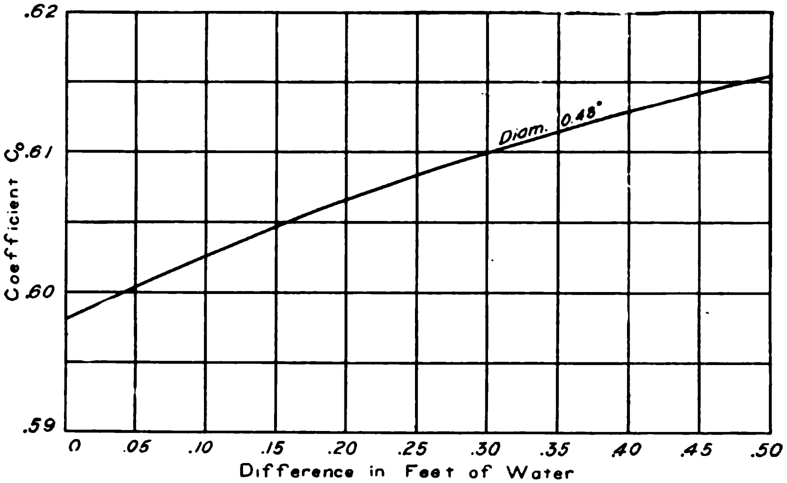

Fig. 16.—Coefficient of Discharge for Air Orifice.

The difference in pressure between the up and down stream sides of the orifice was measured in feet of water by means of the differential bottle gage shown in Figs. 14 and 15. p_u was obtained by the following computation.

(Reading of gage A in feet of mercury × 5.894)

+ (Barometer reading in inches of mercury × 0.4912)

and p_d by the following

p_u — (Bottle gage reading × .4333)

The barometer readings were taken from the autographic record charts of the U. S. Weather Bureau. No correction to these readings was made for a standard temperature, as the difference in elevation between the Weather Bureau station and the Hydraulic Laboratory offset this small correction.

T_u being observed in degrees Centigrade a constant was introduced in the formula; the diameter of the orifice being 0.04 of a foot the formula then reduced to the following one

$$q_a = \frac{p_u}{\sqrt{T_u}} \times c_o \times 3.6144 \sqrt{\left(\frac{p_d}{p_u}\right)^{1.425} - \left(\frac{p_d}{p_u}\right)^{1.712}} \qquad (21)$$

The curve shown in Fig. 17 is plotted to give values of the constant times the radical for different values of $\frac{p_d}{p_u}$ which values multiplied by proper values of the coefficient c_o and $\frac{p_u}{\sqrt{T_u}}$ give the discharge in cubic feet of free air per second. As an illustration, the computations for run 338, the data for which are given on page 63, will be used throughout the following discussion.

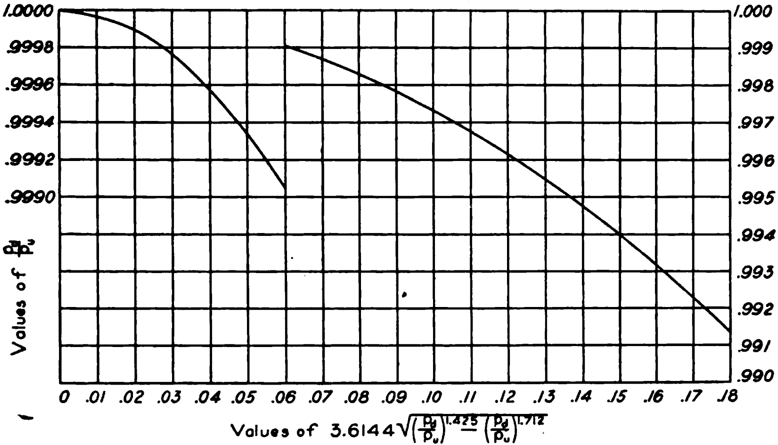

Fig. 17.—Curve for Computing Discharge of Air Through Orifice.

Average difference in pressure in feet of water = .2124

$p_u = (2.425 \times 5.894) + (28.95 \times .4912) = 28.512$

$p_d = 28.512 - (.2124 \times .4333) = 28.4200$

$T_u = 300.3$

$\sqrt{T_u} = 17.329$

$\frac{p_u}{\sqrt{T_u}} = 1.6454$

$\frac{p_d}{p_u} = 0.9967$

$3.6144\sqrt{\left(\frac{p_d}{p_u}\right)^{1.425} - \left(\frac{p_d}{p_u}\right)^{1.712}} = 0.1112$ (Curve Fig. 17).

$c_o = 0.6070$ (Curve Fig. 16)

$q_a = 1.6454 \times 0.6070 \times 0.1112 = .11108$

Theoretical Work Done by Air.—Assuming isothermal expansion of the air in its passage through the eduction pipe,—a very reasonable assumption since the air absorbs heat from the water,—the mechanical work L done between the volumes q_2 and q_1, is theoretically as follows*

$$L = \int p dq = p_1 q_1 \int_{q_1}^{q_2} \frac{dq}{q} = p_1 q_1 \log_e \frac{q_2}{q_1} \qquad (22)$$

The work done by one pound of air in expanding from an initial pressure p_1 to atmospheric pressure, (2116.8 pounds per square foot or 14.7 pounds per square inch) at 60° Fahrenheit, the standard temperature of the Moss formula, is:

$$L = p_1 q_1 \log_e \frac{q_2}{q_1}$$

$$p_1 q_1 = p_2 q_2$$

$$q_1 = \frac{p_2 q_2}{p_1}$$

$$\therefore \; L = p_2 q_2 \log_e \frac{p_1}{p_2} \qquad (23)$$

The volume of one pound of air at 32° Fahrenheit and 14.7 pounds per square inch pressure is 12.3880 cubic feet.† The volume of one pound of air at 60° Fahrenheit and 14.7 pounds per square inch pressure is, since

$$\frac{p_1 q_1}{T_1} = \frac{p_2 q_2}{T_2}$$

$$\frac{14.7 \times 12.3880}{460 + 32} = \frac{14.7 \times q_2}{460 + 60}$$

$$q_2 = 13.093$$

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* Carpenter's Experimental Engineering 6th Edition, p. 712.

† Trautwine, pages 242 and 320.

The formula for L in foot pounds now becomes

$$L = 14.7 \times 144 \times 13.093 \times \log_e \frac{p_1}{14.7 \times 144}$$

or using logarithms to the base 10,

$$L = 14.7 \times 144 \times 13.093 \times 2.30259 \times \log \frac{p_1}{14.7 \times 144}$$

$$L = 63816.886 \log \frac{p_1}{2116.8} \qquad (24)$$

in which p_1 is expressed in pounds per square foot. The curve Fig. 18 was computed by means of this formula for different values of p_1 and was plotted to read pressures in pounds per square inch. In order to get the total work done, the value of work done in expanding from p_1 to atmospheric pressure is added to the work done in expanding from atmospheric to barometric pressure, both values being obtained from the curve. In the 1908 experiments p_1 was measured by a water gage which gave the pressure in feet of water. In the 1909 experiments a mercury gage was used.

$p_1 = (1.376 \times 5.894) + (28.95 \times .4912) = 22.330$

Work done in expanding one pound of air from 22.330 pounds per square inch to 14.7 pounds per square inch = 11600 foot pounds (from curve).

Work done in expanding one pound of air from 14.7 pounds per square inch to 14.22 pounds per square inch = 900 foot pounds (from curve).

Total work = 11600 + 900 = 12500 foot pounds.

Pounds of air used per second = .11108 × .076376 = .008481.

Total work done by .008481 pounds of air in expanding from 22.330 pounds per square inch to 14.22 pounds per square inch = .008481 × 12500 = 106.02 foot pounds per second.

Quantity of Water.—Discharge of water in pounds per second $= \frac{900}{744} = 2.0135$.

Discharge of water in cubic feet per second = 2.0135 × .0160 = .03221.

Discharge of water in U. S. gallons per minute = .03221 × 7.48052 × 60 = 14.4580.

Submergence.—The gage marked water gage in Fig. 10 measured the lift in the 1908 experiments, from which the percentage of submergence was calculated by subtracting the lift from the total length of the pump and dividing the result by the total length. In the 1909 experiments the submergence was measured by the mercury gage B, Fig. 15. For runs 319–440, inclusive, the datum reading on the submergence gage was + 3.00 feet, while for runs 441–500 it was —6.67 feet. By datum reading on the submergence gage is meant the scale reading at the elevation of the air inlet of the pump. The following formula was used to calculate the percentage of submergence for the runs of the 1909 experiments.

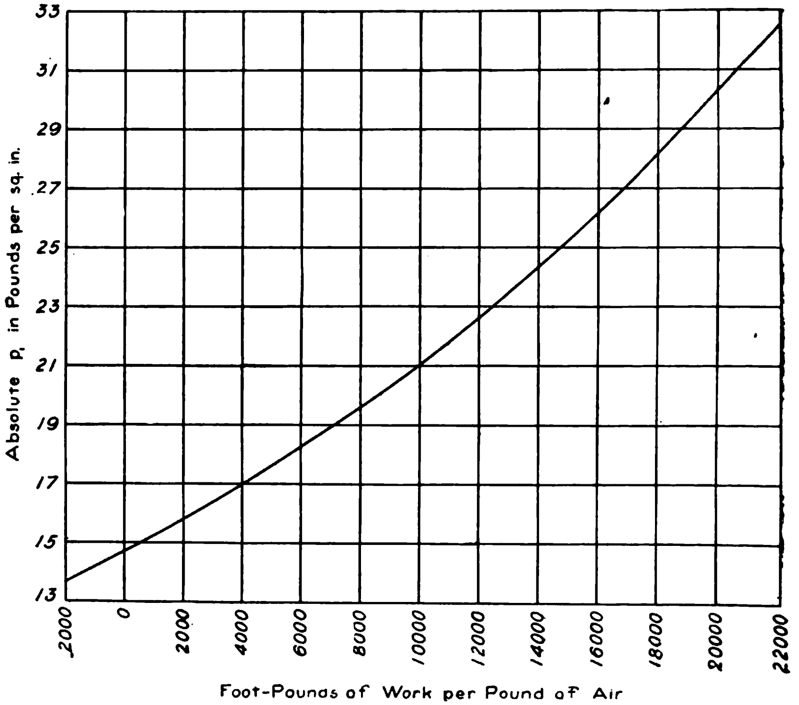

Fig. 18.—Curve for Finding Energy per Pound of Air at Various Pressures.

[(Difference in feet of mercury, gage B, × 13.6) + (Left reading of gage B) — 3.00 feet or + 6.67 feet] ÷ Total length of pump in feet.

Percentage of submergence for run 338=

$$\frac{(1.312 \times 13.6) + 1.104 - 3.00}{19.32} = 82.54$$

Lift.—Observed directly in 1908 experiments. In the 1909 experiments computed from following formula:

Lift=Total length of pump — (Percentage of submergence × total length of pump).

Lift for run 388 = 19.32 — (82.54 × 19.32) = 3.375 feet.

Actual Work Done in Lifting Water.—The product of the discharge in pounds per second and the actual lift in feet was taken as a measure of the actual work done. A few writers on the air lift pump have added the friction head in the eduction pipe to the actual lift, in computing the work done, but in the opinion of the writers the eduction pipe should be considered as a part of the pump. Actual work done in lifting water for run 338 = 2.0135 × 3.375 = 6.7950 foot pounds.

Efficiency—The efficiency of the pump was computed on the basis of the ratio of actual work done in lifting the water to the theoretical work inherent in the air, when expanding from the pressure at the gage to barometric pressure. By reference to Figs. 10 and 15 it will be seen that the gage pressure was measured within a few feet of the foot-piece in the 1908 experiments, but that in the 1909 experiments the pressure was measured at a distance of 20 to 30 feet from the foot-piece in line with the air supply pipe. The efficiency, then, as computed is independent of the efficiency of the compressor and conveying air pipes to the well, but includes a short length of air pipe such as would be necessary to conduct the air from the top of the well to the foot-piece. The loss due to friction in the air pipe and air nozzle is discussed on p. 73. Efficiency for run

$$338 = \frac{6.795}{106.026} = 6.409 \text{ per cent.}$$

Ratio of Volume of Air to Volume of Water.—Obtained by dividing quantities in column 2 of Table I by quantities in column 6. Ratio of volume of air to volume of water for run

$$338 = \frac{.1108}{.03221} = 3.449$$

Velocity of Water in a 1¼-Inch Tail-Piece.—Obtained by dividing the discharge of water in cubic feet per second by the cross sectional area in square feet of a 1¼-inch pipe. Velocity for run 338 $= \frac{.0322}{.00825} = 3.779$ feet per second.

EXPERIMENTAL COEFFICIENT

An experimental coefficient for use in connection with the Lorenz formulas has been computed from the data of the Wisconsin experiments. The method used in making this computation is explained in detail below.

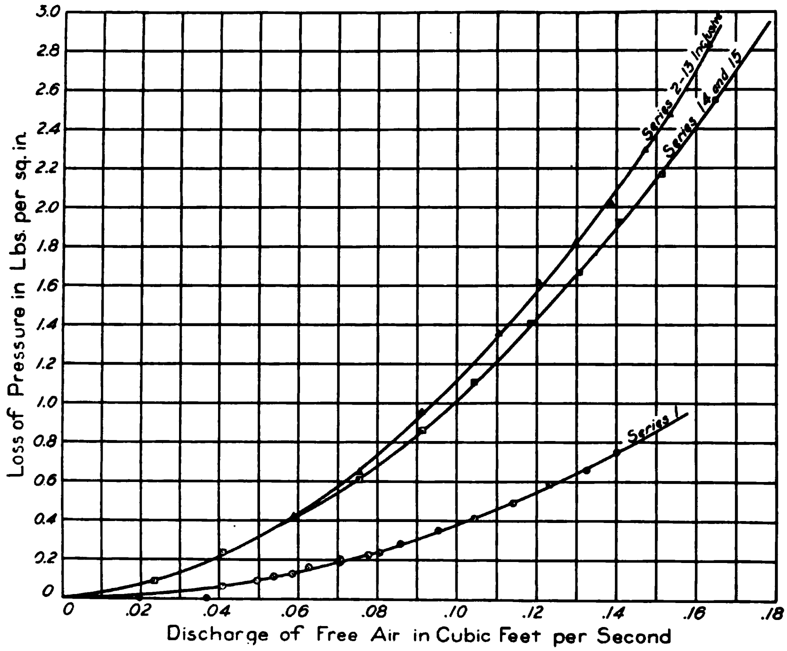

Fig. 19.—Loss Due to Friction in Air Pipe and Air Nozzle.

Loss Due to Friction in Air Pipe and Nozzle.—A considerable loss of energy was caused by the drop in pressure between the point in the air main, where the pressure was measured, and a point in the foot-piece opposite the air inlet. The amount of this drop in pressure was determined by discharging air at various rates through the pump, when all water was drained out of it. The indications of the gage gave the head required to cause the discharge from the air nozzle, and to overcome the frictional resistances in the air pipe with its elbows and other fittings, when discharging into the atmosphere. The curves in Fig. 19 show this loss of pressure for various discharges for three different lengths of air pipe with the Harris foot-piece. The lower curve, for series 1, was plotted from the data of Table 2 and shows the loss under the conditions of the apparatus shown in Fig. 10. The upper curve, for series 2–13, was plotted from the data of Table 3. For some of these runs the length of eduction pipe was 19.32 feet, while during the others it was 26.74 feet, but the length of air pipe was constant. The middle curve, for series 14 and 15, was plotted from the data of Table 4. During these runs the length of eduction pipe was 41.50 feet, but the air pipe was shorter than during series 2–13. The length of air pipe was not measured, but it may be seen from the lower curve that the loss due to the foot-piece was considerable, as in the conditions of the tests represented by this curve, there was only one elbow and less than a foot of air pipe between the gage and the nozzle.

To compute the loss of pressure under operating conditions; that is, when discharging against the pressure p_1 in the foot piece, it was assumed that fluid friction varies very nearly with the square of the velocity and directly with the density. If we let

p_a = the loss of pressure, in pounds per square inch, when discharging into the atmosphere,

p_x = the loss of pressure, in pounds per square inch, when discharging against the pressure p_1 in the foot-piece,

v_a = the velocity of air in the pipe, in feet per second, when discharging into the atmosphere,

v_x = the velocity of air in pipe, in feet per second, when discharging against the pressure p_1,

q_a = the volume of free air discharged, in cubic feet per second,

q_x = the volume of air at pressure p_1, in cubic feet per second,

r = the ratio of compression in atmospheres,

the following proportion may be written

$$p_a : v_a^2 r_a :: p_x : v_x^2 r_x$$

Taking r_a as 1, when discharging into the atmosphere, we have

$$p_x = \frac{p_a v_x^2 r_x}{v_a^2} = \frac{p_a q_x^2 r_x}{q_a^2} = \frac{p_a \left(\frac{q_a}{r_x}\right)^2 r_x}{q_a^2} = \frac{p_a}{r_x}$$

Hence it was only necessary to divide the loss as taken from the curves in Fig. 19 by the ratio of compression, in order to find the loss of pressure when discharging against any pressure p_t existing in the foot-piece.

Loss of Head Due to Entrance.—The entrance to the tail-piece was square edged during the first 318 runs and the loss of head due to entrance was, therefore, assumed to have been equal to one-half of the velocity head of the entering water. The maximum value of the velocity head during this series of runs barely amounted to one per cent. of the total head required to discharge the water. During the 1909 experiments the tail-piece projected into the water and the entrance head has been assumed as equal to the velocity head of the entering water for these runs. The entrance head is therefore expresssd as

$$h_e = \frac{1}{2}\frac{v_t^2}{2g} \quad \text{for flush entrance, and} \qquad (25)$$

$$h_e = \frac{v_t^2}{2g} \quad \text{for inward projecting pipe} \qquad (26)$$

Loss of Head Due to Velocity of Discharge.—The velocity of discharge is much greater than the entrance velocity, due to the fact that the combined volumes of the water and air must be discharged through the outlet end of the pipe, while only the water enters the lower end. The loss due to the velocity of discharge is

$$\frac{v_b^2}{2g} = \frac{\left(\frac{q_w + q_a}{a_p}\right)^2}{2g} \qquad (27)$$

in which v_b = the outlet velocity, in feet per second.

q_w = the discharge of water, in cubic feet per second.

q_a = the discharge of free air, in cubic feet per second.

a_p = area of cross section of eduction pipe, in square feet.

Loss of Head Due to Elbow.—From experiments made in the Hydraulic Laboratory of the University of Wisconsin it was determined that the loss of head due to a 2-inch cast iron elbow was equal to*

$$H = 0.0202\ v^2$$

or

$$H = 1.3\ \frac{v^2}{2g}$$

and from comparison of these experiments with others on different sized pipes, it was concluded that the loss of head was independent of the size of pipe.† In estimating the loss due to the 1¼-inch cast iron elbow in the air lift experiments it has, therefore, been computed as 1.3 the velocity head of the discharge water, or

$$h_o = 1.3\frac{v^2_{\ 0}}{2g} \qquad (28)$$

The loss of head caused by an elbow or bend is largely due to the eddies in the straight pipe following the elbow. The elbow at the top of the air lift pump used in the Wisconsin experiments was followed by only a few inches of straight pipe, so the above formula probably somewhat over estimates the elbow loss.

Loss of Head Due to Pipe Friction and Slip.—The loss of head in the straight part of the eduction pipe varies considerably with the amount of air used, due to the difference in the amount of slip of the water past the air at different velocities of discharge, but it was not possible to estimate the loss due to slip separately from the pipe friction. It was observed in the glass eduction pipe that in addition to the uniform gradual slip of the water past the air bubbles, there were at times sudden disturbances caused by a considerable quantity of water slipping down past a relatively large quantity of air. Loss is occasioned by the necessity of again lifting the water which has slipped down, and also by the internal friction due to the agitation of the water caused by the slip and by the pipe friction.

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* See discussion on curve resistance in water pipes, by Geo. Jacob Davis, Jr. in Trans. Am. Soc. C. E., Vol. LXII, p. 112. (1909).

† Ibid p. 109, effect of pipe diameter.

If h_a = the head produced by the air used per pound of water pumped,

h_p = the combined loss of head due to pipe friction and slip,

v_c = the velocity of the water in the well outside the eduction pipe at the elevation C in Fig. 3 p. 25.

and using the symbols already established for the the other quantities involved, the equation of energy per pound of water pumped, between the outlet and a point in the well outside the eduction pipe at the elevation C, may be written as follows:

$$\frac{v_c^2}{2g} + \frac{p_c}{u_w} + h_c + h_a = \frac{v_b^2}{2g} + \frac{p_b}{u_w} + h_b + h_e + h_o + h_p \qquad (29)$$

In computing the losses the velocity of approach has been neglected and the velocity v_c of the water in the well is taken as 0. To illustrate the method of computing the friction factor, the following data of run No. 338 will be used.

Length of eduction pipe = 19.32 feet.

Depth of submergence = 1.312 × 13.6 + 1.104 — 3.00 = 15.95 feet.

Diameter of tail-piece = 1½ inches, ∴ area = 0.01227 square feet.

Diameter of eduction pipe = 1¼ inches, area = 0.00852 square feet.

Length of pipe = 19.50 feet, including a short nipple at discharge end of eduction pipe, $\therefore \frac{1\ell}{d} = 187$ $= \frac{19.5 \times 12}{1.25}$!

Discharge of water = 0.0322 cubic feet per second, or 2.01 pounds per second.

Quantity of air used = 0.1111 cubic feet per second = 0.1111 ÷ 13.093 = 0.008481 pounds per second.

Total discharge, air and water = 0.1433 cubic feet per second.

Entrance velocity = v_t = 0.0322 ÷ 0.01227 = 2.62 feet per second.

Entrance velocity head = $\frac{v_t^2}{2g}$ = 0.106 feet.

Entrance loss of head = $\frac{v_t^2}{2g}$ = 0.106 feet.

Outlet velocity $= v_b = 0.1433 \div 0.00852 = 16.8$ feet per second.

Outlet velocity head $= \frac{v_b^2}{2g} = \frac{(16.8)^2}{64.32} = 4.389$ feet.

Elbow loss of head $= 1.3 \times 4.389 = 5.706$ feet.

Barometric pressure $= p_b = 28.95 \times 0.4912 = 14.22$ pounds per square inch.

Pressure in air pipe $= p_g = 1.376 \times 5.894 + 28.95 \times 0.4912 = 22.33$ pounds per square inch.

Pressure in foot-piece $= p_f = 22.33$ pounds per square inch minus loss in air pipe and nozzle. Loss when discharging into the atmosphere $= 1.36$ pounds per square inch (from curve Fig. 19). Loss when discharging against the pressure $p_f = 1.36 \div \frac{22.33}{14.7} = .897$ pounds per square inch. $\therefore p_f = 22.33 - .897 = 21.433$ pounds per square inch.

Energy per pound of air used $= 11{,}350$ foot pounds. (Curve Fig. 18.)

Weight of air used $= 0.008481$ pounds.

Energy of air used per second $= 11350 \times 0.008481 = 96.26$ foot pounds per second.

Energy of air per pound of water pumped $= h_a = 96.26 \div 2.01 = 47.89$ foot pounds.

Head due to atmospheric pressure $= 14.22 \times 2.308 = 32.82$ feet.

The pressure head $\frac{p_c}{u_w}$ was equal to the submergence plus the head due to atmospheric pressure. The datum plane was taken at C in Fig. 3 making $h_c = 0$. The values found above may be substituted in the energy equation as follows:

$$0 + (15.95 + 32.82) + 0 + 47.89 = 4.389 + 32.82 + 19.32 + 0.106 + 5.706 + h_p.$$

$$h_p = 34.32 \text{ feet.}$$

$$c_p = \frac{h_p}{\frac{l}{d} \times \frac{v_b^2}{2g}}$$

$$c_p = \frac{34.32}{187 \times 4.389} = .04181$$

In the manner above described the values of the friction factor c_p have been computed for most of the runs of series 1 and for a few runs in some of the other series. The results are given in column 14 of Table 1. These values of the friction factor have been plotted as ordinates against the velocities, designated as v_1, of the water in a 1¼-inch tail piece as abscissas. The velocities in a tail-piece of the same size as the eduction pipe were used instead of the actual velocities in the 1½-inch tail-piece, so as to make it possible to compare the values of c_p with the friction factors for unmixed water flowing in a pipe of the same size as the eduction pipe. In Fig. 20 the numbers adjacent to the points indicate the ratio of volume of free air used to volume of water pumped for each run. Curves have been sketched in to indicate the relation between c_p and v_1 for given ratios of air to water. Before the regular runs were begun on the air lift, a number of runs were made with no air, for the purpose of determining the normal pipe friction in the eduction pipe, which, it will be remembered, was largely made of glass. The friction factors computed from these preliminary runs are also plotted on Fig. 20. It may be seen in this figure that these normal friction values agree very closely with the values computed from Fanning's table of friction factors for clean iron pipe. It may also be seen that when pumping with air the friction factors are very much larger than when only water is flowing through the pipe, but the value of c_p does not increase with the ratio of air to water but instead it rapidly decreases as the ratio increases from small values, apparently reaching a minimum value when an excessively large amount of air is used. This may be due to the fact that when very small amounts of air are used, the air bubbles rise through the water without causing much discharge, the energy of the air being mostly used up in causing eddies in the water in the pipe.

The value of the friction factor, computed as described above, seems to be entirely independent of the percentage of submergence and of the lift. The values, however, decrease with the length of pump, which may be seen by reference to Fig. 20, the points for the 26.74-foot length falling a small distance below the curves, while the points for the 41.50-foot length fall a relatively larger distance below the curves. The curves shown in Fig. 20 are therefore of no practical importance, and it would seem that for a theoretical design of an air lift pump, the law of variation of c_p with both the diameter and length of pump would have to be known.

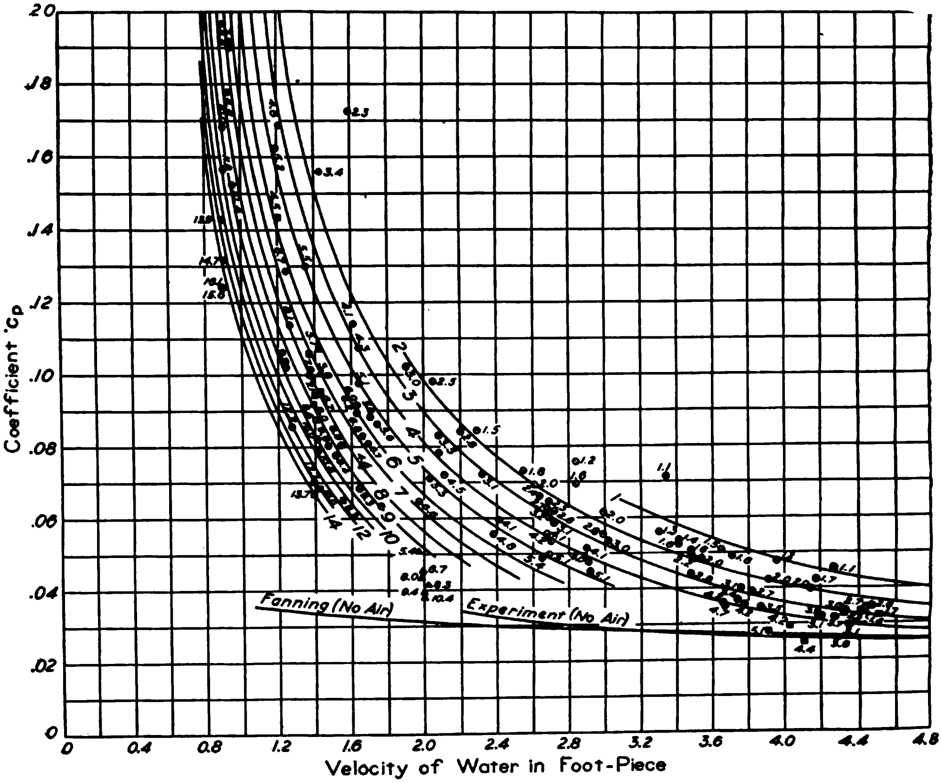


Fig. 20.—Coefficient of Pipe Friction and Slip.

○ = Length of Pump 19.32 Feet
□ = " " " 26.74 "
△ = " " " 41.50 "

EXPERIMENTAL RELATIONS

The experiments made by the writers were planned to determine, first, the effect on the discharge and efficiency of variations in the working conditions of the air lift pump, such as variations in submergence, lift and quantity of air used, and second, the effect on the discharge and efficiency of changes in the structural features of the pump. The first part of the following discussion relates to experiments in which all structural features were the same.

RELATION OF DISCHARGE TO AIR USED

The relation between the quantity of air used and the quantity of water pumped is shown in Fig. 21 for each of the runs of the first series. The small figures, by the plotted points, indicate the percentage of submergence obtaining during the run represented by the point. Curves have been interpolated between the points to indicate the discharge of the water for varying quantities of air under constant conditions of submergence and lift, for each five per cent. submergence from 35 to 115 per cent. These curves were first sketched in freely, interpolating by eye to locate their positions, and were afterwards adjusted to harmonize with the curves in Fig. 22. Each of these curves was plotted by reading horizontally across Fig. 21 the percentage of submergence corresponding to given quantities of air for a constant discharge of water. The trial curves in Fig. 21 were then shifted slightly until the points picked from them fell on smooth curves in Fig. 22.

Fig. 21 shows that for any given percentage of submergence an increase in the amount of free air used, starting with zero quantity, causes an increase in the discharge of water up to a given quantity of air, beyond which the discharge of water decreases with a further increase of air. The quantity of air giving a maximum discharge of water is not a constant for the different percentages of submergence, nor is the ratio of the volumes of air and water constant for the maximum discharge

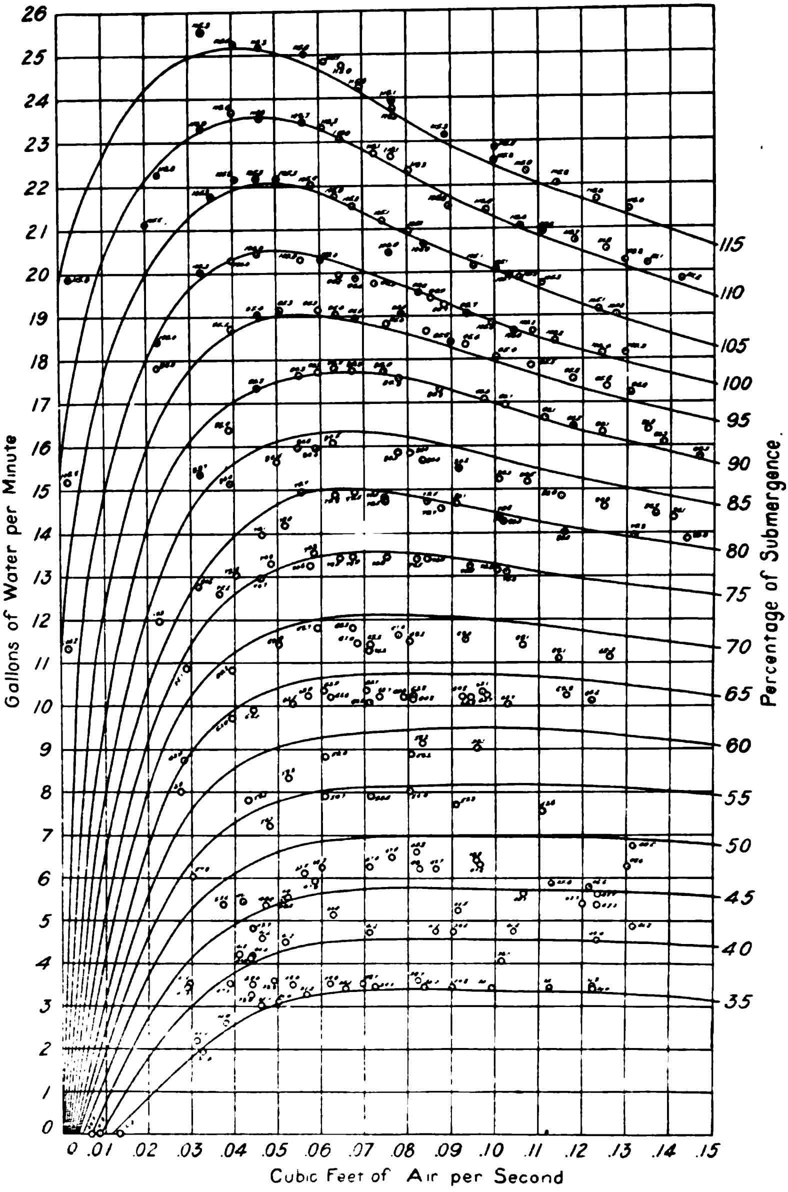


Fig. 21.—Relation of Discharge to Air Used.—Series 1.

at different percentages of submergence. The quantity of air giving the maximum discharge is less, both absolutely and relatively, for high percentages of submergence than it is for low percentages of submergence.

From the curves of Figs. 21 and 22, it appears that for any given rate of air consumption, the discharge of water increases indefinitely as the percentage of submergence increases In this connection these curves are likely to be misleading unless the conditions of the experiments are borne in mind. All of the experiments represented by the points in Fig. 21 were made with a constant length of pump of 19.32 feet. Therefore, an increase in the submergence was necessarily accompanied by a decrease in the lift, the latter becoming zero at 100 per cent., and negative at higher percentages of submergence. That is, at percentages greater than 100 per cent. there was a head on the outlet of the pump which caused a flow without the use of any air, as in the case of artesian wells. In commercial plants using the air lift system of raising water, the pump is of fixed length designed and built to work at some definite percentage of submergence. In many cases the water level in the well is not correctly estimated for working conditions, or the conditions existing at the time of installing the well may be changed by the operation of other wells in relatively close proximity, so that it is often found that the relation of lift to submergence in the case of many pumps is far from what it should be. The large variations in the discharge which may result from such changes in the percentage of submergence are clearly indicated in Figs. 21 and 22.

RELATION OF OUTPUT TO PERCENTAGE OF SUBMERGENCE

As to the question of what is the best percentage of submergence to adopt in the design of a pump to lift the water a definite height, no information is given in Fig. 21, for the reason that it is impossible to determine from this diagram whether the increased discharge of water is due to the increase in the percentage of submergence or to the accompanying decrease in the lift.

In order to throw more light on this subject Figs. 23, 24 and

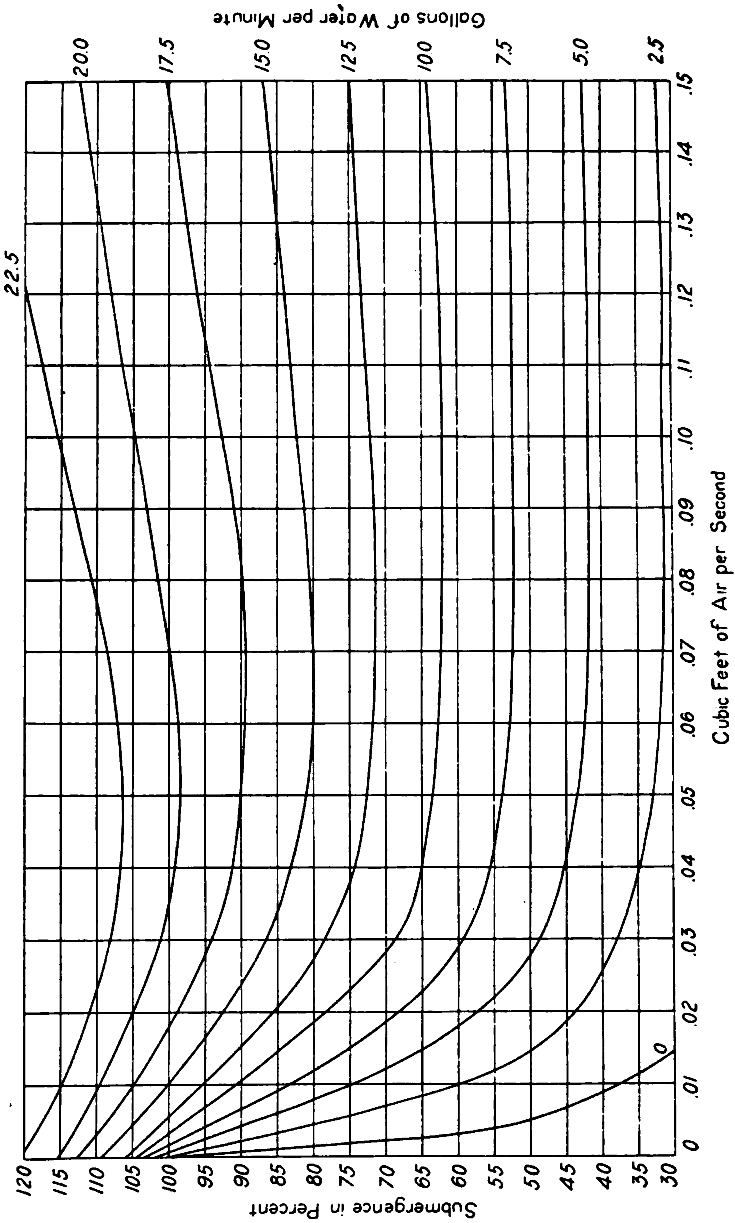


Fig. 22.—Relation of Air Used to Percentage of Submergence.—Series 1.

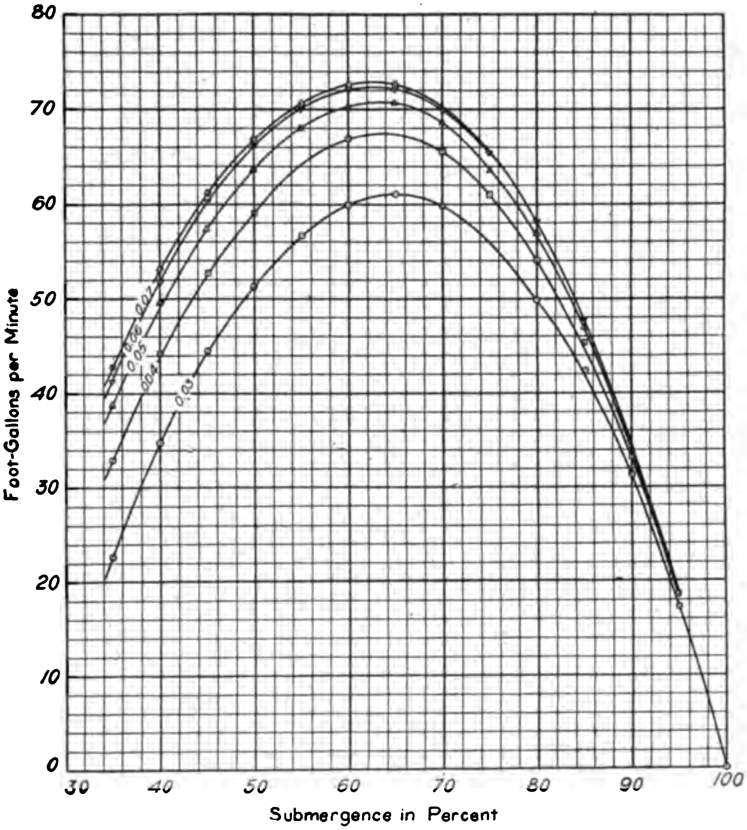


Fig. 23.—Relation of Output to Percentage of Submergence.—Series 1.

25 have been plotted. The method of constructing these curves is as follows: Choosing some definite rate of air consumption, as for example 0.03 second feet, and reading vertically along this air line in Fig. 21, the discharges were obtained for each five per cent. submergence. These quantities were then multiplied by the lift in feet for the corresponding submergences. This gave the work output, expressed in foot gallons, for each five per cent. submergence. In order to avoid confusion, part of the curves, those for the rates of air from 0.03 to 0.07 second feet have been plotted in Fig. 23 and the remainder in Fig 24. The curves are shown only as far as 100 per cent. submergence.

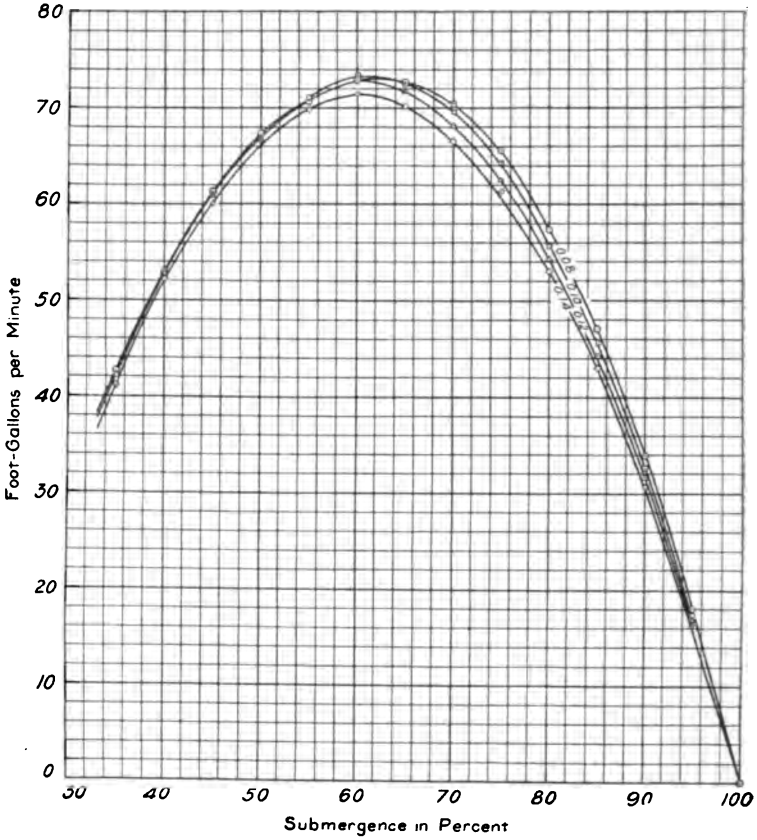


Fig. 24.—Relation of Output to Percentage of Submergence.—Series 1.

Beyond that limit the output is negative on account of the lift being negative, and the curve extends in the general direction indicated by the part near the 100 per cent. submergence point. In-as-much as the discharge increases from zero, while the lift decreases from a maximum value to zero as the percentage of submergence increases, there is necessarily a maximum point on the curve. The maximum points for all the curves lie between 60 and 65 per cent. submergence. The percentage of submergence giving the maximum output is shown for various rates of air consumption by the curve in Fig. 25. In this figure is shown also the relation between air consumption and maximum output.

By inspection of these two curves it may be seen that the maximum output occurred when the rate of air consumption was about 0.09 second feet and the percentage of submergence was about 61. Since the lift corresponding to 61 per cent. submergence was 7.55 feet, the maximum output of 73.2 foot gallons represented a discharge of 0.0215 second feet of water.

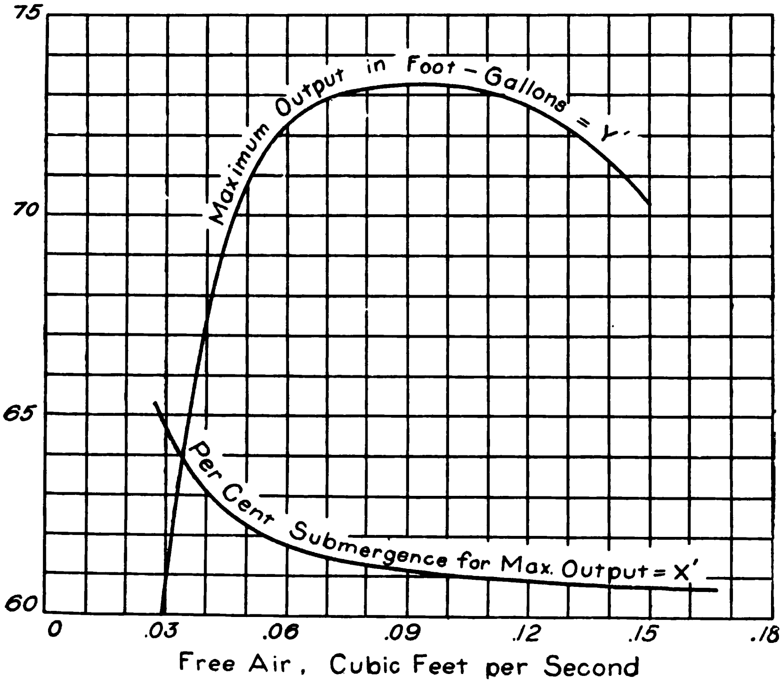


Fig. 25.—Relation of Quantity of Air Used to Maximum Output and to Percentage of Submergence.—Series 1.

Hence, for maximum output the ratio of volumes of air to water was 0.09 ÷ 0.0215 = 4.2. The curves are based on experiments on a single length of pump of 19.32 feet. Some experiments on pumps of different lengths show that for the same air consumption and percentage of submergence the output increases with the length of pump, therefore the outputs in foot gallons shown on Figs. 23, 24 and 25 must be regarded as applying only to the 19.32-foot pump. The experiments on the longer pumps were taken at only two submergences, of approximately 40 and 80 per cent., so curves similar to those of Figs. 23 and 24 could not be drawn for the longer pumps; but the points for these runs plot in such a way as to indicate that the maximum point must be about the same percentage of submergence as for the shorter pump.

The curves of Figs. 23 and 24 seem to be parabolas having the equation

$$(y - y') = m \ (x - x')^n \qquad (30)$$

in which

$y =$ the output in foot gallons.
$y' =$ the maximum output in foot gallons.
$x =$ submergence in per cent.
$x' =$ submergence in per cent. corresponding to maximum output.

By taking the values of x' and y' for any given value of air from Fig. 25, and subtracting them from the values of x and y taken from the curves of Fig. 23 for the same value of air, the values plotted in Fig. 26 were obtained. The symbols with horizontal bars attached represent points of the curve on one side of the vertex while the plain symbols represent points on the other side of the vertex on the same curve. The points near the lower end of the diagram scatter somewhat. These points correspond to points very near the vertex of the parabolas and the divergence from the mean line simply indicates a very small error in the values of x' and y', which were found by trial. The substantial agreement of the points with the line shows that for the 19.32-foot pump, the output varies in accordance with the parabolic law and that for all rates of air consumption the values of m and n are the same. Using the values of m and n determined from the mean line of Fig. 26 the equation of the curves becomes

$$y - y' = 0.028 \, (x - x')^{2.15} \qquad (31$$

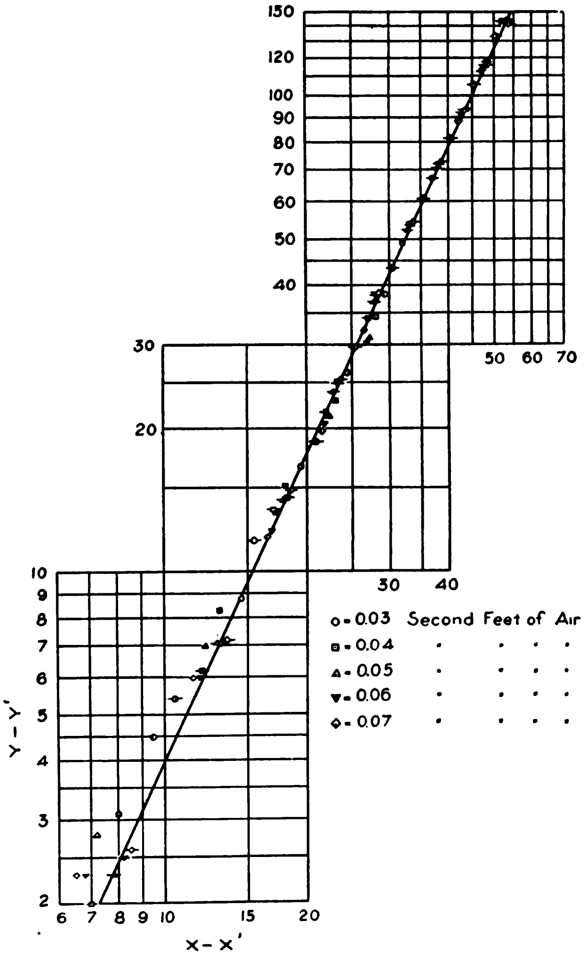


Fig. 26.—Logarithmic Plotting of Output—Percentage of Submergence Curves.—Series 1.

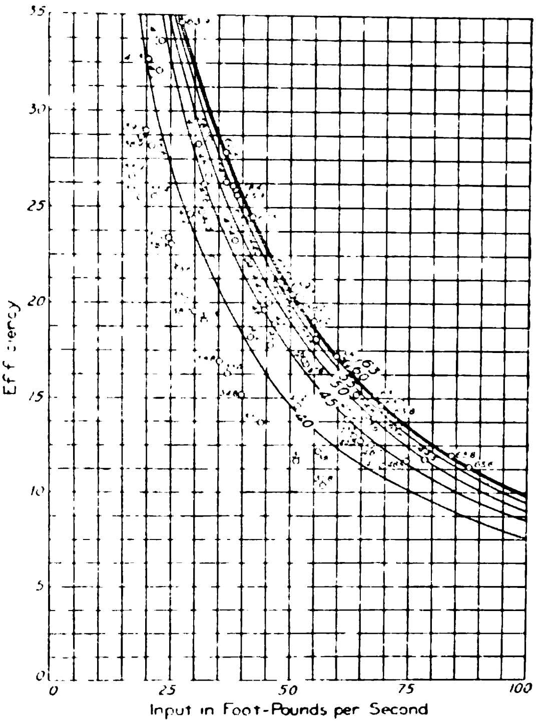


Fig. 27.—Relation of Efficiency to Input with Constant Length of Pump. Runs (1-86) Series 1.

RELATION OF EFFICIENCY TO INPUT AND PERCENTAGE OF SUBMERGENCE WITH CONSTANT LENGTH OF PUMP

If the quantity of free air used were a measure of the work input then the curves of Figs. 23, 24 and 25 would show the submergence to adopt for maximum efficiency at any given rate of air consumption, but such is not the case. An increase in the percentage of submergence, in the case of a pump of fixed length, results in a greater depth of submergence of the air inlet and consequently in an increased air pressure. With a constant consumption of free air the work input varies with the submergence. In order to show the effect of variations in the submergence and input on the efficiency of operation the curves in Figs. 27, 28 and 29 have been drawn.

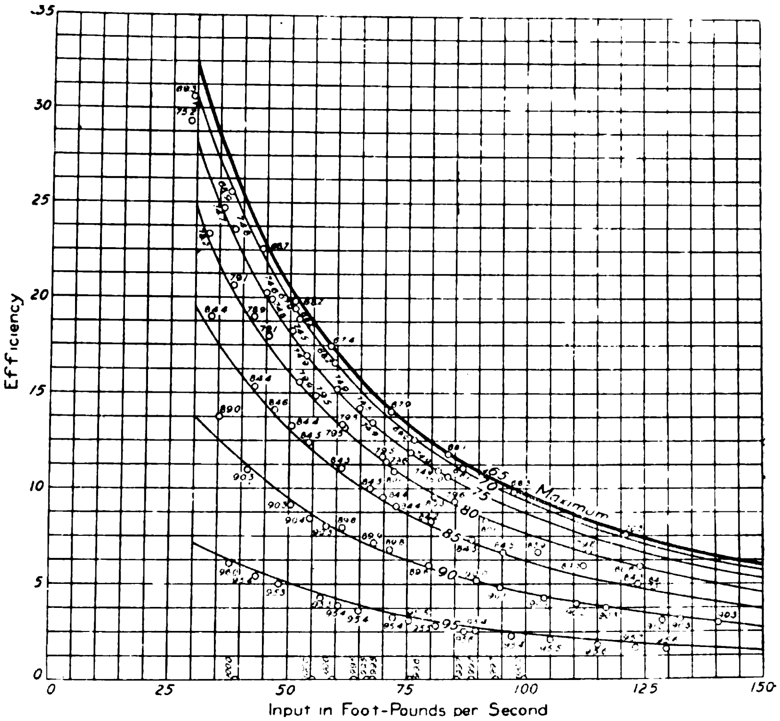


Fig. 28.—Relation of Efficiency to Input with Constant Length of Pump. Runs (87–202) Series 1.

Efficiency and Input.—The efficiencies computed from runs (1—202) of the first series of experiments have been plotted as ordinates against the corresponding inputs in Figs. 27 and 28. In order to avoid confusion of lines the points with percentages of submergence lower than 63 were plotted in Fig. 27 and those above were plotted in Fig. 28. These curves show that for any given constant percentage of submergence and constant length of pump, the efficiency increases very rapidly as the input decreases. Efficiencies higher than those shown on the diagrams have only slight practical importance on account of the small discharge of water obtainable when working at such low rates of input, and the consequently high initial cost of the well in proportion to the discharge.

Efficiency and Percentage of Submergence.—The curves in Figs. 27 and 28 also show that, if the total length of eduction pipe remains constant, the maximum efficiency for a given input or rate of pumping is obtained at about 63 per cent. submergence. This is more clearly shown in Fig. 29, which was drawn from values taken from the average curves of Figs. 27 and 28. It will be seen from these diagrams that with a given input,

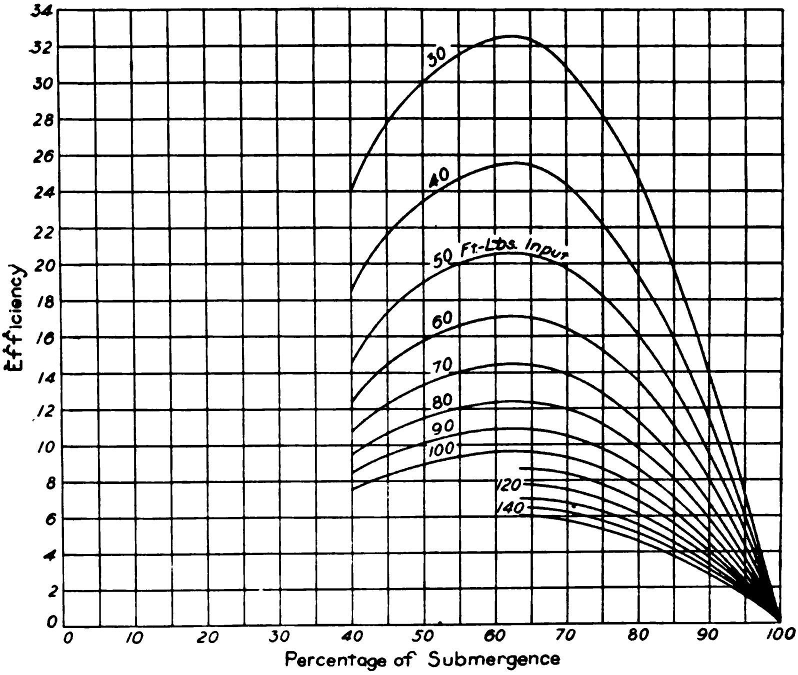


Fig. 29.—Relation of Efficiency to Percentage of Submergence with Constant Length of Pump.—Series 1.

the efficiency first increases with the percentage of submergence until a maximum is reached at about 63 per cent., and then decreases with a further increase in the percentage of submergence; becoming zero at 100 per cent. submergence and the efficiencies having negative values when the percentage of submergence is greater than 100, because for such cases the lift has a negative value. The efficiency curves are not drawn for submergences greater than 100 per cent. The total length of eduction pipe, however, remaining constant, the lift varies as the percentage of submergence is changed.

RELATION OF EFFICIENCY TO INPUT AND PERCENTAGE OF SUBMERGENCE WITH A CONSTANT LIFT

In comparing the results of the Wisconsin experiments there are five variables to be considered; namely, the lift, the submergence, the discharge, and the quantity and pressure of the air used. In the previous discussion the effect of varying some of these factors has been shown, but on account of the fact that the lift varied inversely as the submergence, the methods of comparison used on the preceding pages do not give definite information as to the best submergence to use with a constant lift. The practical problem in most deep well propositions is to determine at what percentage of submergence the maximum efficiency will be obtained for a given lift. The curves of

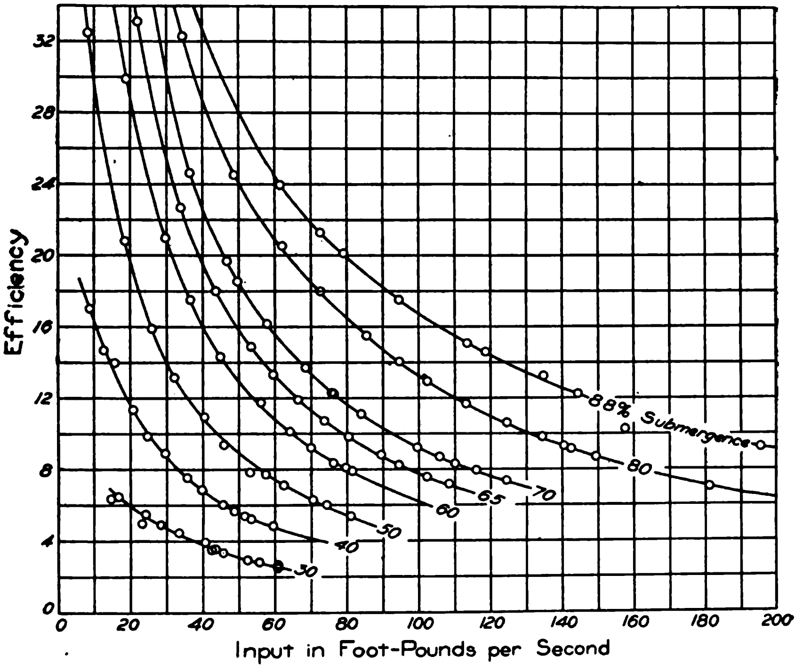
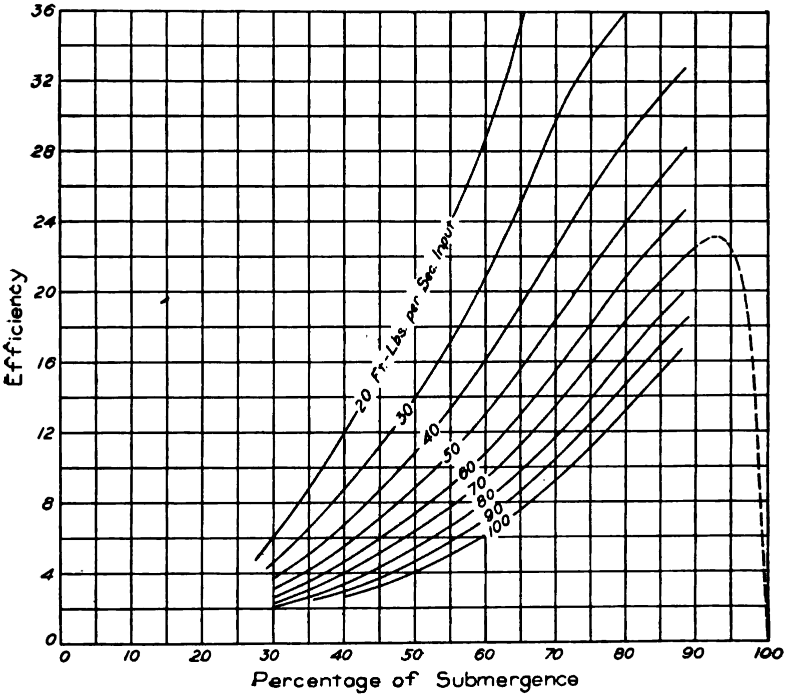


Fig. 30.—Relation of Efficiency to Input with a Constant Lift.—Series 20.

Figs. 30 and 31, plotted from the data of series 20, show this information. In this series the lift was kept constant at five feet and the percentage of submergence was varied by changing the length of pump.

Efficiency and Input.—Fig. 30 shows that for any given constant percentage of submergence and constant lift, the efficiency increases as the input decreases, which checks the conclusion drawn from the curves of Figs. 27 and 28.



Fif. 31.—Relation of Efficiency to Percentage of Submergence with a Constant Lift.—Series 20.

Efficiency and Percentage of Submergence.—Fig. 30 also shows that with a constant lift, the efficiency increases as the percentage of submergence increases, for all rates of input and all practical percentages of submergence. This result is quite different from that obtained when the length of pump remained constant and lift varied, as may be seen by comparing the curves of this figure with those of Figs. 27 and 28. The above conclusions may perhaps be more plainly seen by reference to Fig. 31, which was plotted for the purpose of comparison with the curves of Fig. 29. The curves of Fig. 31 show a tendency to reverse after passing the ordinate at 90 per cent. submergence. This may be understood if one reflects that between 90 and 100 per cent. the length of pump increases very rapidly with any constant lift, reaching infinity at a submergence of 100 per cent. Hence, the efficiency will decrease very rapidly as the percentage of submergence is increased from a point near 90 per cent. submergence to a submergence of 100 per cent., as indicated by the broken curve for an input of 70 foot pounds.

Relation of Discharge and Efficiency to Lift

In discussions of the air lift the statement is frequently made that an increase in the lift causes a reduction in the discharge and in the efficiency, other conditions remaining constant. That this statement is not true in all cases is proved by the curves shown in Fig. 32, in which are plotted the results of three series

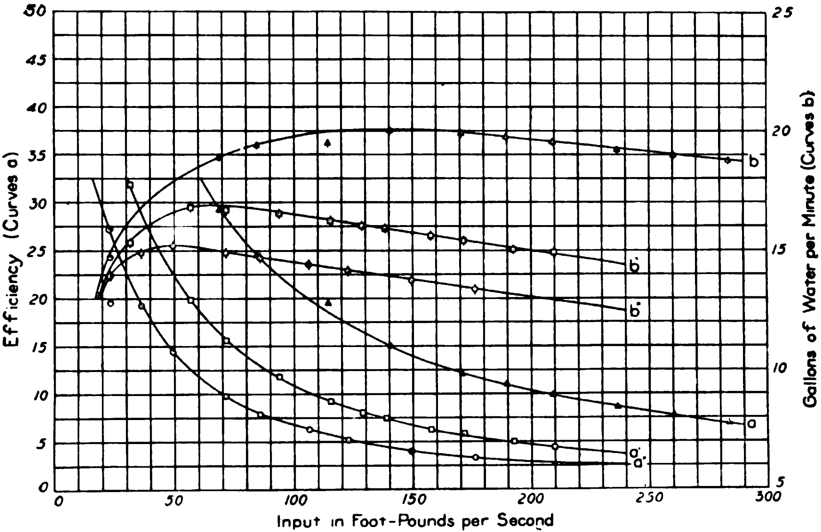


Fig. 32.—Relation of Discharge and Efficiency to Lift.

○ = Series 4—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 19.32 Feet—Average Lift 3.29 Feet—Average Submergence 82.97 Per Cent.

□ = Series 11—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 26.74 Feet—Average Lift 4.74 Feet—Average Submergence 82.40 Per Cent.

△ = Series 14—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 41.50 Feet—Average Lift 7.66 Feet—Average Submergence 81.54 Per Cent.

of experiments made in order to show the effect of varying the lift, other conditions remaining the same. The same foot-piece and size of eduction pipe were used in all these series, the submergence was kept nearly constant, at about 82 per cent., and the lift was varied by adding extra pipe to the discharge end of the eduction pipe. This diagram shows, that for a given input, the discharge and hence the efficiency, increases with the lift and the length of eduction pipe; the percentage of submergence and all other conditions, except the length of the pipe, remaining constant.

A similar result was obtained at a smaller percentage of submergence, as is shown in Fig. 33. In this diagram the discharge curve for a lift of 15.73 feet lies below that for a lift of 10.88 feet, although for a given input the efficiency is greater for the larger lift. This apparent disagreement with the law seeming to hold for the other series of runs plotted, was doubt-

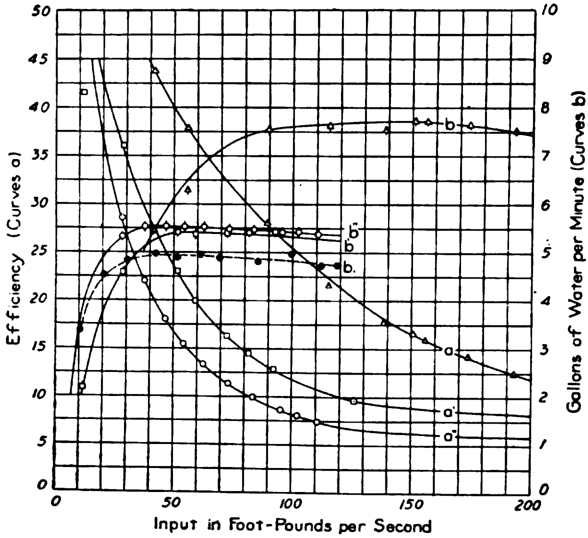


Fig. 33.—Relation of Discharge and Efficiency to Lift.

⊙ = Series 7—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 19.32 Feet—Average Lift 10.88 Feet—Average Submergence 43.66 Per Cent.

▣ = Series 10—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 26.74 Feet—Average Lift 15.73 Feet—Average Submergence 41.19 Per Cent.

△ = Series 15—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 41.50 Feet—Average Lift 23.47 Feet—Average Submergence 43.45 Per Cent.

● = Series 6—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 19.32 Feet—Average Lift 11.40 Feet—Average Submergence 40.97 Per Cent.

less due to the fact that the percentage of submergence was different in the two series of runs. It was difficult in setting up the apparatus and adjusting the valves to get exactly the same percentage of submergence in the different series, or even to maintain the percentage constant for all the runs of a series. To show the difference in the discharge due to the variation in the percentage of submergence the discharges for series 6 have been plotted in Fig. 33 and a dash line has been drawn through them. The average percentage of submergence for these runs was 40.97 and the lift 11.40 feet. By comparing this curve with that for series 10, which was made with practically the same percentage of submergence, it may be seen that the discharge with the higher lift of 15.73 feet was considerably greater than the discharge with the lift of 11.40 feet. The discharge curve for the 15.73-foot lift would therefore have plotted higher, between the curves for the 10.88-foot and the 23.47-foot lifts, if the conditions of submergence had been the same in the three series of runs.

The reason for an increase in the discharge occurring when the length of pump is increased may be understood by studying the theoretical relations of the various quantities involved, as expressed in equation (17) of the Lorenz theory (see p. 28). By transposing this equation and restoring u_b which had been cancelled out from equation (12) it may be made to read

$$q_w = \frac{\left(2.303 \frac{p_b}{u_b} \log_{10} \frac{p_l}{p_b}\right) q_b u_b \frac{2g a_p^2}{u_w}}{2gh_l a_p^2 + \left(1 + c_p \frac{l}{d}\right)\left(q_b + q_w\right)^2 + c_e q_w^2} \tag{32}$$

To adapt this formula to the conditions of the Wisconsin experiments, 1.3 must be added to the quantities in the first parenthesis of the denominator to take account of the loss due to the elbow at the top of the eduction pipe. With this change, equation (32) becomes

$$q_w = \frac{\left(2.303 \frac{p_b}{u_b} \log_{10} \frac{p_l}{p_b}\right) q_b u_b \frac{2g a_p^2}{u_w}}{2gh_l a_p^2 + \left(2.3 + c_p \frac{l}{d}\right)\left(q_b + q_w\right)^2 + c_e q^2} \tag{33}$$

Curves showing the relation of the discharge, q_w, to the lift,

h_1, have been computed by means of this formula for three different percentages of submergence, and are plotted in Fig. 34. The conditions assumed were as follows:

Area of eduction pipe = 0.00852 square feet.

Percentages of submergence = 40, 60, and 80 per cent.

Barometric pressure = 14.3 pounds per square inch. (The approximate average pressure during series 1.)

Input at piezometer on air pipe = 100 foot pounds.

In the numerator of equation (33) the expression

$$q_b\, u_b \left(2.303\, \frac{p_b}{u_b} \log_{10} \frac{p_1}{p_b}\right)$$

represents the input at the air inlet in the foot-piece. In the 1909 experiments the pressure, from which the input was computed, was measured at a point on the air pipe some distance from the foot-piece. The pressure, so measured, therefore, includes the loss in a section of air pipe of length sufficient to reach from the top of the well to the foot-piece. This loss should be charged to the pump, for such a length of air pipe is inevitable. Hence, in computing the quantity of air used at any given rate of pumping and at any lift, the gage pressure, p_g, should be used instead of the pressure, p_1, in the foot-piece.

Putting $q_b\, u_b \left(2.303\, \frac{p_b}{u_b} \log_{10} \frac{p_g}{p_b}\right) = 100$, the assumed input, the quantity of air used per second was computed for the gage pressures corresponding to the various lifts and percentages of submergence. The gage pressure, p_g, was computed, for the assumed conditions, by adding 14.3 pounds per square inch, to 0.434 times the depth of submergence in feet, plus the loss of head due to friction in the air pipe and nozzle, minus the loss due to entrance and velocity in the foot-piece. Since the loss of head due to friction in the air-pipe and nozzle is a function of the volume of air used, it was necessary to first assume the volume of air in order to estimate the approximate loss of head due to air friction from the curve of Fig. 19 as explained on page 75. By making a few approximations in this way, the proper value of p_g and the corresponding value of q_b were found. An assumed value of the discharge q_w was used to compute the ratio of volume of free air to volume of water and the ratio of discharge to area of eduction pipe which were used as arguments in estimating the friction factor c_p from Fig. 20. By making a few such approximations, corresponding values of c_p, q_a, and q_w were found which satisfied equation (33).

With constant percentage of submergence, an increase in the lift causes a proportionate increase in the depth of submergence and, therefore, a proportionate increase in the pressure. As the air pressure increases with the lift, the quantity of free air per given input will decrease as the lift increases. The numerator of equation (33), therefore, increases slightly as the lift increases, because the loss in the air pipe is less with the smaller volume of air and consequent lower velocity. Also, since q_w is small compared with q_b, an increase in the lift decreases the value of the second parenthesis in the denominator of equation (33). c_p varies with the velocity of the water in the foot-piece and with the ratio of volume of free air to volume of water. It was found, in computing, that the relation of these quantities was such as to make c_p increase with the lift and that the last term in the denominator, which is a function of the entrance loss, was practically negligible. The middle term of the denominator proved to be the controlling one in the determination of the shape of the discharge-lift curve.

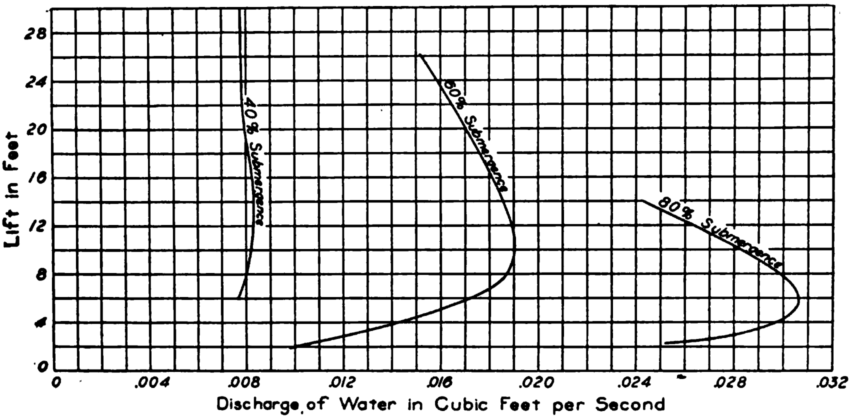


Fig. 34.—Relation of Discharge to Lift with Constant Input.

With the first parenthesis increasing in value and the second decreasing with the lift, evidently their product must have a maximum value at some point. The points of maximum discharge, as shown by the curves of Fig. 34, do not agree with the indications of Figs. 32 and 33, which show the discharge increasing up to higher lifts. This discrepancy is explained by the fact that for a given velocity of water in the foot-piece and given ratio of volumes of air to water the friction factor c_p decreases with the length of pump, as pointed out on page 79. In-as-much as the law of this variation is not known, the value of c_p was necessarily assumed independent of the lift, and in making the computations the values were taken from the curves of Fig. 20. It may be seen, however, by reference to equation (33), that if c_p decreases with the lift, other things being constant, the maximum discharge would have occurred at a higher lift under the assumed conditions and the curves of Fig. 34 would have tended to check the experimental results.

Effect of Compressed Air Outside of Pump

When the Harris foot-piece was purchased the manufacturers furnished it with fittings for introducing compressed air into the well casing outside of the pump, but the apparatus as used in the 1908 experiments did not permit any runs under these conditions. The statement is made in the Harris patent (No. 758360) that "the chief feature consists in combining the idea of introducing compressed air or gas into the casing to act upon the surface of the water or other fluid to force the same downward and the use of a suction means, such as an ejector or similar means, for drawing up the fluid and expelling it." Through correspondence we were informed by the manufacturers of the Harris pump that "the object in the use of this outside pressure is to steady the flow of water and retard the usual surging up and down occasioned when air is liberated in the well. The idea is not to put on excessive pressure, and as to the amount of pressure to be used, it depends entirely upon conditions and necessarily has to be adjusted on the ground to meet the conditions encountered." They further stated in this connection that "we find in many cases that by simply filing a leakage screw allowing a small volume of air to leak into the casing that it affords sufficient amount to work in harmony with the air pressure leading directly to the pump, while in other cases we find that it is often necessary to carry 25 or 30 pounds on the outside in order to assist in forcing the water through the discharge pipe."

In view of the foregoing it was deemed desirable to make some experiments with compressed air outside of the pump, and the apparatus as set up for the 1909 experiments was arranged with this object in view. Series 3, 4, 6, 7, 8, 9 and 10 were made for this purpose. The results are plotted in Figs. 35, 36 and 37, which show the efficiency and discharge curves for different rates of energy input under different conditions as to air outside of the pump. In series 3 the air was introduced above the surface of the water in the well and was

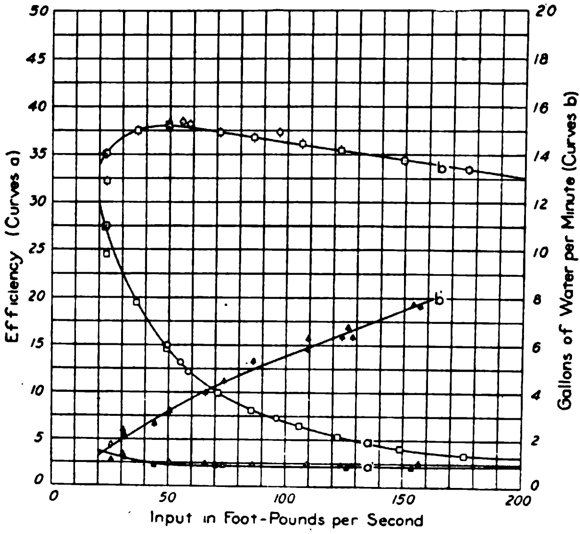


Fig. 35.—Curves Showing the Effect of Compressed Air Outside of Pump.

⊙ = Series 3—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 19.32 Feet—Average Lift 3.44 Feet—Average Submergence 82.19 Per Cent.—Compressed Air in Casing Connected with Air Main.

□ = Series 4—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 19.32 Feet—Average Lift 3.29 Feet—Average Submergence 82.97 Per Cent.—Compressed Air in Casing Shut Off from Air Main.

△ = Series 5—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 19.32 Feet—Average Lift 3.02 Feet—Average Submergence 84.37 Per Cent.—Annular Air Tube System.

in connection with the air main throughout the run, the valve in the casing air pipe (see Fig. 15) being open, so that the air pressure in the casing remained constant, and in case of any leakage from the casing or connections the amount lost was included in the amount of air recorded as supplied to the pump. In series 4 the air was admitted through the casing air pipe to the surface of the water in the well before the run started, and after the proper submergence was obtained and the pump was working steadily the air in the casing was shut off from the main by closing the valve in the casing air-pipe. In case of any air-leakage under these conditions the water rose in the well, thereby keeping the submergence fairly constant during the run. By means of a glass gage connecting with the well near the top and bottom, marked well gage in Fig. 15, the elevation of the water in the well could be noted. During series 3 and 4 the water level in the well stood at times at various elevations lower than the air inlet, but during none of the runs did the water surface fall low enough to allow air to enter the bottom of the tail-piece. The submergence in both cases remained practically constant and there was no appreciable leakage from the casing. The points belonging to the two series of runs lie on the same curve in Fig. 35, indicating that there was no difference in the discharge under the two conditions, as was to be expected.

In the same figure the efficiency and discharge curves are plotted for series 5. In this series the same apparatus was used but the supply to the Harris foot-piece was shut off, the foot-piece, however, remaining in the well and the well pumped according to the annular air tube system described on page 34, all the air used entering through the casing air pipe. As may be seen from the diagram the efficiency for this method of pumping is very low, but this series of runs, however, should not be taken as a criterion for the efficiency of the annular tube system, as the apparatus was not properly designed for this system of pumping. Obviously the annular space between the 1¼-inch discharge pipe and the six inch casing was too large for efficient pumping.

Fig. 36 shows the results of series 6 and 7. In both series the length of eduction pipe was 19.32 feet and the 1¼-inch Harris foot-piece was used. In series 6 the casing was open to the atmosphere so that the submergence was due entirely to the water in the well, while in series 7 compressed air was introduced above the surface of the water in the well and was shut off from the air main. It will be noticed, however, that the average submergence differed by 2.69 per cent., which undoubtedly accounts for the higher efficiency for the higher percentage of submergence. In order to show what difference the percentage of submergence makes, the points plotted as triangles were interpolated from the curves shown in Fig. 27 for a submergence of 43.66 per cent. The curves in Fig. 27 were plotted from the first series of experiments in which the length of eduction pipe and foot-piece were the same as used in the series under discussion, but the submergence was obtained by

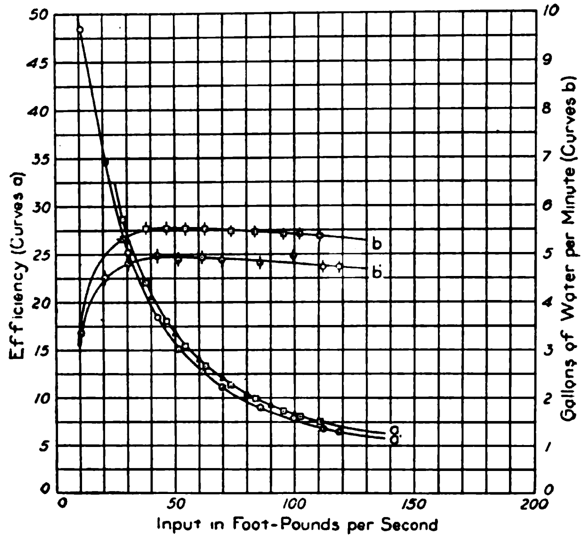


Fig. 36.—Curves Showing the Effect of Compressed Air Outside of Pump.

⊙ = Series 6—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 19.32 Feet—Average Lift 11.40 Feet—Average Submergence 40.97 Per Cent.—Casing Open to Atmosphere.

⊡ = Series 7—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 19.32 Feet—Average Lift 10.88 Feet—Average Submergence 43.66 Per Cent.—Compressed Air in Casing Shut Off from Air Main.

△ = Series 1—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 19.32 Feet—Average Lift 10.89 Feet—Average Submergence 43.66 Per Cent.—Submergence Obtained by Means of Overflow Hose Shown in Fig. 3.

means of the overflow hose shown in Fig. 10 corresponding very closely to the conditions of series 6. As will be noticed from the diagram, the points fall very closely on the curve so that the conclusion may be drawn that had the percentage of submergence remained constant no difference would have been noticed in the efficiencies of the two series of runs. With the same size and length of pump and the same percentage of submergence, the discharge per given input would necessarily be equal for the two series, if the efficiencies were equal.

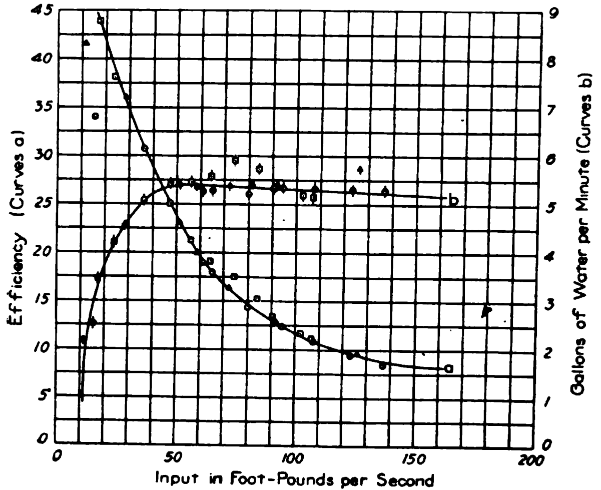


Fig. 37.—Curves Showing the Effect of Compressed Air Outside of Pump.

⊙ = Series 8—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 26.74 Feet—Average Lift 15.70 Feet—Average Submergence 41.29 Per Cent.—Casing Open to Atmosphere.

□ = Series 9—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 26.74 Feet—Average Lift 16.02 Feet—Average Submergence 40.08 Per Cent.—Compressed Air in Casing Connected to Air Main.

△ = Series 10—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 26.74 Feet—Average Lift 15.73 Feet—Average Submergence 41.19 Per Cent.—Compressed Air in Casing Shut Off from Air Main.

This conclusion is further justified by a study of Fig. 37. In this figure the results of three series of runs have been plotted, which differ only in the method of obtaining the submergence, the length of eduction pipe being 26.74 feet and the same size and style of foot-piece being used for the three series. The submergences remained more nearly constant in this case than in the preceding one, and the points are seen to lie very nearly on the same curve. In view of the results of these series of experiments the conclusion seems justified that, other conditions remaining constant, there is no advantage to be gained by introducing compressed air above the surface of the water in the well.

EFFECT OF TYPE OF FOOT-PIECE

Figs. 38 and 39 were plotted from results of comparative tests on three different types of foot-pieces. The three different types used were the Harris, the Indiana, and a Tee pump which are shown in section in Figs. 11 and 12, and are described on page 57. The conditions during the different tests were kept as constant as possible, although, as previously stated, it was difficult to maintain the percentage of submergence the same for the different series. Fig. 38 shows the results of three series at a submergence of about 42.5 per cent. The highest efficiency for a given input is obtained either by the Tee or Indiana pump. The points for the Indiana series do not lie on a smooth curve, so no curve was drawn for this series, but they will average pretty close to the line drawn for the Tee pump. The percentages of submergence for the Indiana and Tee pumps were practically the same, while the percentage for the Harris pump was about 1 per cent. higher. This higher percentage of submergence was, however, in favor of the Harris pump, which may be seen by reference to Fig. 27.

In Fig. 39 are shown the results of three series of runs with the same foot-pieces, but at a higher percentage of submergence than shown in the preceding figure. It will be seen from this figure that the highest efficiency for a given input is obtained by the Indiana pump. The percentages of submergence for the Harris and Indiana pumps are nearly the same with the difference in favor of the Harris pump. The percentage of submergence for the Tee pump is, however, 2.2 per cent higher than the Indiana pump. Correcting for this difference would make the efficiency curve of the Tee pump coincide very nearly with that of the Indiana pump. The conclusion to be drawn from the curves shown in Figs. 38 and 39 is that both the Indiana and Tee pumps showed a slight advantage over the Harris pump, while the Indiana and Tee pump showed practically the same efficiency, as nearly as could be seen from the curves. It should, however, be said in favor of the Harris pump that the Indiana pump was a 1½-inch foot-piece which was reduced to 1¼-inch at the entrance to the eduction pipe.

The opinion of the writers on this point is that the type of foot-piece has very little effect on the efficiency of the pump, so long as the air is introduced in an efficient manner and the full cross-sectional area of the eduction pipe is realized for the passage of the water. Anything in the shape of a nozzle to increase the kinetic energy of the air is detrimental.

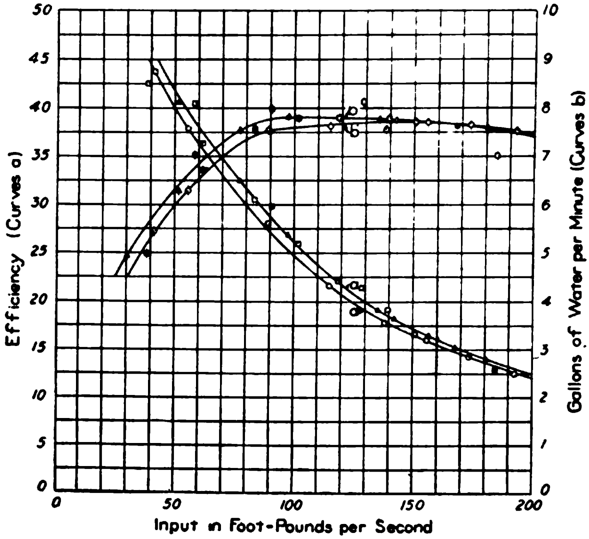


Fig. 38.—Curves Showing the Effect of the Type of Foot-Piece.

⊙ = Series 15—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 41.50 Feet—Average Lift 23.47 Feet—Average Submergence 48.45 Per Cent.
⊡ = Series 16—1¼-Inch Indiana Foot-Piece—Length of Eduction Pipe 42.08 Feet—Average Lift 24.20 Feet—Average Submergence 42.50 Per Cent.
△ = Series 19—Tee Pump—Length of Eduction Pipe 41.60 Feet—Average Lift 23.96 Feet—Average Submergence 42.40 Per Cent.

EFFECT OF DIVERGING OUTLET

Fig. 40 shows the effect of increasing the diameter of the eduction pipe at its upper end, thus decreasing the velocity of discharge and conserving part of the kinetic energy of the velocity head. The two series of experiments were performed under similar conditions, with the exception that 7.5 feet of 2-inch pipe and a 2-inch elbow replaced 7.5 feet of 1¼-inch pipe and a 1¼-inch elbow at the upper end of the eduction pipe. The enlargement from the 1¼-inch pipe to the 2-inch pipe was made by means of a standard cast-iron reducer. The enlargement of the upper part of the pipe caused a large increase in the discharge and hence in the efficiency. This diagram illustrates forcibly the necessity of keeping the velocity at the outlet as small as possible. Had the experiment been made with a more gradual enlargement, increasing the diam-

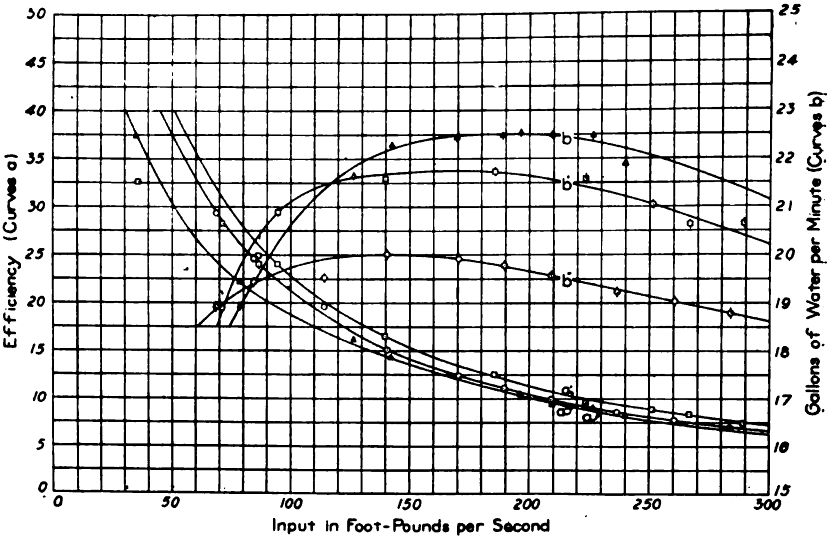


Fig. 39.—Curves Showing the Effect of Type of Foot-Piece.

⊙ = Series 14—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 41.50 Feet—Average Lift 7.66 Feet—Average Submergence 81.54 Per Cent.

⊡ = Series 17—1¼-Inch Indiana Foot-Piece—Length of Eduction Pipe 42.06 Feet—Average Lift 7.63 Feet—Average Submergence 81.95 Per Cent.

△ = Series 18—Tee Pump—Length of Eduction Pipe 41.60 Feet—Average Lift 6.58 Feet—Average Submergence 84.18 Per Cent.

eter at the outlet to a larger diameter, still better results would doubtless have been secured.

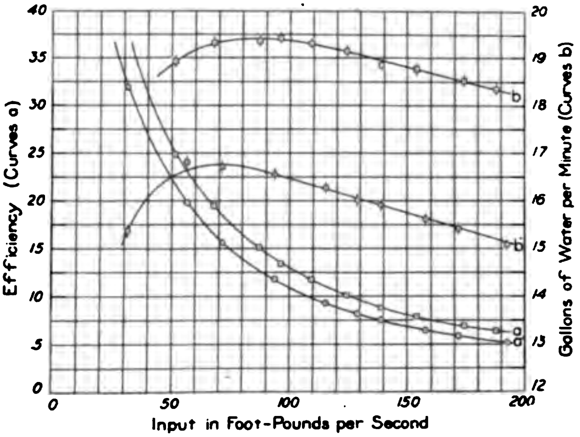


Fig. 40.—Curves Showing the Effect of a Diverging Outlet.

⊙ = Series 11—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 26.74 Feet—Average Lift 4.74 Feet—Average Submergence 82.40 Per Cent.

⊡ = Series 13—1¼-Inch Harris Foot-Piece—Length of Eduction Pipe 26.74 Feet—Average Lift 4.79 Feet—Average Submergence 82.23 Per Cent.—7.5 Feet of 1¼-Inch Pipe at Upper End of Eduction Pipe Replaced by 2-Inch Pipe.

This method of piping the well may be used to advantage when the lower part of the well is of a smaller bore than the upper part.

STUDY OF THE SIZE OF AIR BUBBLES

Reference has been made, on a previous page, to the fact that there is a difference of opinion as to the desirability of having the air in small bubbles in the rising column of water.

Experiments made with small glass tubes generally show the bubbles to be of a practically uniform size and of the form shown in Fig. 3 (b); the horizontal cross-sectional area of the bubble being about one-half the area of the pipe bore. Under these conditions the bubbles rise with a uniform motion and with very little disturbance of the intermediate layers of water. The action in a large pipe under operating conditions of velocity, etc., are, however, quite different.

The sixteen feet of glass pipe forming part of the eduction pipe used in the Wisconsin experiments afforded a good opportunity to study the action of the air in a pump of commercial size, under the conditions of the first experiments. The Harris foot-piece used in this series of experiments was designed, as shown in Figs. 11 and 12, to discharge the air, in the form of a thin shell, through an annular slit in the upper end of the nozzle just below the entrance to a contracted tube or sleeve; the evident purpose of the design being to promote such an intimate mixture of the air and water as would result in the production of small bubbles. If such action occurred, its effect was largely lost before the mixture of air and water rose to the beginning of the glass pipe, which was about a foot above the air nozzle.

When the air was admitted to the pump at a very low rate, so that it simply rose through the water without pumping, the bubbles took the form of a single convex lens, with the convex side upward. The under side of the bubbles was a practically level water surface, and the edges of the bubbles were quite sharp. The diameter of the bubble appeared, through the cylindrical glass walls, to be nearly as great as the diameter of the pipe bore. Such bubbles rose through the water with very steady uniform motion. Between such bubbles were occasional smaller ones, in which the surface tension was great enough to cause them to tend toward an oblate spheroid in form, though their motion was so irregular as to cause them to go through violent contortions.

As the rate of air admission was increased the longitudinal axis of the bubbles was lengthened; the shape of the bubbles becoming ovoid, with the bottom still flat, but the edges no longer sharp. With a further increase in the rate of air admission this general form of bubble persisted until the vertical axis became five or six diameters long. A greater rate of air admission caused a discharge of water which altered the conditions as to actual velocities, pipe friction, etc.

Under the conditions of pumping it was difficult to observe the form of the bubbles precisely on account of the velocity with which they shot through the pipe. Under practically all rates of pumping the bubbles appeared to be quite uniform in length near the foot-piece, being about a foot, or about ten diameters, long. The lower end of the bubble was quite flat, but the upper end appeared to have lost the regularity of form which was so noticeable at slow rates of air admission. The bubbles, in short, appeared as cylindrical pistons of air practically filling the pipe. Between the large air bubbles were pistons of water, of length about equal to that of the air pistons. In the lower part of these pistons of water were numerous small bubbles of air about the size of peas. The upper part of the water piston, which consisted principally of water which had slipped down past the next large air piston above, was clear and free from bubbles.

As the pistons approached the upper end of the glass pipe there were frequent sudden disturbances caused by one of the water pistons losing its equilibrium and slipping bodily down past the air piston below it, thus making two adjacent water and air pistons of double the average length.

Large bubbles, having greater buoyancy per unit area of surface, rise more rapidly through a liquid than do small bubbles. The large bubbles, therefore, overtake the smaller ones and coalesce with them. Likewise eddies in the water cause bubbles to impinge upon one another and coalesce. It appears from the Wisconsin experiments, with the Harris foot-piece, that the conditions of air admission and flow in the foot-piece were such as to cause most of the bubbles to coalesce within a space of a few pipe diameters above the foot-piece. The conditions of the experiments with the Indiana and Tee foot-pieces did not admit of inspection of the air in rising through the eduction pipe.

Experiments made by Professor E. Josse, in which he used a section of glass pipe in air lift pumps of 79 and 78 mm. diameter respectively, showed the action of the air and water to be exactly similar to that occurring in the somewhat smaller pump used in the present experiments, with the exception that his description* would indicate that there are many more small bubbles in the water pistons which are described as being foam-like. In his experiments the foot-piece was quite similar to our Tee foot-piece, but the air was admitted to the eduction pipe through a narrow annular slit around the pipe wall.

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* Prof. E. Josse, Druckluft-Wasserheber, Zeit. des Ver. Deutscher Ingenieure, Band 42, seite 981, Sept. 3, 1898.

RESULTS OF PREVIOUS EXPERIMENTS

While there have been many tests made on actual wells, the facilities for varying the conditions of operation or for making accurate measurements of the quantities have been limited, and knowledge of comparative tests is, therefore, confined chiefly to laboratory tests. Brief outlines of those tests with which the writers have become familiar will be found in the succeeding paragraphs.

Browne and Behr Experiments.—The experiments were performed on a 10-inch well with a depth of 55 feet. The diameter of the eduction pipe used was three inches, with lengths of 51.6, 88.3, and 128.2 feet. The diameter of the air pipe was 1 inch, and a Pohle foot-piece with a nozzle 5/8 inches in diameter was used. The water was measured by means of a weir and the air by volumetric measurement, checked by the flow through an orifice. The efficiency, based on the least work theoretically required to compress the air, so that it did not include the compressor, ranged from 2 to 53 per cent. The conclusions derived from these experiments were:

(1) Efficiency for a given lift and submergence was greatest when the pressure in the receiver did not greatly exceed the head due to submergence.

(2) Efficiency increased with the percentage of submergence.

Josse's Experiments.—Experiments performed at the technical institute at Charlottenburg, Germany. The comparative tests undertaken were: (1) with a lift having a discharge pipe 119.75 feet long and corrugations of 2.76 and 3.07 inches in minimum and maximum diameter, (2) with a lift having a smooth discharge pipe of the same length and 2.76 inches in diameter, and (3) with a lift having a smooth discharge pipe of the same length and 3.07 inches in diameter. Three comparative tests were made under above conditions, differing in the percentage of submergence. The water was measured by volumetric measurement and the amount of air computed from the number of strokes of the air piston and cards taken from the air cylinder, which gave the volumetric efficiency

Computations checked by volumetric measurement and found to be sufficiently accurate. The efficiency was calculated from the indicated work done in the air cylinder and the lift and volume of water raised.

Conclusions: Under similar conditions the smooth pipe gave an efficiency of about 45 per cent., the corrugated pipe about 25.7 per cent. and about half as much water delivered in the case of the corrugated pipe. However, the foot-pieces used with the two types of discharge pipe differed, the smooth pipe having a foot-piece which discharged the air all around the circumference of the pipe, while in the case of the corrugated pipe the air was introduced by a simple U bend at the bottom of the air pipe, with its free end in the center of the discharge pipe. In order to test the influence of this foot-piece, a series of experiments were made using the U bend foot-piece with the smooth discharge pipe. It was found that at high rates of pumping there was practically no difference in the discharge, but that at normal rates the foot-piece discharging the air around the circumference of the pipe discharged about 25 per cent. more water.

A series of runs in which the submergence, lift, type of foot-piece and size of discharge pipe remained constant, showed that by increasing the amount of air the water discharged increased to a certain point and then decreased.

A series of experiments were also made on actual wells from which Professor Josse deduced the following conclusions: (1) If submergence and lift be kept constant, the amount of air per volume of water will not vary much with the size of the pump. (2) With increasing lift the volume of air per volume of water increases, and hence the efficiency decreases. (3) Other things being equal, an increase in the area of the discharge pipe of 20 per cent. only increased the discharge 1.2 per cent.

The efficiencies varied from 20 to 45 per cent. in the laboratory tests, and from 22 to 28 per cent. in operating wells.

Kelly's Experiments.—Experiments performed at Preesall, Lancashire, on actual wells. Water measured by volumetric measurement. Air measured by piston displacement and indicator cards were taken from steam and air cylinders. Efficiencies were calculated from the ratio of the work done in raising the water to the work indicated in the air cylinders of the compressor.

Same general result obtained in regard to efficiency, discharge and ratio of volume of air to volume of water, when one well was working alone or when two or three were working together. One well was piped according to the side inlet system and three wells were piped according to the annular tube system, the air entering the eduction pipes through their open ends. The size of air and eduction pipe varied for the different wells. In the well piped according to the side inlet system, the upper end of eduction pipe was enlarged from 4 to 6 inches in diameter in order to reduce the velocity of discharge. Total length of eduction pipe varied from 433 feet to 323 feet for the different wells. The highest efficiency was about 40 per cent., obtained when three wells were working together.

Conclusions:

(1) Highest efficiency obtained at lowest rate of working.

(2) Discharge increases as the rate of working increases, with a tendency to decrease after the rate of working reaches a certain point.

(3) A percentage of submergence of 60 gave a better efficiency than one of 50, other conditions remaining the same.

(4) A well piped with 5-inch eduction and 7-inch air pipe gave higher efficiencies than one piped with a 4-inch eduction and 6-inch air pipe, other conditions remaining the same.

(5) The well piped with the side inlet system and diverging outlet appeared to give better results than the other wells, the cross sectional area of the eduction pipe remaining equal.

(6) The action of the compressed air in an air lift may be either similar to that of a piston in a cylinder, it may form an emulsion with the water, or it may produce a combination of both, the result depending on the rate of working. The piston like layers were obtained with the higher rates of working.

Darapsky and Schubert Analysis of Experiments.—An analysis of Prof. Josse's and other experiments, performed by the firm of Desenisz and Jacobi, from which the authors deduce an empirical formula for computing the amount of air required per volume of water pumped, and from which by the aid of a theoretical analysis, they deduce the following conclusions:

(1) The lift and velocity remaining constant the efficiency increases as the percentage of submergence increases.

(2) The lift and percentage of submergence remaining constant the efficiency decreases as the velocity increases.

(3) The submergence and velocity remaining constant the efficiency decreases as the lift increases.

(4) The percentage of submergence and velocity remaining constant the efficiency decreases slightly with the lift.

A few experiments with glass tubes showed that by adding a very short length of pipe to the pump as a tail-piece, the discharge as compared with the discharge without a tail-piece, was first diminished, then increased and again diminished as the length of the tail-piece was increased, the supply of air to the pump remaining constant. In the case of a 1.88-inch pipe the maximum discharge was obtained when the length of the tail-piece was about twelve times the diameter.

Westinghouse Air Brake Co's. Experiments.—An extensive series of tests comprising nearly 1,800 different experiments, covering nearly 400 different combinations of discharge pipe, diameter, lift and submergence, were made by the Westinghouse Air Brake Co. The results and conclusions of the tests have been published, but no numerical data or curves showing the relation of discharge, air consumption, etc., have been given to the public. The experiments were all made on an actual well, 6 inches in diameter and 174 feet deep. The discharge was weighed and amount of air computed by observing the initial and final pressures in a tank of known volume. The following conclusions, taken from an article in the Engineering News of June 18, 1908, indicate what these tests showed.

(1) The rate of delivery of water, and the air consumption per gallon, with fixed size of discharge pipe, are practically constant for all lifts, provided the ratio of lift to submergence is maintained constant.

(2) With a discharge pipe of given diameter, the delivery decreases and the air consumption per gallon increases as the ratio of lift to submergence increases.

(3) With a fixed ratio of lift to submergence, the air consumption per gallon decreases as the size of the discharge pipe increases.

(4) The least air pressure that will give continuous flow is the proper pressure to use. A slightly lower pressure gives intermittent delivery, and the amount delivered is much decreased, though the air consumption per gallon is slightly lower than with continuous flow. With pressure higher than just enough to give continuous flow, the delivery is increased somewhat, but the air consumption per gallon delivered is increased in greater ratio; and with further increase in air pressure a point of maximum delivery is reached, beyond which the delivery is decreased in amount. The sound of the discharge is a reliable guide to proper regulation of the air supply.

(5) It appears from (2) that by increasing the submergence, i. e. locating the foot-piece deeper down in the water, for a given lift, the air consumption is progressively reduced. But as the required air pressure is increased with the greater depth, a cubic foot of air represents greater power. A curve representing the variation of horsepower required per gallon of water delivered, with depth varying, shows that the power first decreases with increasing depth, then reaches a ~~max~~minimum and thence increases. The ratio of lift to submergence at this minimum point may be called the "economical ratio."

(6) For a given size of discharge pipe the economical ratio decreases as the lift increases; i. e. the submergence should be increased in greater ratio than the lift. For a given lift, the economical ratio increases (submergence decreases) as the size of discharge pipe increases.

(7) A tail-piece or projection of the discharge pipe below the air inlet is essential in starting, as it tends to prevent the air from backing down into the well and rising in the casing outside the discharge pipe.

(8) Anything in the shape of a jet or pipe introduced into the discharge pipe to serve as an air inlet has no value, and is, in fact, detrimental by forming an obstacle to the free passage of water.

(9) The size of the air pipe is determined only by consid-

erations of friction loss required to force the air through the pipe.

DUTY TESTS

The published results of duty tests on air lift plants are few in number. The plant is usually so arranged that a duty test is rather difficult to make, as the compressor takes steam from the same main as the force pumps. In order to give the reader an idea of the performance of an air lift plant figured on a duty basis, the following published tests are herewith appended:

"The plant at Atlantic City, N. J., showed a satisfactory duty ranging roughly from 20,000,000 to 25,000,000 foot pounds work per 1,000 pounds of dry steam. The pumping plant comprised a Rand duplex flywheel compressor, having 10-inch and 16-inch cross compound steam cylinders of 12-inch stroke, 13x12-inch air cylinder, Corliss inlet valves, and poppet discharge valves; a compressor of the same type with 11-inch and 18x14-inch stroke steam end and 16x14-inch air end, having Meyer adjustable valves on the steam cylinders; a Wainwright surface condenser; a Deane combined wet vacuum and circulating pump, with 5½x7-inch steam end and 6x7-inch pumps; an air receiver 28 inches by 8 feet; and galvanized piping to the wells, with valves. The smaller compressor was to be in reserve. The contract price for the pumping equipment complete was $8,250. Plant consisted of 13 wells with a capacity of about 5,450,000 gallons per 24 hours. The average lift is about 27 feet and submergence about 60 per cent."

"A comparison between the efficiency of wells pumped with air lift and steam deep well pumps at Waukesha, Wis., showed an efficiency of between 16 and 18 per cent. for air lift based on I. H. P. in steam cylinder and an efficiency of 74.8 per cent. for deep well pumps based on I. H. P. of engine."

Air Lift.

Duty per 100 pounds of coal......	8,200,000 foot pounds
Duty per 1,000 pounds of steam...	11,940,000 foot pounds

Deep Well Pumps.

Duty per 100 pounds of coal......	34,500,000 foot pounds
Duty per 1,000 pounds of steam...	53,300,000 foot pounds

In a paper read before the American Water Works Association, Mr. D. W. Mead estimates the duty of an air lift plant for different types of compressors as follows, assuming the efficiency (based on I. H. P. in steam cylinder) of the air lift to vary from 15 to 25 per cent.

Type of Compressor	Steam consumption, pounds dry steam per I. H. P.	Duty, in million foot pounds with	
		15 per cent. efficiency	25 per cent. efficiency
Compound Corliss compressor	16 to 20	19 to 30	31 to 25
Simple condensing Corliss compressor	22 to 28	13 to 10	22.5 to 18
Simple Corliss compressor	35 to 40	9.5 to 8.5	14 to 12
Well designed high pressure compressor	40 to 60	8.5 to 5	12 to 8
Small straight line compressor	50 to 80	6 to 4.5	10 to 6

CONCLUSION

A comparison of the advantages and disadvantages of the air lift pump shows that there is a field of usefulness of sufficient magnitude to make it an important apparatus deserving of further theoretical and experimental study. From a study of the foregoing discussion and accompanying data, ideas in regard to improvement in the design will no doubt be suggested to the designer or experimenter. However, a full realization of the complexity of the action must be appreciated, as also the number of variables which enter into the problem. In order to facilitate the study of results, the variables which may affect a particular type and size of pump are again given at this place. They are (1) percentage of submergence, (2) lift, (3) discharge, (4) volume of air, (5) pressure of air. The conclusions which may justifiably be deduced from the Wisconsin experiments are given below, and hold only for the particular type, size and length of pump on which the experiments were performed. The inference, however, may be drawn that these conclusions would hold for other types and sizes.

(1) The central air tube pump has the greatest theoretical capacity for a given size of well.

(2) The coefficient of pipe friction and slip decreases as the discharge increases, and decreases as the ratio of volume of air to volume of water increases. (See Fig. 20.)

(3) The coefficient of pipe friction and slip varies with the length of pump, but seems to be independent of the percentage of submergence and of the lift.

(4) The length of pump, the percentage of submergence, and therefore, the lift remaining constant, there is a definite quantity of air causing the maximum discharge. This quantity of air for maximum discharge, as also the ratio of volume of air to volume of water, differs for different percentages of submergence and lift, the length of the pump remaining constant. (See. Fig. 21.)

(5) The length of pump remaining constant, the maximum output (e. g. foot gallons) occurs at about the same percent-

age of submergence for all rates of air consumption, being at from 61 to 65 per cent. for the pump used in the Wisconsin experiments. At other submergences the output varies as the ordinates of a parabola having a vertical axis. Under these conditions the lift does not remain constant as the percentage of submergence varies. (See Figs. 23 and 24.)

(6) The length of pump and percentage of submergence remaining constant, and therefore constant lift, the efficiency increases as the input decreases, that is, the highest efficiencies are obtained at the lowest rates of pumping. (See Figs. 27, 28, and 30.)

(7) By varying the percentage of submergence, and therefore the lift, the length of pump remaining constant, the maximum efficiency is obtained at approximately 63 per cent. submergence for all rates of input or discharge. (See Figs. 27, 28, and 29.)

(8) The lift remaining constant, the efficiency increases as the percentage of submergence increases, for all rates of input and all practical percentages of submergence. (See Figs. 30 and 31.)

(9) With the same size and type of pump, the percentage of submergence remaining constant, the efficiency increased as the lift increased for the small lifts experimented on, that is, up to about 24 feet. From a theoretical study, however, the indications are that a point will be reached from which the efficiency will decrease as the lift increases. (See Figs. 32, 33 and 34.)

(10) Other conditions remaining constant, there is no advantage to be gained by introducing compressed air above the surface of the water in the well. (See Figs. 35, 36, and 37.)

(11) The type of the foot-piece has very little effect on the efficiency of the pump, so long as the air is introduced in an efficient manner and the full cross sectional area of the eduction pipe is realized for the passage of the liquid. Anything in the shape of a nozzle to increase the kinetic energy of the air is detrimental. (See Figs. 38 and 39.)

(12) A diverging outlet which will conserve the kinetic energy of the velocity head increases the efficiency. (See Fig. 40.)

TABLE I

SERIES I

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_l	c_p
1	.04407	18.007	21.843	.5766	.00923	4.143	43.77	10.865	6.265	28 684	4.775	1.08	.2199
2	.04429	18.061	22.222	.5834	.00933	4.187	44.39	10.743	6.208	28.209	4.747	1.09	.2200
3	.18057	19.104	108.380	.6867	.01098	4.928	43.47	10.923	7.501	6.921	16.446	1.29	
4	.12338	18.445	64.830	.7500	.01200	5.386	43.26	10.963	8.222	12.683	10.282	1.41	.0788
5	.12004	18.480	63.570	.7519	.01203	5.390	43.26	10.962	8.242	12.965	9.979	1.41	.0822
6	.09154	18.231	45.860	.7326	.01172	5.260	42.50	11.110	8.139	17.747	7.811	1.37	.1000
7	.06307	18.213	31.477	.7194	.01151	5.166	44.01	10.817	7.782	24.723	5.480	1.35	.1300
8	.03174	18.182	15.731	.3068	.00491	2.204	44.37	10.748	3.298	20.961	6.465	0.58	.7382
9	.04428	18.388	22.961	.6745	.01079	4.843	48.34	9.982	6.733	29.325	4.104	1.27	.1821
10	.05609	18.009	32.108	.8333	.01333	5.984	47.85	10.076	8.396	26.150	4.433	1.56	.1188
11	.07671	18.554	41.130	.9091	.01454	6.526	47.91	10.064	9.148	22.243	5.276	1.70	.0883
12	.09592	18.657	52.970	.8969	.01435	6.441	48.15	10.018	8.984	16.962	6.685	1.69	.0810
14	.03910	18.501	21.173	.4934	.00789	3.541	47.31	10.180	5.023	23.724	4.956	0.93	.3458
15	.08244	18.664	45.518	.9259	.01481	6.647	49.25	9.806	9.079	19.946	5.567	1.74	.0863
16	.13004	18.846	74.540	.8734	.01397	6.270	48.57	9.936	8.678	11.643	9.308	1.64	.0384
17	.12377	18.642	68.110	.7813	.01250	5.610	47.73	10.098	7.890	11.584	9.901	1.47	.0802
18	.12160	18.574	67.140	.8048	.01288	5.781	45.09	10.435	8.298	12.508	9.441	1.51	.0778
19	.11300	18.552	62.130	.8214	.01314	5.897	45.88	10.456	8.589	13.825	8.600	1.54	.0805
20	.09660	18.581	53.360	.8820	.01413	6.342	47.64	10.116	8.923	16.760	6.837	1.66	.0830
21	.08652	18.480	46.752	.8715	.01394	6.257	47.65	10.114	8.814	18.854	6.207	1.63	.0895
22	.07158	18.451	38.433	.8772	.01403	6.297	47.35	10.172	8.923	23.220	5.102	1.64	.0978
23	.05665	18.511	30 825	.8584	.01373	6.163	47.47	10.150	8.712	28.264	4.126	1.61	.1141

SERIES I—Continued

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_1	w_w	q_w	q_g	s	h_1	l_o	e	$\frac{q_a}{q_w}$	v_1	c_p
24	.05198	18.414	27.747	.7722	.01235	5.543	48.02	10.042	7.754	27.945	4.209	1.46	.1396
25	.04198	18.588	23.245	.7649	.01224	5.494	47.12	10.216	7.814	33.618	3.430	1.43	.1761
26	.02961	18.628	16.509	.4838	.00774	3.474	47.49	10.146	4.908	29.731	3.826	0.90	.4143
27	.13172	18.306	69.660	.6757	.01081	4.852	41.15	11.370	7.682	11.027	12.185	1.27	.0857
28	.10438	18.019	51.660	.6612	.01059	4.753	41.20	11.360	7.511	14.540	9.857	1.24	.1020
29	.08644	17.917	41.755	.6678	.01068	4.793	41.20	11.360	7.586	18.168	8.094	1.26	.1138
30	.07118	17.840	33.710	.6601	.01056	4.739	41.20	11.360	7.498	22.243	6.741	1.24	.1287
31	.05195	17.845	24.597	.6289	.01006	4.515	41.22	11.356	7.142	29.036	5.164	1.18	.1626
32	.03002	18.075	15.075	.4926	.00788	3.537	41.23	11.354	5.592	37.103	3.810	0.93	.3155
33	.04664	17.954	22.707	.6411	.01026	4.605	41.20	11.360	7.283	32.075	4.546	1.21	.1689
34	.04134	18.032	20.524	.5899	.00944	4.237	41.25	11.350	6.695	32.622	4.379	1.10	.2119
35	.13191	19.009	78.380	.9389	.01502	6.741	49.46	9.765	9.168	11.697	8.782	1.76	.0636
36	.10667	18.411	56.170	.7859	.01257	5.642	49.73	9.712	7.633	13.588	8.486	1.48	.0902
37	.08281	18.505	44.525	.8696	.01391	6.243	48.08	10.030	8.722	19.590	5.953	1.63	.0915
38	.06049	18.560	32.893	.8753	.01400	6.283	48.23	10.002	8.754	26.615	4.321	1.64	.1075
39	.05265	18.495	28.226	.7663	.01226	5.503	49.12	9.820	7.532	26.686	4.294	1.44	.1434
40	.04789	18.628	26.405	0.7519	.01203	5.399	48.96	9.862	7.416	28.087	3.981	1.41	.1578
41	.12336	18.502	66.320	0.6359	.01017	4.564	48.41	9.968	6.339	9.559	12.130	1.20	.1051
42	.11093	19.155	67.730	1.0498	.01680	7.541	53.58	8.968	9.414	13.899	6.603	1.97	.0648
43	.09123	19.068	54.830	1.0811	.01730	7.765	53.29	9.022	9.753	17.789	5.274	2.03	.0711
44	.08085	19.098	48.752	1.1237	.01798	8.070	54.40	8.810	9.900	20.307	4.497	2.11	.0722

SERIES I—Continued

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_1	w_w	q_w	q_g	s	h_1	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
45	.07195	19.071	43.245	1.1080	.01773	7.957	54.38	8.814	9.766	22.584	4.058	2.08	.0782
46	.06109	19.070	36.715	1.1049	.01768	7.935	54.72	8.748	9.666	26.327	3.456	2.08	.0831
47	.04831	18.989	28.573	1.0102	.01616	7.253	53.96	8.894	8.984	31.443	2.989	1.90	.1022
48	.03775	18.919	22.055	0.7533	.01205	5.408	53.64	8.950	6.742	30.571	3.133	1.42	.1886
49	.09612	19.193	59.790	1.2658	.02025	9.089	58.06	8.102	10.255	17.152	4.747	2.38	.0557
50	.08354	19.202	52.130	1.2780	.02045	9.178	58.20	8.076	10.321	19.800	4.085	2.41	.0596
51	.08096	19.203	50.510	1.2423	.01988	8.923	58.18	8.080	10.037	19.878	4.073	2.34	.0642
52	.06112	19.206	38.137	1.2384	.01981	8.891	58.43	8.032	9.946	26.083	3.085	2.32	.0723
53	.05291	19.206	33.013	1.1662	.01866	8.375	58.21	8.074	9.415	28.523	2.836	2.19	.0841
54	.04347	19.212	27.158	1.0928	.01748	7.845	57.96	8.122	8.875	32.680	2.487	2.05	.0981
55	.03097	19.213	19.347	0.8429	.01349	6.055	57.84	8.144	6.864	35.479	2.296	1.58	.1727
56	.10173	17.312	56.050	0.5690	.00910	4.084	38.01	11.976	6.815	12.158	11.180	1.07	.1016
57	.09045	17.382	38.823	0.6678	.01068	4.794	39.24	11.738	7.837	20.193	8.469	1.26	.0900
58	.08222	17.378	35.286	0.5031	.00805	3.613	38.76	11.832	5.952	16.868	10.214	0.95	.1526
59	.06988	17.383	29.994	0.4949	.00792	3.555	38.74	11.836	5.857	19.526	8.823	0.93	.1780
60	.06207	17.383	26.641	0.4960	.00795	3.568	38.77	11.830	5.878	22.065	7.808	0.94	.1901
61	.04920	17.384	21.117	0.5015	.00804	3.608	38.82	11.820	5.939	28.127	6.120	0.94	.2118
62	.04412	17.384	18.937	0.4587	.00734	3.294	38.87	11.810	5.417	28.605	6.011	0.86	.2598
63	.05371	17.384	23.053	0.4926	.00788	3.537	38.93	11.798	5.812	25.212	6.816	0.93	.2137
64	.04413	17.384	20.065	0.4926	.00788	3.537	39.00	11.786	5.806	28.937	5.601	2.93	.2778
65	.07917	19.844	55.450	1.4286	.02286	10.260	63.77	7.000	10.000	18.035	3.463	2.69	.0613

SERIES I—Continued

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_l	c_p
66	.07067	19.592	47.600	1.4494	.02319	10.408	63.67	7.022	10.177	21.380	3.048	2.72	.0586
67	.11658	20.108	84.090	1.4337	.02294	10.296	63.78	6.997	10.032	11.930	5.082	2.69	.0498
68	.09816	19.964	69.200	1.4337	.02294	10.296	63.68	7.017	10.060	14.537	4.279	2.69	.0545
69	.09712	19.905	67.940	1.4388	.02304	10.341	63.06	7.136	10.266	15.113	4.215	2.70	.0536
70	.09265	19.823	64.000	1.4236	.02278	10.224	64.80	6.800	9.680	15.126	4.067	2.68	.0558
71	.08114	19.771	55.590	1.4184	.02269	10.184	64.82	6.798	9.642	17.345	3.576	2.66	.0601
72	.08055	19.764	55.180	1.4337	.02294	10.296	62.12	7.318	10.491	19.013	3.511	2.69	.0576
73	.07391	19.742	50.410	1.4286	.02286	10.260	62.72	7.202	10.288	20.410	3.233	2.69	.0602
74	.06245	19.677	42.118	1.4235	.02278	10.224	62.55	7.235	10.299	24.453	2.742	2.68	.0625
75	.05394	19.702	36.580	1.4036	.02246	10.081	62.48	7.250	10.176	27.820	2.402	2.64	.0662
76	.03148	19.790	21.626	No	water		64.85	6.792		pumped			
77	.12212	20.165	87.810	1.4134	.02261	10.148	63.55	7.013	9.914	11.330	5.401	2.65	.0489
78	.10305	20.056	72.950	1.4036	.02246	10.081	63.65	7.023	9.857	13.537	4.588	2.64	.0550
79	.08105	19.838	55.520	1.4286	.02286	10.200	63.80	6.995	9.903	18.000	3.546	2.69	.0586
80	.09444	20.373	69.960	1.4235	.02278	10.224	63.84	6.988	9.947	14.218	4.146	2.68	.0611
81	.07146	19.837	48.947	1.4084	.02253	10.112	63.88	6.978	9.827	20.077	3.172	2.64	.0639
82	.06102	19.783	41.427	1.4494	.02319	10.408	63.93	6.970	10.103	24.387	2.631	2.72	.0621
83	.05726	19.781	38.878	1.4286	.02286	10.200	63.94	6.968	9.954	25.604	2.505	2.69	.0646
84	.04501	19.783	30.559	1.3888	.02222	9.973	63.99	6.958	9.663	31.620	2.026	2.61	.0694
85	.02858	19.849	19.633	1.2196	.01951	8.757	63.90	6.975	8.507	43.333	1.465	2.29	.0848
86	.03994	19.827	27.360	1.3559	.02169	9.735	63.86	6.983	9.468	34.003	1.842	2.55	.0731

SERIES I—Continued

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_1	l_o	e	$\frac{q_a}{q_w}$	v_l	c_p
87	.12640	20.499	97.310	1.5564	.02490	11.175	68.30	6.125	9.538	9.797	5.077	2.92	.0450
88	.11485	20.354	86.570	1.5504	.02481	11.135	68.08	6.168	9.563	11.046	4.629	2.91	.0477
89	.10194	20.284	76.210	1.5444	.02471	11.090	68.00	6.183	9.549	12.580	4.126	2.90	.0517
90	.06883	20.236	51.100	1.6001	.02560	11.490	67.88	6.205	9.928	19.429	2.689	3.01	.0581
91	.08070	20.184	59.440	1.6065	.02570	11.534	68.21	6.142	9.866	16.597	3.140	3.02	.0536
92	.07168	20.046	51.840	1.5936	.02550	11.445	68.22	6.140	9.784	18.874	2.811	2.99	.0555
93	.14772	21.011	120.840	1.5936	.02550	11.445	70.30	5.788	9.144	7.567	5.793	2.99	......
94	.10643	20.678	83.390	1.5936	.02550	11.445	68.06	6.172	9.835	11.793	4.174	2.99	.0508
95	.09305	20.517	71.350	1.6130	.02581	11.584	67.94	6.195	9.992	14.005	3.606	3.03	.0524
96	.07808	20.351	58.560	1.6265	.02602	11.678	67.44	6.290	10.230	17.470	3.001	3.05	.0540
97	.07172	20.368	53.890	1.7167	.02747	12.328	71.20	5.566	9.554	17.729	2.612	3.23	.0515
98	.07137	20.264	52.820	1.5747	.02519	11.306	70.16	5.765	9.077	17.185	2.833	2.96	.0602
99	.06761	20.386	50.970	1.6461	.02634	11.822	68.23	6.138	10.104	19.825	2.567	3.09	.0563
100	.05948	20.273	44.108	1.6461	.02634	11.822	68.70	6.048	9.955	22.570	2.258	3.09	.0566
101	.05053	20.252	37.372	1.5936	.02550	11.445	68.87	6.015	9.585	25.648	1.982	2.99	.0618
102	.03959	20.252	29.282	1.5094	.02415	10.838	69.26	5.940	8.965	30.617	1.639	2.84	.0695
103	.02760	20.230	20.360	1.1155	.01785	8.011	69.10	5.970	6.659	32.707	1.546	2.10	.1360
104	.00707	20.620	5.425	No	water		85.71	2.751		pumpe	d		
105	.00221	20.356	1.634	"	"		69.56	5.882		"			
106	.02240	15.217	2.823	"	"		24.87	14.514		"			
107	.02240	16.134	5.551	"	"		25.40	14.412		"			

Series I—Continued

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_o	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_l	c_p
108	.01307	17.106	4.848	No	water		31.18	13.296		pumped			
109	.00697	17.934	3.284	"	"		41.54	11.294		"			
110	.00216	20.350	1.595	"	"		70.30	5.738		"			
111	.00857	17.501	3.600	"	"		41.03	11.394		"			
112	.00428	19.178	2.621	"	"		51.45	9.380		"			
113	.00426	19.120	2.585	"	"		58.25	8.064		"			
114	.00679	18.478	3.628	"	"		46.37	10.362		"			
115	.00212	19.455	1.363	"	"		62.00	7.342		"			
116	.10044	20.930	81.390	1.8348	.02935	13.173	74.90	4.850	8.898	10.933	3.422	3.45	.0440
117	.10294	20.920	83.330	1.8264	.02922	13.114	75.00	4.830	8.821	10.585	3.523	3.43	.0436
118	.09423	20.852	75.570	1.8518	.02962	13.294	74.92	4.846	8.973	11.875	3.181	3.48	.0442
119	.08458	20.814	67.510	1.8692	.02991	13.424	74.88	4.852	9.069	13.433	2.828	3.52	.0454
120	.07523	20.794	59.930	1.8780	.03005	13.487	74.91	4.848	9.104	15.192	2.504	3.54	.0471
121	.06751	20.776	53.670	1.8780	.03005	13.487	74.88	4.852	9.112	16.978	2.247	3.54	.0485
122	.05883	20.677	46.145	1.8868	.03019	13.550	74.77	4.874	9.196	19.928	1.949	3.56	.0483
123	.04866	20.679	38.203	1.8517	.02962	13.294	74.84	4.860	8.999	23.558	1.643	3.48	.0509
124	.03660	20.677	28.708	1.7544	.02807	12.598	75.15	4.800	8.421	29.335	1.304	3.30	.0560
125	.04616	20.584	35.817	1.8100	.02896	12.997	74.66	4.894	8.857	24.727	1.594	3.41	.0524
126	.02924	20.580	29.666	1.5152	.02424	10.879	75.13	4.804	7.279	24.535	1.206	2.84	.0758
127	.05779	20.588	44.885	1.8517	.02962	13.294	74.59	4.910	9.092	20.256	1.951	3.48	.0496
128	.06459	20.643	50.460	1.8692	.02991	13.424	74.50	4.926	9.208	18.250	2.160	3.52	.0483

SERIES I—Continued

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	Y_1	c_p
129	.08207	20.718	64.750	1.8692	.02991	13.424	74.54	4.918	9.193	14.197	2.744	3.52	.0452
130	.01955	20.680	15.365	No	water		75.15	4.800		pumpe	d		
131	.11572	20.746	91.560	1.9513	.03122	14.013	79.93	3.878	7.567	8.264	3.707	3.67	.0350
132	.10184	20.753	80.600	1.9902	.03184	14.292	80.00	3.864	7.690	9.534	3.199	3.74	.0369
133	.09086	20.764	72.030	2.0409	.03265	14.654	80.14	3.836	7.828	10.808	2.783	3.84	.0374
134	.14397	21.580	123.820	1.9325	.03092	13.877	80.55	3.758	7.262	5.865	4.657	3.63	....
135	.13171	21.575	113.170	1.9418	.03107	13.944	79.84	3.896	7.565	6.685	4.240	3.66	.0360
136	.10076	21.400	84.870	2.0000	.03200	14.362	79.55	3.950	7.900	9.308	3.149	3.76	.0401
137	.08777	21.000	70.450	2.0305	.03248	14.577	79.55	3.950	8.020	11.383	2.702	3.82	.0391
138	.08420	21.248	69.640	2.0513	.03282	14.730	79.45	3.970	8.143	11.693	2.566	3.85	.0409
139	.07495	21.196	61.650	2.0513	.03282	14.730	79.52	3.958	8.119	13.170	2.284	3.85	.0424
140	.07439	21.193	61.140	2.0620	.03299	14.806	79.52	3.958	8.161	13.348	2.255	3.88	.0420
141	.06756	21.188	55.470	2.0833	.03333	14.958	79.54	3.954	8.237	14.850	2.027	3.91	.0422
142	.06330	21.184	51.070	2.0720	.03316	14.883	79.78	3.908	8.099	15.583	1.909	3.90	.0433
143	.05553	21.162	45.421	2.0833	.03333	14.958	79.72	3.918	8.162	17.970	1.666	3.91	.0435
144	.05171	21.067	42.335	1.9763	.03162	14.192	78.93	4.070	8.043	19.000	1.635	3.71	.0491
145	.04637	21.071	38.000	1.9481	.03117	13.989	79.11	4.034	7.858	20.681	1.488	3.67	.0510
146	.04024	21.004	32.670	1.8127	.02900	13.016	78.22	4.208	7.627	23.347	1.388	3.41	.0537
147	.02275	21.003	18.470	1.6667	.02668	11.974	79.19	4.020	6.700	36.276	0.853	3.14	.0789
148	.00708	21.034	5.769	No	water		79.06	4.046		pumpe	d		
149	.14084	21.899	126.830	2.0000	.03200	14.363	84.10	3.072	6.144	4.845	4.402	3.70	....

SERIES I—Continued

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tall piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
150	.13687	21.874	123.020	2.0135	.03221	14.456	84.27	3.040	6.121	4.976	4.250	3.78	.0355
151	.12498	21.853	111.860	2.0340	.03254	14.605	83.95	3.100	6.305	5.637	3.841	3.82	.0375
152	.11500	21.781	102.330	2.0691	.03310	14.856	83.90	3.110	6.435	6.288	3.474	3.89	.0382
153	.10738	21.749	95.130	2.1127	.03380	15.170	84.50	2.994	6.325	6.649	3.177	3.97	.0384
154	.10107	21.528	88.600	2.1277	.03403	15.228	84.28	3.038	6.464	7.296	2.970	3.99	.0386
155	.09155	21.483	79.850	2.1584	.03453	15.497	84.18	3.056	6.596	8.261	2.651	4.05	.0393
156	.08325	21.466	72.540	2.1819	.03491	15.667	84.44	3.006	6.558	9.041	2.385	4.10	.0403
157	.08042	21.436	69.770	2.2060	.03529	15.838	84.39	3.014	6.649	9.530	2.279	4.15	.0397
158	.07755	21.414	67.110	2.2142	.03542	15.897	84.33	3.028	6.704	9.989	2.189	4.16	.0398
159	.07085	21.385	61.090	2.2223	.03556	15.960	84.28	3.038	6.751	11.053	1.992	4.18	.0404
160	.06289	21.375	54.170	2.2389	.03582	16.077	84.47	3.000	6.716	12.397	1.756	4.21	.0411
161	.05470	21.353	46.913	2.2223	.03556	15.960	84.56	2.984	6.631	14.135	1.538	4.18	.0424
162	.05868	21.353	50.500	2.2223	.03556	15.960	84.40	3.014	6.698	13.264	1.656	4.18	.0420
163	.04977	21.328	42.610	2.1740	.03478	15.610	84.43	3.008	6.539	15.346	1.431	4.08	.0462
164	.03899	21.315	33.320	2.1054	.03368	15.116	84.44	3.006	6.329	18.995	1.158	3.96	.0477
165	.03178	21.322	27.187	1.7753	.02840	12.746	84.37	3.020	5.361	19.720	1.119	3.34	.0716
166	.00215	21.535	1.885	No	water		85.04	2.890		pumped			
167	.14682	22.350	140.500	2.1899	.03503	15.722	90.29	1.876	4.108	2.924	4.192	4.11	......
168	.11795	22.135	110.440	2.3077	.03692	16.570	90.21	1.892	4.366	3.953	3.195	4.33	.0386
169	.13497	22.306	128.640	2.2815	.03650	16.382	91.02	1.736	3.961	3.079	3.698	4.28	.0817
170	.13884	22.324	132.440	2.2389	.03582	16.077	90.31	1.872	4.191	3.165	3.876	4.21	.0822

SERIES I—Continued

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
171	12436	22.172	116.740	2.2729	.03636	16.319	90.06	1.920	4.364	3.738	3.421	4.27	.0334
172	.11134	22.078	103.670	2.3167	.03706	16.633	90.10	1.912	4.429	4.273	3.004	4.35	.0344
173	.10229	22.019	94.450	2.3623	.03780	16.965	90.09	1.916	4.526	4.792	2.706	4.44	.0346
174	.09712	21.975	89.450	2.3810	.03809	17.095	89.96	1.940	4.619	5.164	2.550	4.47	.0348
175	.08709	21.918	79.540	2.4097	.03855	17.302	89.78	1.974	4.757	5.980	2.259	4.53	....
176	.07804	21.886	71.170	2.4491	.03918	17.584	89.78	1.974	4.835	6.793	1.992	4.60	....
177	.07448	21.872	67.870	2.4692	.03951	17.733	89.86	1.960	4.840	7.130	1.885	4.64	....
178	.06739	21.856	61.200	2.4692	.03951	17.733	89.80	1.970	4.864	7.948	1.706	4.64	....
179	.06334	21.859	57.850	2.4794	.03967	17.805	90.34	1.868	4.631	8.006	1.597	4.66	....
180	.05520	21.837	50.340	2.4592	.03935	17.661	90.32	1.870	4.599	9.135	1.403	4.63	....
181	.05963	21.840	54.380	2.4692	.03951	17.733	90.43	1.850	4.568	8.400	1.509	4.64	....
182	.04527	21.831	41.215	2.4195	.03871	17.373	90.32	1.870	4.524	10.977	1.170	4.54	......
183	.03220	21.823	29.290	2.1353	.03416	15.332	89.72	1.986	4.241	14.478	0.943	4.02	.0536
184	.03905	21.753	35.252	2.2814	.03650	16.382	88.99	2.126	4.850	13.759	1.070	4.28	.0454
185	.00218	21.774	1.971	No	water		89.67	1.996		pumpe	d		
186	.13092	22.648	129.490	2.4002	.03840	17.234	95.63	0.844	2.026	1.564	3.410	4.51	....
187	.12525	22.595	123.300	2.4194	.03871	17.373	95.65	0.840	2.032	1.648	3.236	4.55	....
188	.11753	22.529	114.900	2.4491	.03918	17.584	95.55	0.860	2.106	1.833	3.000	4.60	....
189	.10805	22.458	104.900	2.4897	.03983	17.876	95.47	0.874	2.176	2.073	2.713	4.67	....
190	.10038	22.385	96.900	2.5106	.04017	18.028	95.44	0.880	2.209	2.280	2.490	4.72	....
191	.09302	22.333	89.300	2.5533	.04085	18.334	95.37	0.894	2.283	2.556	2.277	4.80	....

SERIES I—Continued

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
192	.09000	22.377	86.810	2.5643	.04103	18.414	95.60	0.850	2.180	2.511	2.194	4.82	....
193	.08445	22.327	80.940	2.5974	.04156	18.653	95.48	0.872	2.265	2.798	2.032	4.89	....
194	.07875	22.306	75.360	2.6549	.04248	19.066	95.47	0.874	2.320	3.079	1.854	4.98	....
195	.07531	22.283	71.900	2.6202	.04192	18.814	95.39	0.890	2.332	3.244	1.796	4.92	....
196	.06806	22.255	64.770	2.6433	.04229	18.982	95.38	0.892	2.358	3.641	1.609	4.97	....
197	.06359	22.230	60.420	2.6549	.04248	19.066	95.44	0.880	2.336	3.867	1.497	4.98	....
198	.05979	22.213	56.720	2.6667	.04266	19.147	95.30	0.908	2.421	4.269	1.402	5.01	....
199	.05071	22.203	48.020	2.6667	.04266	19.147	95.34	0.900	2.400	4.998	1.189	5.01	....
200	.04565	22.187	43.200	2.6549	.04248	19.066	95.42	0.884	2.347	5.433	1.075	4.98	....
201	.03973	22.185	37.597	2.6088	.04174	18.733	95.50	0.870	2.270	6.036	0.952	4.90	....
202	.02253	22.210	21.353	2.4794	.03967	17.805	95.96	0.780	1.934	9.057	0.568	4.66	....
203	.00221	22.284	21.114	No	water		94.36	1.090		pumpe d			
204	.12955	23.034	132.370	2.5237	.04038	18.123	100.29	—0.054	....	....	3.209	4.74	....
205	.12427	23.007	126.890	2.5237	.04038	18.123	100.35	—0.068	....	....	3.078	4.74	....
206	.11348	22.954	115.090	2.5642	.04103	18.415	100.23	—0.046	....	....	2.766	4.81	....
207	.10414	22.833	104.580	2.6060	.04161	18.674	100.02	—0.002	....	....	2.503	4.89	....
208	.10875	22.832	109.230	2.5974	.04156	18.652	100.02	—0.002	....	....	2.617	4.89	....
209	.09923	22.821	99.520	2.6231	.04197	18.837	99.96	+0.006	0.016	0.000	2.364	4.93	....
210	.08862	22.779	88.600	2.6846	.04295	19.277	99.70	+0.058	0.156	0.002	2.063	5.04	....
211	.09365	22.781	93.700	2.6490	.04238	19.021	99.74	+0.050	0.133	0.001	2.210	4.97	....
212	.08556	22.773	85.480	2.7027	.04324	19.406	99.69	+0.060	0.162	0.002	1.979	5.08	....

SERIES I—Continued

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift in feet.	Output in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_1	w_w	q_w	c_g	s	h_1	l_o	e	$\frac{q_a}{q_w}$	v_1	c_p
213	.08246	22.733	75.770	2.7211	.04354	19.542	99.62	+0.072	0.196	0.003	1.804	5.11	....
214	.07269	22.709	66.680	2.7492	.04399	19.743	99.54	+0.088	0.242	0.004	1.652	5.16	....
215	.06837	22.679	67.780	2.7683	.04429	19.878	99.50	+0.096	0.266	0.004	1.544	5.20	....
216	.06458	22.669	63.970	2.7779	.04444	19.945	99.48	+0.100	0.278	0.004	1.453	5.22	......
217	.06021	22.657	59.460	2.8270	.04523	20.300	99.95	+0.010	0.028	0.000	1.331	5.31	......
218	.05598	22.654	55.280	2.8270	.04523	20.300	99.99	+0.002	0.006	0.000	1.238	5.31	......
219	.03971	22.633	39.155	2.8270	.04523	20.300	99.95	+0.010	0.028	0.001	0.878	5.31	......
219½	.04560	22.203	43.115	2.8471	.04555	20.443	100.03	—0.006	....	....	1.001	5.35	......
220	.03271	22.191	30.905	2.7875	.04460	20.017	100.31	—0.060	....	....	0.733	5.24	....
221	.02280	22.178	21.525	2.5642	.04103	18.414	99.96	+0.008	0.021	0.001	0.556	4.81	....
222	.12788	23.403	135.000	2.6471	.04236	19.007	105.21	—1.008	....	....	3.020	4.97	......
223	.12374	22.377	130.420	2.6606	.04266	19.147	105.11	—0.988	....	....	2.901	5.01	....
224	.11073	23.309	116.030	2.7493	.04399	19.743	105.25	—1.014	....	....	2.517	5.16	....
225	.10584	23.257	110.340	2.7682	.04429	19.878	105.17	—1.000	....	....	2.390	5.20	....
226	.10312	23.234	107.270	2.7778	.04444	19.945	105.06	—0.978	....	....	2.320	5.22	....
227	.10031	23.216	104.270	2.7973	.04475	20.084	105.07	—0.980	....	....	2.242	5.25	....
228	.09511	23.188	95.010	2.8070	.04491	20.157	105.06	—0.978	....	....	2.118	5.27	....
229	.07579	23.148	78.320	2.8470	.04555	20.443	104.95	—0.958	....	....	1.664	5.35	....
230	.08397	23.121	86.490	2.8778	.04604	20.664	104.92	—0.950	....	....	1.824	5.41	....
231	.08059	23.087	82.780	2.9199	.04671	20.964	105.05	—0.974	....	....	1.725	5.48	....
232	.07440	23.070	76.380	2.9521	.04723	21.198	105.09	—0.982	....	....	1.575	5.54	....

SERIES I—Continued

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
233	.06761	23.044	69.250	2.9963	.04794	21.517	105.17	−1.000	....	....	1.410	5.62	....
234	.06339	22.992	64.630	3.0304	.04849	21.763	105.00	−0.984	....	....	1.307	5.69	....
235	.05810	22.929	58.620	3.0653	.04904	22.010	105.42	−1.048	....	....	1.185	5.75	....
236	.05008	22.927	50.520	3.0890	.04942	22.180	105.30	−1.024	....	....	1.013	5.78	....
237	.04558	22.923	45.950	3.0890	.04942	22.180	105.33	−1.030	....	....	0.922	5.78	....
238	.04068	22.921	41.010	3.0890	.04942	22.180	105.31	−1.026	....	....	0.823	5.78	....
239	.03528	22.919	35.540	3.0304	.04849	21.763	105.26	−1.016	....	....	0.728	5.69	....
240	.01998	22.920	20.128	2.9413	.04706	21.121	105.47	−1.058	....	....	0.425	5.52	....
241	No	air		1.5504	.02481	11.135	105.19	−1.002		used		2.91	....
242	.13492	23.512	143.150	2.8125	.04500	20.196	111.07	−2.140	....	....	2.998	5.28	....
243	.14263	23.523	151.430	2.7610	.04417	19.824	111.17	−2.160	....	....	3.229	5.18	....
244	.12553	23.861	136.980	2.8572	.04572	20.520	110.95	−2.116	....	....	2.746	5.37	....
245	.12973	23.891	141.970	2.8214	.04514	20.260	110.84	−2.096	....	....	2.874	5.30	....
246	.11867	23.801	128.960	2.8847	.04615	20.713	110.69	−2.064	....	....	2.572	5.42	....
247	.11115	23.738	120 200	2.9127	.04662	20.923	110.57	−2.042	....	....	2.384	5.47	....
248	.10592	23.684	113.970	2.9317	.04691	21.054	110.39	−2.008	....	....	2.258	5.51	....
249	.09823	23.638	105.170	2.9803	.04769	21.404	110.35	−2.000	....	....	2.060	5.60	....
250	.08954	23.521	95.130	3.0000	.04800	21.543	109.56	−1.848	....	....	1.866	5.63	....
251	.08043	23.519	85.390	3.1142	.04983	22.364	110.53	−1.996	....	....	1.164	5.85	....
252	.07665	23.489	81.190	3.1579	.05052	22.674	110.14	−1.960	....	....	1.517	5.93	....
253	.07263	23.451	76.660	3.1690	.05070	22.754	110.14	−1.960	....	....	1.433	5.95	....

SERIES I—Continued

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_1	w_w	q_w	q_g	s	h_1	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
254	.06489	23.434	68.440	3.2143	.05143	23.081	110.01	—1.934	....	....	1.262	6.04	....
255	.06090	23.414	64.050	3.2493	.05199	23.333	110.33	—1.996	....	....	1.171	6.10	....
256	.05619	23.390	59.050	3.2729	.05233	23.486	109.71	—1.878	....	....	1.074	6.14	....
257	.04621	23.382	48.530	3.2848	.05255	23.585	109.96	—1.924	....	....	0.879	6.17	....
258	.03994	23.374	41.911	3.2969	.05275	23.675	110.63	—2.048	....	....	0.757	6.19	....
259	.03288	23.390	34.555	3.2493	.05199	23.333	110.96	—2.116	....	....	0.633	6.10	....
260	.02289	23.412	23.902	3.1036	.04965	22.284	110.57	—2.042	....	....	0.461	5.83	....
261	No	air		2.0980	.03357	15.066	109.80	—1.892		used		3.94	....
262	.12304	24.287	137.660	3.0213	.04834	21.695	116.03	—3.098	....	....	2.545	5.67	....
263	.13075	24.364	147.180	2.9852	.04776	21.435	115.97	—3.086	....	....	2.738	5.61	....
264	.11404	24.235	127.070	3.0677	.04908	22.027	115.81	—3.056	....	....	2.324	5.76	....
265	.10711	24.224	119.200	3.1057	.04969	22.303	115.85	—3.064	....	....	2.156	5.83	....
266	.09998	24.161	110.740	3.1447	.05031	22.580	115.76	—3.026	....	....	1.987	....	....
267	.10022	24.110	110.520	3.1849	.05096	22.872	115.47	—2.990	....	....	1.967	5.98	....
268	.08880	24.024	97.180	3.2260	.05162	23.167	115.33	—2.962	....	....	1.720	6.06	....
269	.07680	24.050	84.340	3.3115	.05298	23.778	116.25	—3.140	....	....	1.450	6.22	....
270	.07653	24.040	83.970	3.3333	.05333	23.935	116.06	—3.102	....	....	1.435	6.26	....
271	.06921	24.022	75.750	3.3785	.05406	24.263	115.77	—3.048	....	....	1.280	6.35	....
272	.06533	24.019	71.500	3.4485	.05517	24.762	115.94	—3.080	....	....	1.184	6.48	....
273	.06115	23.968	66.680	3.4604	.05537	24.851	115.65	—3.024	....	....	1.104	6.50	....
274	.05670	23.958	61.800	3.4845	.05575	25.022	115.61	—3.018	....	....	1.017	6.54	....

SERIES I—Continued

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per spuare inch.	Input, in foot pounds per second.	Dischafge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
275	.04641	23.922	50.400	3.5090	.05614	25.197	115.43	—2.982	....	....	0.827	6.59	....
276	.04044	23.902	43.855	3.5213	.05634	25.286	115.47	—2.990	....	....	0.718	6.61	....
277	.03309	23.881	35.840	3.5590	.05694	25.555	115.33	—2.960	....	....	0.581	6.68	....
278	No	air		2.7550	.04408	19.783	115.82	—3.057		used		5.17	
306	.12283	17.824	57.130	0.4672	.00762	3.420	35.75	12.413	5.911	10.347	16.220	0.89	.1243
307	.12272	17.833	57.360	0.4793	.00767	3.442	35.86	12.392	5.939	10.354	16.000	0.90	.1241
308	.11278	17.745	51.420	0.4793	.00767	3.442	35.14	12.532	6.006	11.680	14.705	0.90	.1319
309	.09949	17.617	43.905	0.4785	.00766	3.438	35.09	12.540	6.000	13.637	12.990	0.89	.1430
310	.08357	17.592	36.955	0.4793	.00767	3.442	35.04	12.550	6.015	16.277	10.897	0.90	.1683
311	.09033	17.595	39.943	0.4800	.00768	3.447	34.81	12.505	6.045	15.135	11.763	0.90	.1567
312	.07275	17.579	32.112	0.4870	.00779	3.496	35.28	12.507	6.092	18.972	9.339	0.91	.1920
313	.06595	17.553	28.860	0.4796	.00767	3.442	35.20	12.520	6.004	20.804	8.599	0.90	.1968
314	.05679	17.514	24.628	0.4598	.00736	3.303	35.22	12.515	5.754	23.363	7.716	0.87	.2267
315	.05016	17.494	21.605	0.4396	.00703	3.155	85.44	12.473	5.484	25.382	7.135	0.82	.2593
316	.04648	17.608	20.695	0.4255	.00681	3.056	35.74	12.415	5.283	25.530	6.825	0.80	.2981
317	.03816	17.007	16.992	0.3643	.00583	2.617	35.87	12.390	4.514	26.562	6.545	0.68	.4215
318	.03290	17.001	20.442	0.2725	.00436	1.957	35.92	12.380	3.374	16.504	7.546	0.51	....

SERIES 2

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input in foot pounds per second.	Discharge of water in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
319	.06355	17.457	26.938	0.3239	.00518	2.325	30.00	13.524	4.381	16.261	12.268	....	....
320	.05166	21.501	44.700	2.1391	.03422	15.356	83.10	3.265	6.984	15.622	1.510	....	....
321	.07371	21.849	66.320	2.1506	.03441	15.445	83.45	3.198	6.878	10.370	2.142	....	....
322	.09043	22.143	83.910	2.1040	.03366	15.108	83.30	3.226	6.787	8.088	2.677	....	....
323	.11443	22.762	112.790	2.0202	.03232	14.506	83.50	3.188	6.441	5.711	3.541	....	....
324	.15172	23.735	162.980	1.8958	.03033	13.613	83.64	3.162	5.995	3.678	5.002	....	....
325	.04865	21.513	42.093	2.1740	.03478	15.612	83.50	3.188	6.931	16.466	1.399	....	....
326	.05167	21.578	45.090	2.1666	.03466	15.558	83.50	3.188	6.907	15.318	1.490	....	....

SERIES 3

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

Average lift 3.44 feet

Average Submergence 82.19 per cent

Supply of Compressed Air in Well Casing Connected with Air Main

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ inch tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_t	c_p
327	.05765	21.471	49.705	2.1052	.03368	15.118	81.73	3.530	7.431	14.950	1.712	....	....
328	.05768	21.483	49.772	2.1097	.03375	15.149	81.76	3.524	7.434	14.937	1.709	....	....
329	.06303	21.612	55.230	2.1391	.03422	15.360	82.48	3.385	7.241	13.411	1.842	....	......
330	.10627	21.936	96.470	2.0774	.03324	14.921	82.55	3.372	7.004	7.261	3.197	....	....
331	.06635	21.712	58.835	2.1220	.03395	15.238	82.55	3.372	7.155	12.161	1.954	....	....

SERIES 4

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

Average Lift 3.29 feet

Average Submergence 82.97 per cent

Supply of Compressed Air in Well Casing Shut off from Air Main

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_l	c_p
332	.02726	21.257	23.127	1.7898	.02864	12.855	83.64	3.162	5.659	24.468	0.952	....	....
333	.02727	21.251	23.115	1.9523	.03123	14.017	83.16	3.254	6.353	27.485	0.873	....	....
334	.04258	21.281	36.257	2.0810	.03329	14.943	82.54	3.375	7.023	19.372	1.279	....	....
335	.05678	21.452	49.327	2.1202	.03392	15.226	82.54	3.375	7.156	14.506	1.674	....	....
336	.07903	21.788	71.280	2.0777	.03324	14.920	82.54	3.375	7.012	9.837	2.378	....	....
337	.09331	21.971	85.790	2.0479	.03276	14.704	82.54	3.375	6.912	8.056	2.848	....	....
338	.11108	22.330	106.020	2.0135	.03221	14.458	82.54	3.375	6.796	6.409	3.449	....	....
339	.12428	22.684	122.530	1.9738	.03158	14.174	83.02	3.286	6.486	5.293	3.936	....	....
340	.14421	23.202	149.000	1.9200	.03072	13.788	83.36	3.215	6.173	4.143	4.694	....	....
341	.16332	23.703	176.120	1.8673	.02987	13.407	83.36	3.215	6.004	3.409	5.465	....	....
342	.02706	21.251	22.937	1.9503	.03120	14.003	83.36	3.215	6.270	27.336	0.867	....	....

SERIES 5

1¼ inch Annular Air Tube System

Length of Eduction Pipe 19.32 feet

Average Lift 3.02 feet

Average Submergence 84.37 per cent

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds qer second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
343	.03372	22.130	31.545	0.2956	.00473	2.123	83.70	3.149	0.931	2.951	7.129	......	......
344	.05340	22.294	50.940	0.4434	.00709	3.182	85.54	2.794	1.239	2.432	7.532	......	......
345	.08921	22.418	86.120	0.7380	.01181	5.300	86.00	2.705	1.996	2.318	7.554	......	......
346	.11202	22.542	109.380	0.8776	.01404	6.301	84.47	3.001	2.634	2.408	7.978	......	......
347	.12910	22.559	126.280	0.9404	.01504	6.750	84.73	2.950	2.774	2.197	8.584	......	......
348	.15506	22.707	153.930	1.0808	.01729	7.700	84.10	3.072	3.321	2.158	8.968	......	......
349	.02775	21.968	25.663	0.2430	.00389	1.746	85.07	2.885	0.701	2.732	7.134	......	......
350	.04712	22.105	44.045	0.3658	.00585	2.626	86.16	2.675	0.978	2.222	8.055	......	......
351	.07736	22.300	73.850	0.6236	.00998	4.479	86.36	2.636	1.644	2.226	7.751	......	......
352	.07427	22.166	70.760	0.5484	.00877	3.936	85.28	2.844	1.560	2.204	8.469	......	......
353	.11310	22.307	109.220	0.8128	.01300	5.835	84.27	3.040	2.471	2.262	8.700	......	......
354	.13194	22.360	128.130	0.8865	.01418	6.364	82.48	3.385	3.001	2.343	9.304	......	......
355	.16106	22.401	156.890	1.0688	.01710	7.675	80.19	3.828	4.091	2.609	9.418	......	......
356	.12755	22.366	123.950	0.8892	.01423	6.387	82.96	3.292	2.927	2.302	8.964	......	......
357	.06910	22.195	66.000	0.5542	.00887	3.981	85.48	2.805	1.554	2.355	7.791	......	......
358	.03324	21.947	30.910	.3322	.00531	2.383	83.98	3.095	1.028	3.326	6.200	......	......

SERIES 6

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

Average Lift 11.40 feet

Average Submergence 40.97 per cent

Well Casing open to Atmosphere

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
359	.16245	20.205	118.600	.6603	.01056	4.739	40.36	11.522	7.608	6.415	15.384	....	....
360	.15727	20.040	112.020	.6584	.01053	4.726	40.36	11.522	7.586	6.773	14.934	....	....
361	.14641	19.735	99.620	.6904	.01105	4.959	41.12	11.376	7.853	7.883	13.251	....	....
362	.13339	19.393	85.770	.6714	.01074	4.820	40.43	11.509	7.727	9.009	12.419	....	....
363	.11566	19.045	69.870	.6801	.01088	4.883	41.11	11.378	7.738	11.075	10.620	....	....
364	.10565	18.821	61.410	.6879	.01101	4.941	40.90	11.419	7.854	12.790	9.596	....	....
365	.09236	18.600	51.420	.6819	.01091	4.896	40.84	11.430	7.794	15.156	8.465	....	....
366	.07979	18.421	42.718	.6911	.01106	4.964	41.04	11.392	7.873	18.432	7.215	....	....
367	.06059	18.108	30.240	.6700	.01072	4.811	41.10	11.380	7.624	25.212	5.052	....	....
368	.04214	18.020	20.630	.6307	.01009	4.528	41.38	11.326	7.144	34.637	4.177	....	....
369	.02258	17.943	10.795	.4668	.00747	3.353	42.06	11.194	5.225	48.402	3.023	....	....

SERIES 7

1¼ inch Harris Pump

Length of Eduction Pipe 19.32 feet

Average Lift 10.88 feet

Average Submergence 43.66 per cent

Supply of Compressed Air in Well Casing shut off from Air Main

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
370	.15286	20.158	110.730	.7500	.01200	5.386	43.36	10.943	8.208	7.411	12.738	....	....
371	.14442	19.988	102.250	.7566	.01211	5.435	43.50	10.916	8.259	8.077	11.926	....	....
372	.13800	19.818	95.200	.7533	.01205	5.408	43.50	10.916	8.223	8.638	11.453	....	....
373	.12812	19.602	83.840	.7627	.01220	5.475	43.63	10.891	8.306	9.907	10.503	....	....
374	.11721	19.333	73.180	.7646	.01223	5.489	43.63	10.891	8.327	11.379	9.584	....	....
375	.10433	19.118	62.750	.7695	.01231	5.525	43.63	10.891	8.380	13.355	8.474	....	....
376	.09344	18.924	54.160	.7702	.01232	5.529	43.76	10.866	8.369	15.452	7.585	....	....
377	.08294	18.753	46.535	.7715	.01234	5.538	43.76	10.866	8.382	18.015	6.722	....	....
378	.07054	18.559	37.980	.7707	.01233	5.534	43.69	10.879	8.384	22.076	5.722	....	....
379	.05435	18.376	28.142	.7407	.01185	5.318	43.69	10.879	8.058	28.632	4.587	....	....

SERIES 8

1¼ inch Harris Pump

Length of Eduction Pipe 26.74 feet

Average Lift 15.70 feet

Average Submergence 41.29 per cent

Well Casing open to Atmosphere

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
381	.02708	18.661	15.559	.3528	.00564	2.531	43.92	14.997	5.291	34.007	4.908	0.66	.0854
383	.05711	19.363	36.595	.7094	.01135	5.094	40.76	15.842	11.238	30.710	5.032	....	....
384	.08941	19.770	61.190	.7340	.01174	5.269	40.76	15.842	11.628	19.005	7.617	1.38	.0943
385	.09389	19.858	65.140	.7377	.01180	5.296	40.76	15.842	11.687	17.942	7.957	1.39	.0917
386	.11082	20.135	80.280	.7278	.01104	5.225	40.76	15.842	11.530	14.363	9.520	1.37	.1058
387	.12442	20.424	94.540	.7451	.01192	5.349	41.16	15.735	11.724	12.402	10.438	1.40	.0779
388	.13729	20.683	107.980	.7423	.01188	5.333	41.16	15.735	11.680	10.818	11.557	....	....
389	.15024	21.013	123.230	.7396	.01183	5.309	41.16	15.735	11.638	9.444	12.700	1.39	.0700
390	.16155	21.267	136.950	.7374	.01180	5.297	41.16	15.735	11.603	8.473	13.689	1.39	.0667

SERIES 9

1¼ inch Harris Pump

Length of Eduction Pipe 26.74 feet

Average Lift 16.02 feet

Average Submergence 40.08 per cent

Supply of Compressed Air in Well Casing Connected with Air Main

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
391	.16016	19.557	107.040	.7180	.01149	5.157	37.73	16.652	11.955	11.168	13.930	....	....
392	.15329	19.557	102.420	0.7240	.01158	5.197	37.98	16.585	12.007	11.723	13.239	....	....
393	.13558	19.557	90.640	0.7443	.01191	5.345	38.70	16.392	12.201	13.461	11.384	....	....
394	.12313	19.695	84.150	0.7995	.01279	5.740	39.93	16.062	12.842	15.261	9.627	1.50	.0574
395	.10836	19.736	74.500	0.8229	.01316	5.906	40.57	15.892	13.078	17.540	8.234	1.55	.0611
396	.09476	19.689	64.770	0.7779	.01244	5.584	40.48	15.917	12.382	19.119	7.617	....	....
397	.08192	19.695	56.060	0.7592	.01215	5.453	41.31	15.695	11.914	21.252	6.743	1.43	.0351
398	.06941	19.677	47.337	0.7549	.01208	5.421	41.17	15.732	11.875	25.086	5.746	1.42	.1070
399	.03867	19.194	23.978	0.5848	.00936	4.201	41.37	15.678	9.168	38.236	4.131	....	....
400	.02735	19.170	17.127	0.4817	.00771	3.460	41.61	15.614	7.521	43.917	3.547	....	....

SERIES 10

1¼ inch Harris Pump

Length of Eduction Pipe 26.74 feet

Average Lift 15.73 feet

Average Submergence 41.19 per cent

Supply of Compressed Air in Well Casing Shut off from Air Main

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	q_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
401	.01865	19.022	11.537	0.3038	.00486	2.181	40.97	15.785	4.796	41.567	3.838	....	....
402	.04614	19.034	28.611	0.6384	.01021	4.582	39.63	16.143	10.306	36.025	4.519	1.20	.1434
403	.07618	19.553	51.610	0.7530	.01205	5.409	41.01	15.775	11.878	23.015	6.322	....	....
404	.08477	19.717	58.975	0.7474	.01196	5.368	41.12	15.745	11.768	19.955	7.089	1.40	.0956
405	.10125	19.925	72.620	0.7510	.01201	5.390	41.06	15.761	11.836	16.298	8.431	1.41	.0875
406	.11055	20.142	81.810	0.7530	.01205	5.408	41.27	15.706	11.827	14.456	9.175	....	....
407	.12091	20.360	92.140	0.7544	.01207	5.418	41.45	15.657	11.810	12.818	10.018	1.42	.0688
409	.14969	21.150	126.280	0.8007	.01281	5.749	42.99	15.245	12.207	9.667	11.684	....	....

SERIES 11

1¼ inch Harris Pump

Length of Eduction Pipe 26.74 feet

Average Lift 4.74 feet

Average Submergence 82.40 per cent

Supply of Compressed Air in Well Casing Shut off from Air Main

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, to pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
410	.02958	23.922	31.741	2.1421	.03427	15.382	82.35	4.738	10.148	31.970	0.863	....	....
411	.05231	24.058	46.750	2.3443	.03751	16.835	81.99	4.818	11.294	19.903	1.395	4.41	.0334
412	.06542	24.141	71.465	2.3288	.03726	16.723	81.99	4.818	11.219	15.698	1.756	....	....
413	.08416	24.388	93.740	2.3073	.03692	16.570	81.99	4.818	11.116	11.858	2.280	4.33	.0331
414	.10138	24.641	115.150	2.2683	.03629	16.287	82.38	4.713	10.691	9.284	2.794	....	....
415	.11222	24.771	128.770	2.2322	.03571	16.028	82.42	4.702	10.495	8.150	3.143	4.19	.0317
416	.11977	24.901	138.673	2.2166	.03546	15.916	82.42	4.702	10.422	7.516	3.378	....	....
417	.13375	25.137	157.680	2.1740	.03478	15.611	82.72	4.622	10.047	6.373	3.846	....	....
418	.14417	25.278	171.580	2.1475	.03436	15.422	82.58	4.659	10.004	5.831	4.196	4.03	.0294
419	.15856	25.550	192.480	2.0996	.03359	15.078	82.58	4.659	9.782	5.083	4.721	....	....
420	.16968	25.832	209.940	2.0798	.03327	14.933	83.01	4.544	9.450	4.502	5.101	3.91	.0279

SERIES 12

1¼ inch Harris Pump

Length of Eduction Pipe 26.74 feet

Average Lift 4.48 feet

Average Submergence 83.26 per cent

Supply of Compressed Air in Well Casing Connected with Air Main

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_l	c_p
421	.16317	24.712	186.370	2.1658	.03465	15.553	82.49	4.683	10.142	5.442	4.709	....	....
422	.15415	24.741	176.400	2.1868	.03499	15.705	82.47	4.688	10.253	5.812	4.406	4.11	.0247
423	.14477	24.730	165.560	2.2125	.03540	15.889	82.47	4.688	10.372	6.265	4.090	....	....
424	.13803	24.844	159.990	2.2821	.03651	16.386	83.86	4.317	9.851	6.157	3.781	4.29	.0260
425	.13260	24.821	153.400	2.2855	.03657	16.413	83.90	4.307	9.843	6.417	3.626	....	....
426	.11900	24.791	137.400	2.3079	.03692	16.570	83.85	4.318	9.965	7.253	3.223	....	....
427	.11414	24.796	131.860	2.3125	.03700	16.606	84.00	4.279	9.895	7.504	3.085	4.35	.0299
428	.09308	24.608	106.060	2.3414	.03746	16.813	83.71	4.357	10.202	9.619	2.485	....	....
429	.06418	24.260	71.120	2.4001	.03840	17.235	83.51	4.410	10.584	14.882	1.671	4.51	.0328
430	.04296	23.942	46.425	2.2523	.03604	16.178	82.33	4.726	10.643	22.927	1.192	....	....

SERIES 13

1¼ inch Harris Pump

Length of Eduction Pipe 26.74 feet

Average Lift 4.79 feet

Average Submergence 82.23 per cent

7.5 feet of 2-inch Pipe at Upper End of Eduction Pipe

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
431	.04783	23.751	51.890	2.6378	.04220	18.941	81.70	4.895	12.911	24.883	1.134	....	....
432	.06223	23.857	68.090	2.6902	.04307	19.331	81.60	4.922	13.241	19.449	1.445	....	....
433	.07868	24.011	87.120	2.6972	.04315	19.367	81.60	4.922	13.274	15.237	1.824	....	....
434	.08575	24.175	96.240	2.7064	.04330	19.434	82.14	4.773	12.918	13.425	1.981	....	....
435	.09630	24.306	109.170	2.6881	.04301	19.303	82.14	4.773	12.830	11.753	2.239	....	....
436	.10791	24.500	124.140	2.6668	.04267	19.153	82.44	4.698	12.528	10.092	2.529	....	....
437	.11888	24.706	138.920	2.6265	.04202	18.859	82.68	4.632	12.165	8.756	2.829	....	....
438	.12994	24.883	158.870	2.6145	.04183	18.775	82.68	4.632	12.109	7.869	3.106	....	....
439	.14444	25.137	174.020	2.5808	.04129	18.532	82.68	4.632	11.952	6.869	3.498	....	....
440	.15295	25.390	187.650	2.5544	.04087	18.343	82.68	4.632	11.832	6.306	3.743	....	....

SERIES 14

1¼ inch Harris Pump

Length of Eduction Pipe 41.50 feet

Average Lift 7.66 feet

Average Submergence 81.54 per cent

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	I_i	w_w	q_w	q_g	s	h_l	I_o	e	$\frac{q_a}{q_w}$	v_i	c_p
441	.04628	29.119	68.780	2.6318	.04211	18.900	81.57	7.650	20.182	29.270	1.099	....	....
442	.05673	29.160	84.550	2.7047	.04827	19.421	81.53	7.670	20.748	24.582	1.311	....	....
443	.07710	29.080	114.120	2.7174	.04348	19.515	80.24	8.200	22.281	19.525	1.773	....	....
444	.09321	29.431	140.730	2.7895	.04463	20.032	81.71	7.590	21.171	15.048	2.089	....	....
445	.11178	29.656	170.630	2.7721	.04435	19.905	81.68	7.602	21.073	12.350	2.520	....	....
446	.12360	29.786	189.750	2.7549	.04408	19.784	81.68	7.602	20.940	11.035	2.804	....	....
447	.13504	29.974	209.100	2.7230	.04356	19.551	81.68	7.602	20.700	9.899	3.100	5.12	.0184
448	.15065	30.268	236.490	2.6775	.04284	19.228	81.68	7.602	20.353	8.607	3.517	5.02	.0166
449	.16503	30.374	260.350	2.6510	.04241	19.033	81.68	7.602	20.153	7.741	3.892	4.98	.0173
450	.17810	30.598	283.800	2.6162	.04186	18.788	81.96	7.487	19.588	6.902	4.255	4.91	.0217

SERIES 15

1¼ inch Harris Pump

Length of Eduction Pipe 41.50 feet

Average Lift 23.47 feet

Average Submergence 43.45 per cent

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
452	.04608	22.184	41.703	0.7579	.01213	5.445	41.99	24.075	18.245	43.755	3.799	1.43	.1001
453	.06119	22.260	55.850	0.8763	.01402	6.292	41.88	24.121	21.138	37.850	4.365	1.65	.0745
454	.09139	22.906	89.060	1.0502	.01680	7.540	42.67	23.793	24.984	28.052	5.441	1.97	.0506
455	.11442	23.259	114.990	1.0613	.01698	7.621	43.08	23.372	24.808	21.573	6.789	1.99	.0462
456	.13427	23.536	138.320	1.0515	.01682	7.549	43.75	23.345	24.548	17.745	7.983	1.98	.0431
457	.14235	23.814	150.900	1.0752	.01720	7.719	43.97	23.252	25.001	16.567	8.276	2.02	.0414
458	.14586	23.897	155.720	1.0732	.01717	7.706	44.13	23.187	24.882	15.979	8.495	2.02	.0410
459	.15958	24.120	173.410	1.0652	.01704	7.647	44.19	23.161	24.670	14.227	9.365	2.00	.0391
460	.17384	24.350	192.570	1.0468	.01675	7.518	44.82	22.900	23.970	12.447	10.389	1.98	.0378

SERIES 16

1¼ inch Indiana Pump

Length of Eduction Pipe 42.08 feet

Average Lift 24.20 feet

Average Submergence 42.50 per cent

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_l	c_p
461	.06385	22.504	61.980	0.9346	.01495	6.710	42.78	24.079	22.504	36.308	4.272	....	....
462	.08435	22.734	83.580	1.0533	.01685	7.563	42.40	24.239	25.525	30.540	5.006	....	....
463	.10107	22.893	101.700	1.0839	.01734	7.782	42.25	24.304	26.342	25.903	5.829	....	....
464	.11530	23.135	118.600	1.0859	.01737	7.795	42.41	24.236	26.318	22.178	6.638	....	....
465	.13223	23.447	139.650	1.0842	.01735	7.787	42.19	24.327	26.375	18.837	7.022	....	....
466	.03937	22.562	38.830	0.6931	.01109	4.977	43.56	23.753	16.462	42.400	3.550	....	....
467	.05899	22.556	58.300	0.9791	.01506	7.028	42.79	24.077	23.573	40.435	3.767	....	....
468	.08997	22.775	90.630	1.1111	.01778	7.980	42.22	24.313	27.018	29.807	5.000	....	....
469	.12171	23.346	128.870	1.1309	.01809	8.119	42.16	24.340	27.527	21.300	6.728	....	....
470	.16194	24.236	184.150	0.9761	.01562	7.010	42.25	24.313	23.730	12.886	10.368	....	....

SERIES 17

1¼ inch Indiana Pump

Length of Eduction Pipe 42.08 feet

Average Lift 7.63 feet

Average Submergence 81.95 per cent

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Imput, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
471	.02344	29.439	35.582	1.4842	.02375	10.659	81.48	7.797	11.573	32.525	0.987	......	......
472	.05683	29.514	86.570	2.7549	.04407	19.780	81.48	7.797	21.480	24.814	1.290	......	......
473	.06171	29.657	94.610	2.9091	.04655	20.893	81.48	7.797	22.682	23.975	1.326	......	......
474	.04650	29.568	71.010	2.6282	.04205	18.873	81.88	7.627	20.045	28.230	1.106	......	......
475	.09030	29.875	139.820	3.0043	.04807	21.574	81.81	7.653	22.993	16.445	1.879	......	......
476	.11778	30.179	185.180	3.0259	.04841	21.727	81.82	7.652	23.153	12.503	2.433	......	......
477	.14039	30.497	223.820	3.0078	.04812	21.597	83.07	7.127	21.435	9.577	2.918	......	......
478	.15713	30.545	251.080	2.9306	.04689	21.046	81.98	7.587	22.235	8.855	3.352	......	......
479	.16622	30.639	266.750	2.8738	.04598	20.638	81.53	7.771	22.330	8.371	3.615	......	......
480	.17769	30.957	289.040	2.8758	.04601	20.650	83.00	7.533	21.661	7.494	3.862	......	......

SERIES 18

1¼ inch Tee Pump

Length of Eduction Pipe 41.60 feet

Average Lift 6.58 feet

Average Submergence 84.18 per cent

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Imput, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_l	c_p
481	.02299	29.628	34.903	1.9931	.03189	14.313	84.28	6.540	13.034	37.345	0.721	....	....
482	.05188	29.598	78.650	2.6369	.04219	18.937	84.11	6.605	17.417	22.146	1.230	....	....
483	.08346	29.573	126.380	3.0145	.04823	21.648	83.71	6.777	20.427	16.163	1.730	....	....
484	.09323	29.739	142.220	3.1033	.04965	22.283	84.28	6.540	20.295	14.270	1.878	....	....
485	.11101	29.792	169.760	3.1251	.05000	22.441	84.01	6.645	20.765	12.232	2.220	....	....
486	.12285	29.898	188.850	3.1300	.05008	22.478	84.30	6.529	20.435	10.821	2.453	....	....
487	.12708	29.909	196.050	3.1373	.05020	22.532	84.28	6.542	20.524	10.469	2.532	....	....
488	.13555	30.033	209.630	3.1325	.05011	22.491	84.18	6.580	20.610	9.832	2.705	....	....
489	.14628	30.092	226.750	3.1325	.05011	22.491	84.26	6.556	20.535	9.056	2.920	....	....
490	.15770	29.757	239.990	3.0489	.04878	21.893	84.40	6.487	19.777	8.240	3.194	....	....

SERIES 19

1¼ inch Tee Pump

Length of Eduction Pipe 41.60 feet

Average Lift 23.96 feet

Average Submergence 42.40 per cent

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{g_w}$	v_i	c_p
491	.03234	21.930	30.072	0.6857	.01097	4.924	42.74	23.823	16.333	54.307	2.948	......	......
492	.05557	21.912	51.610	0.8762	.01402	6.292	42.48	23.929	20.965	40.620	3.964	......	......
493	.08162	22.083	77.170	1.0500	.01680	7.540	42.62	23.870	25.063	32.480	4.859	......	......
494	.10163	22.255	97.700	1.0883	.01741	7.813	42.07	24.103	26.230	26.847	5.838	......	......
495	.12077	22.384	117.600	1.0839	.01734	7.782	42.36	23.978	25.990	22.101	6.965	......	......
496	.13632	22.567	135.350	1.0817	.01731	7.769	42.39	23.965	25.920	19.150	7.876	......	......
497	.14213	22.673	142.200	1.0808	.01729	7.759	42.33	23.993	25.930	18.235	8.221	......	......
498	.15487	22.797	156.960	1.0742	.01719	7.715	42.33	23.992	25.771	16.419	9.009	......	......
499	.16434	22.879	167.800	1.0635	.01701	7.635	42.26	24.021	25.548	15.225	9.662	......	......
500	.17430	23.032	180.380	1.0552	.01688	7.576	42.46	23.939	25.260	14.004	10.326	......	......

SERIES 20

1¼ inch Tee Pump

Variable Length of Eduction Pipe

Constant Lift of Five feet

Variable Submergence

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q}{q_w}$	v_l	c_p
501	.17902	16.433	60.830	0.3049	.00488	2.190	30.00	5.00	1.524	2.510	37.968	....	....
502	.16960	16.350	55.700	0.3125	.00500	2.244	30.00	"	1.562	2.802	33.920	....	....
503	.16610	16.244	52.325	0.3012	.00485	2.177	30.00	"	1.506	2.878	34.251	....	....
504	.15602	16.091	45.873	0.3044	.00482	2.163	30.00	"	1.522	3.318	32.372	....	....
505	.14962	16.047	43.429	0.3077	.00492	2.208	30.00	"	1.538	3.543	30.412	....	....
506	.12849	15.820	33.305	0.2950	.00472	2.118	30.00	"	1.475	4.421	27.224	....	....
507	.11937	15.649	28.262	0.2755	.00441	1.979	30.00	"	1.387	4.910	27.068	....	....
508	.11257	15.407	23.040	0.2294	.00367	1.647	30.00	"	1.147	4.978	30.670	....	....
509	.07932	15.284	14.843	0.1887	.00302	1.355	30.00	"	0.943	6.357	26.267	....	....
510	.17903	16.826	60.830	0.3192	.00511	2.293	30.00	"	1.596	2.624	35.040	....	....
511	.12482	16.690	40.513	0.3178	.00508	2.280	30.00	"	1.589	3.920	24.574	....	....
512	.14870	16.412	42.588	0.2960	.00474	2.127	30.00	"	1.480	3.475	31.372	....	....
513	.10607	15.948	24.097	0.2650	.00424	1.903	30.00	"	1.325	5.500	25.017	....	....
514	.08495	15.694	16.548	0.2142	.00343	1.539	30.00	"	1.071	6.470	24.768	....	....
515	.06883	15.586	12.361	0.1686	.00270	0.990	30.00	"	0.843	6.280	47.800	....	....
516	.08477	30.620	134.890	3.5600	.05700	25.583	88.00	"	17.800	13.220	1.487	....	....
517	.16847	31.551	278.850	3.5100	.05620	25.224	88.00	"	17.550	6.295	2.998	....	....
518	.16530	31.544	273.600	3.5350	.05660	25.403	88.00	"	17.675	6.465	2.921	....	....
519	.15052	31.364	247.140	3.5880	.05740	25.760	88.00	"	17.440	7.065	2.622	....	....
520	.12154	30.898	195.670	3.6000	.05760	25.850	88.00	"	18.000	9.210	2.110	....	....

SERIES 20—Continued

1¼ inch Tee Pump

Variable Length of Eduction Pipe

Constant Lift of Five feet

Variable Submergence

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_l	c_p
521	.09058	30.628	144.220	3.5350	.05660	25.403	88.00	5.00	17.675	12.222	1.601	....	....
522	.07174	30.463	113.400	3.4220	.05480	24.596	88.00	"	17.110	15.100	1.309	....	....
523	.05023	30.365	79.020	3.1620	.05070	22.756	88.00	"	15.810	20.150	0.991	....	....
524	.04619	30.365	72.660	3.0940	.04950	22.170	88.00	"	15.470	21.280	0.933	....	....
525	.03906	30.316	61.373	2.9400	.04700	21.095	88.00	"	14.700	23.980	0.831	....	....
526	.05982	30.463	94.560	3.3100	.05300	23.788	88.00	"	16.550	17.500	1.129	....	....
527	.07484	30.537	118.590	3.4600	.05540	24.867	88.00	"	17.300	14.600	1.351	....	....
528	.09855	30.761	157.850	3.2380	.05180	23.250	88.00	"	16.190	10.270	1.903	....	....
529	.13623	31.151	221.870	3.2640	.05230	23.473	88.00	"	16.320	8.070	2.005	....	....
530	.13872	23.941	149.380	2.5910	.04152	18.637	80.00	"	12.955	8.672	3.341	....	....
531	.13235	23.888	142.510	2.6112	.04185	18.784	80.00	"	13.056	9.161	3.163	....	....
532	.13120	23.862	140.300	2.6180	.04196	18.832	80.00	"	13.090	9.330	3.127	....	....
533	.12688	23.771	134.450	2.6387	.04229	18.980	80.00	"	13.194	9.815	3.001	....	....
534	.11800	23.713	124.500	2.6387	.04229	18.980	80.00	"	13.194	10.600	2.790	....	....
535	.10855	23.587	113.160	2.6457	.04240	19.030	80.00	"	13.228	11.690	2.560	....	....
536	.09870	23.507	102.330	2.6528	.04251	19.082	80.00	"	13.264	12.962	2.322	....	....
537	.09180	23.458	94.650	2.6528	.04251	19.082	80.00	"	13.264	14.014	2.159	....	....
538	.08328	23.385	85.380	2.6457	.04240	19.030	80.00	"	13.228	15.492	1.964	....	....
539	.07144	23.295	72.700	2.6180	.04196	18.831	80.00	"	13.090	18.008	1.703	....	....
540	.06130	23.225	62.020	2.5511	.04089	18.352	80.00	"	12.755	20.575	1.499	....	....

SERIES 20—Continued

1¼ inch Tee Pump

Variable Length of Eduction Pipe

Constant Lift of Five feet

Variable Submergence

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_l	c_p
541	.04929	23.157	49.520	1.9610	.03143	14.104	80.00	5.00	9.805	19.805	1.568	....	....
542	.03443	23.092	34.380	2.2222	.03562	15.984	80.00	"	11.111	32.323	0.967	....	....
543	.04877	23.128	48.920	2.4040	.03849	17.278	80.00	"	12.020	24.575	1.267	....	....
544	.16384	24.281	181.110	2.5643	.04110	18.447	80.00	"	12.822	7.080	3.987	....	....
545	.15805	20.577	124.330	1.8293	.02932	13.155	70.00	"	9.146	7.357	5.392	....	....
546	.14968	20.446	116.010	1.8422	.02952	13.250	70.00	"	9.211	7.941	5.070	....	....
547	.14369	20.363	110.010	1.8383	.02946	13.221	70.00	"	9.191	8.356	4.877	....	....
548	.13967	20.283	105.810	1.8383	.02946	13.221	70.00	"	9.191	8.688	4.741	....	....
549	.13330	20.202	99.820	1.8383	.02946	13.221	70.00	"	9.191	9.210	4.525	....	....
550	.10339	20.092	76.200	1.8678	.02990	13.418	70.00	"	9.329	12.243	3.458	....	....
551	.11514	20.013	83.980	1.8728	.03001	13.470	70.00	"	9.364	11.150	3.837	....	....
552	.10590	19.889	75.780	1.8658	.02990	13.418	70.00	"	9.329	12.310	3.542	....	....
553	.09700	19.797	68.520	1.8728	.03001	13.470	70.00	"	9.364	13.670	3.232	....	....
554	.08380	19.661	57.910	1.8728	.03001	13.470	70.00	"	9.364	16.170	2.792	....	....
555	.07253	19.595	49.630	1.8451	.02957	13.257	70.00	"	9.225	18.590	2.453	....	....
556	.05421	19.500	36.510	1.7986	.02882	12.935	70.00	"	8.993	24.630	1.881	....	....
557	.06866	19.583	46.930	1.8451	.02957	13.244	70.00	"	9.225	19.670	2.322	....	....
558	.02937	20.416	22.650	1.4750	.02364	10.610	70.00	"	7.375	32.560	1.242	....	....
559	.15917	19.480	108.300	1.5580	.02497	11.208	65.00	"	7.790	7.192	6.376	....	....
560	.15302	19.383	102.250	1.5480	.02481	11.136	65.00	"	7.740	7.570	6.168	....	....

SERIES 20—Continued

1¼ inch Tee Pump

Variable Length of Eduction Pipe

Constant Lift of Five feet

Variable Submergence

1	2	3	4	5	6	7	8	9	10	11	12	3	14
Number of run.	Quantity of free air in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
561	.14462	19.244	94.430	1.5530	.02489	11.172	65.00	5.00	7.765	8.234	5.810	....	....
562	.13850	19.178	89.460	1.5770	.02528	11.347	65.00	"	7.885	8.814	5.480	....	....
563	.12730	19.028	80.220	1.5770	.02528	11.347	65.00	"	7.885	9.828	5.036	....	....
564	.11986	18.906	73.680	1.5720	.02519	11.306	65.00	"	7.860	10.670	4.760	....	....
565	.10973	18.791	66.200	1.5770	.02528	11.347	65.00	"	7.885	11.912	4.342	....	....
566	.10100	18.676	59.620	1.5820	.02535	11.378	65.00	"	7.910	13.268	3.984	....	....
567	.09206	18.585	53.440	1.5970	.02559	11.486	65.00	"	7.985	14.942	3.598	....	....
568	.07798	18.433	43.780	1.5770	.02528	11.347	65.00	"	7.885	18.010	3.019	....	....
569	.06224	18.299	33.980	1.5430	.02473	11.100	65.00	"	7.715	22.702	2.517	....	....
570	.04100	18.212	22.080	1.4620	.02343	10.516	65.00	"	7.310	33.107	1.750	....	....
571	.02666	18.131	14.047	1.2340	.01978	8.869	65.00	"	6.170	43.950	0.634	....	....
572	.14218	18.708	81.440	1.2820	.02053	9.214	60.00	"	6.410	7.871	6.926	....	....
573	.14089	18.649	79.890	1.2920	.02070	9.290	60.00	"	6.460	8.087	6.807	....	....
574	.14441	18.605	80.900	1.2920	.02070	9.290	60.00	"	6.460	7.985	6.977	....	....
575	.13696	18.555	76.090	1.2765	.02045	9.178	60.00	"	6.382	8.388	6.697	....	....
576	.12870	18.454	70.030	1.2887	.02065	9.267	60.00	"	6.443	9.201	6.233	....	....
577	.12064	18.343	64.270	1.3020	.02086	9.362	60.00	"	6.510	10.130	5.784	....	....
578	.10852	18.201	55.940	1.3160	.02110	9.470	60.00	"	6.580	11.763	5.144	....	....
579	.09120	18.013	44.940	1.2903	.02068	9.281	60.00	"	6.451	14.856	4.410	....	....
580	.07681	17.865	36.530	1.2820	.02055	9.223	60.00	"	6.410	17.550	3.738	....	....

SERIES 20—Continued

1¼ inch Tee Pump

Variable Length of Eduction Pipe

Constant Lift of Five feet

Variable Submergence

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air, in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_l	c_p
581	.06479	17.742	29.810	1.2500	.02003	8.990	60.00	5.00	6.250	20.968	3.235	....	....
582	.04248	17.624	18.980	1.1363	.01821	8.178	60.00	"	5.681	29.985	2.333	....	....
583	.03024	17.005	11.250	0.9757	.01563	7.015	60.00	"	4.878	43.365	1.935	....	....
584	.15900	17.896	74.070	0.8870	.01422	6.382	50.00	"	4.435	5.994	11.180	....	....
585	.16970	17.997	81.000	0.8811	.01412	6.337	50.00	"	4.405	5.386	12.020	....	....
586	.15541	17.789	70.630	0.8870	.01422	6.382	50.00	"	4.435	6.280	10.930	....	....
587	.14218	17.679	62.440	0.8870	.01422	6.382	50.00	"	4.435	7.103	9.980	....	....
588	.13527	17.563	57.330	0.8811	.01412	6.337	50.00	"	4.405	7.684	9.586	....	....
589	.12915	17.434	52.970	0.8788	.01408	6.319	50.00	"	4.394	7.840	9.174	....	....
590	.11692	17.308	45.990	0.8646	.01386	6.220	50.00	"	4.323	9.390	8.439	....	....
591	.10560	17.196	40.300	0.8824	.01414	6.346	50.00	"	4.412	10.940	7.482	....	....
592	.08915	17.003	32.000	0.8427	.01351	6.064	50.00	"	4.213	13.170	6.236	....	....
593	.07646	16.905	26.010	0.8310	.01332	5.978	50.00	"	4.155	15.900	5.740	....	....
594	.05662	16.806	18.810	0.7834	.01256	5.637	50.00	"	3.917	20.820	4.580	....	....
595	.02661	16.632	8.270	0.5376	.00862	3.867	50.00	"	2.688	32.500	3.089	....	....
596	.15290	17.075	59.550	0.5731	.00918	4.120	40.00	"	2.865	4.814	16.655	....	....
597	.14323	16.922	53.330	0.5556	.00890	3.995	40.00	"	2.778	5.209	16.094	....	....
598	.14001	16.898	51.590	0.5556	.00890	3.995	40.00	"	2.778	5.385	15.781	....	....
599	.13358	16.778	48.710	0.5510	.00883	3.963	40.00	"	2.755	5.656	15.128	....	....
600	.13065	16.733	45.400	0.5495	.00881	3.954	40.00	"	2.747	6.053	14.830	....	....

SERIES 20—Continued

1¼ inch Tee Pump

Variable Length of Eduction Pipe

Constant Lift of Five feet

Variable Submergence

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of run.	Quantity of free air in cubic feet per second.	Absolute pressure of air at gage, in pounds per square inch.	Input, in foot pounds per second.	Discharge of water, in pounds per second.	Discharge of water, in cubic feet per second.	Discharge of water, in gallons per minute.	Submergence, in per cent.	Lift, in feet.	Output, in foot pounds per second.	Efficiency, in per cent.	Ratio of volume of air to volume of water.	Velocity of water in a 1¼ in. tail piece.	Coefficient of slip and pipe friction.
	q_a	p_g	l_i	w_w	q_w	q_g	s	h_l	l_o	e	$\frac{q_a}{q_w}$	v_i	c_p
601	.12152	16.588	39.910	0.5450	.00878	3.918	40.00	”	2.725	6.828	13.920	....	....
602	.11320	16.450	35.660	0.5391	.00864	3.878	40.00	”	2.695	7.559	13.102	....	....
603	.09990	16.356	29.760	0.5319	.00852	3.824	40.00	”	2.659	8.936	11.727	....	....
604	.08929	16.205	24.890	0.4914	.00788	3.536	40.00	”	2.457	9.871	11.330	....	....
605	.07830	16.107	20.930	0.4762	.00763	3.424	40.00	”	2.381	11.375	10.263	....	....
606	.06226	15.994	15.690	0.4386	.00703	3.155	40.00	”	2.193	13.980	8.856	....	....
607	.05360	15.864	12.590	0.3704	.00594	2.666	40.00	”	1.852	14.710	9.320	....	....
608	.03978	15.796	8.900	0.3058	.00490	2.199	40.00	”	1.529	17.065	8.119	....	....

TABLE II

LOSS DUE TO FRICTION IN AIR PIPE AND NOZZLE WHEN DISCHARGING INTO THE ATMOSPHERE

Series I

Run.	Quantity of free air in cubic feet per second.	Loss of head in pounds per square inch.
279	.13253	.658
280	.14018	.750
281	.12336	.585
282	.11439	.490
283	.09630	.355
284	.10438	.416
285	.07792	.225
286	.08059	.238
287	.08597	.282
288	.07052	.208
289	.06264	.160
290	.07025	.186
291	.05847	.126
292	.04926	.091
293	.05392	.117
294	.04071	.061
295	.03697	.004
296	.01951	.001

TABLE III

LOSS DUE TO FRICTION IN AIR PIPE AND NOZZLE WHEN DISCHARGING INTO THE ATMOSPHERE

Series 2-13 inclusive

Run.	Quantity of free air in cubic feet per second.	Loss of head in pounds per square inch.
1	.05860	0.4185
2	.07510	0.6424
3	.09098	0.9490
4	.11085	1.3556
5	.12047	1.6090
6	.12978	1.8095
7	.13847	2.0924
8	.14710	2.2986
9	.15867	2.4695
10	.16303	2.8527

TABLE IV

LOSS DUE TO FRICTION IN AIR PIPE AND NOZZLE WHEN DISCHARGING INTO THE ATMOSPHERE

Series 14 and 15

Run.	Quantity of free air in cubic feet per second.	Loss of head in pounds per square inch.
1	.02347	0.0825
2	.04092	0.2358
3	.07531	0.6071
4	.09119	0.8605
5	.10425	1.1022
6	.11822	1.4086
7	.13095	1.6621
8	.14068	1.9215
9	.15158	2.1750
10	.16493	2.5521

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AN INVESTIGATION OF THE AIR LIFT PUMP

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